The Advantage to Hiding One’s Hand: Speculation and Central Bank Intervention in the Foreign Exchange Market

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Abstract

We analyze an asymmetric information model of sterilized intervention in the foreign exchange market. We characterize an equilibrium in which a central bank with “inside information” about its exchange rate target trades with risk averse speculators who have private information about future spot rates. The model identifies circumstances in which “perverse” responses to intervention will be observed i.e. the domestic currency depreciates when the central bank purchases it, and it provides conditions under which the exchange rate will be highly sensitive to intervention. The model also provides an explanation for two forms of “policy secrecy”: (i) secrecy about the scale of an intervention operation is always desirable, (ii) secrecy about the target is sometimes desirable.

Key Words: foreign exchange, central bank, exchange rate target, intervention, currency speculation

JEL classification: E58, F31, G15, G18

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Running Headline: The Advantage to Hiding One’s Hand

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"After frantic speculative activity in 1987, currency markets treated the stabilization efforts of the Group of Seven with notably more respect last year, after being caught in a costly central bank 'bear trap' early in January. At that time, sudden, coordinated intervention to prop up the dollar when it had been oversold forced commercial banks to cover open positions at considerable cost”.


Central bank intervention has been a common occurrence in foreign exchange markets¹, and as the quotation above illustrates, there is an obvious element of strategic interaction associated with such activity. Speculators in these markets have certain beliefs about the objectives of central bank intervention policy, and will adjust their behavior according to their beliefs. The central bank in turn will want to take account of the anticipated reactions of speculative traders in formulating its policy.

There has been considerable debate, both in the policy arena and among academics, about whether intervention can be effective, and if so, why.² And yet there has been little attempt to provide a formal analysis of the strategic aspects of the issue, nor of the potentially important role played by information asymmetries.³

In the standard monetary models of exchange rate determination, sterilized intervention has no effect, because it is designed to have no influence upon interest rates. Recent empirical evidence, however, suggests that sterilized intervention has had some impact upon exchange rates (Dominguez (1990), Dominguez and Frankel, (1993)). It has been argued (Mussa (1981), Dominguez (1992b)) that sterilized intervention has an effect because it is used to serve a signalling function. The analysis proceeds in the spirit of financial signalling models (e.g. Ross (1977)) by suggesting that interventions affect exchange rates because they are used to signal future changes in monetary policy. A current purchase of the domestic currency signals a

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¹For some recent history, see Funabashi (1988) who describes the secret intervention agreements negotiated as part of the Plaza Agreement and the Louvre Accord. Dominguez (1992a) reports the scale and frequency of German and U.S. intervention over the period 1983-1990.
³Exceptions are the analyses of Stein (1989) and Dominguez (1992b) in the context of the foreign exchange market. Cukierman and Meltzer (1986) address similar issues in the context of domestic monetary policy.
future monetary tightening, and so is associated with an anticipatory current appreciation. But there are difficulties with a position which seeks to explain all interventions as signals. An immediate implication is that intervention should be consistently profitable on average, since this is required for the signal to be credible. Although the evidence on this issue is somewhat mixed, there have undoubtedly been extended periods when central banks have made very large losses.\(^4\) A recent paper by LeBaron (1995) provides directly contradictory evidence, suggesting that during periods of intervention it is the speculators who make money at the expense of the central bank. The signalling hypothesis also predicts that intervention should always have unidirectional effects: that is to say, a purchase of the domestic currency should be followed by subsequent appreciation. The empirical evidence does not support this prediction. Finally, it is difficult to reconcile the reluctance of central banks to provide any kind of detailed and timely intelligence about their interventions with the signalling hypothesis.

The purpose of this paper is to provide an alternative model to explain the empirical puzzles described above. We do not directly address the normative issue concerning the justification for central bank intervention, but rather assume that the bank has a short run target for the exchange rate. Nevertheless, our model is general enough to accommodate the two distinct views as to why a central bank might find it desirable to engage in intervention operations. The first holds that central banks use it as a means of bringing about a target level for the exchange rate which may be inconsistent with market fundamentals in the short run.\(^5\) The second holds that, on the contrary, the objective of targeting is to counteract the effects of destabilizing speculation and to prevent misalignments. The essential feature common to both these points of view is that the central bank has a different opinion from the market about where the spot exchange rate "ought" to be.

\(^4\) Obstfeld (1990) cites the case of Germany which lost over DM 9 billion on its reserves in 1987, substantially increasing the public sector deficit and causing the government significant political embarrassment.

\(^5\) Reasons for the existence of exchange rate targets are advanced in Dornbusch (1980) and in Williamson (1985). These include a desire to trade off internal and external balance, which may be inconsistent policy objectives in the short run. In the U.S., for example, the intervention activities of the Fed, in contrast to its decisions on domestic monetary policy, are subject to the scrutiny of the Treasury.
This makes the central bank an insider in the sense that it possesses potentially payoff-relevant information about its target which is not available to the rest of the market. In this paper we model the interaction between the central bank as such an "informed insider" and rational speculators. We introduce this information asymmetry into a standard model of rational speculative trade in the forward market. Previous attempts to explain the effect of intervention in this framework have suffered from the difficulty that the typical size of daily intervention is very small (of the order of millions of dollars) relative to total net currency demand (of the order of billions of dollars), or the outstanding stock of assets (of the order of trillions of dollars). It is implausible to suppose that the impact of such relatively insignificant changes in the worldwide asset mix as are brought about by intervention would have a significant effect on exchange rates. However, the introduction of asymmetric information and strategic behavior leads to sharply altered conclusions.

We obtain the following results. First, sterilized intervention is effective only because rational speculators, on average, gain at the expense of the central bank. This is not surprising because speculators, even if they have different information, will not trade unless they have something to gain (Milgrom and Stokey (1982)). Second, as the equilibrium spot exchange rate now reveals some information about the current target, which is useful for predicting the future spot rate, it is possible for the spot rate to become highly sensitive to interventions. Indeed, we show that there exist circumstances in which the response to intervention is “perverse” (the domestic currency depreciates when the central bank purchases it). This can occur because a lower spot exchange rate has the effect not only of lowering the current forward rate, but also of lowering the expected value of the future spot rate. The decrease in speculative demand for forward currency caused by the latter effect may, in some situations, be nearly equal to the increase in demand caused by the former effect. If this happens, small changes in quantity caused by intervention are associated with large changes in the exchange rate. If the latter effect dominates the former, we obtain an upward sloping speculative demand curve - the “perverse” response which has been documented in the empirical literature on
intervention. We show that a perverse response is likely to arise when the market has a relatively precise estimate of the exchange rate target and a relatively imprecise estimate of fundamentals.

Our third result arises when we allow the central bank the opportunity to commit credibly to releasing some information about its target to the market. We show that there is a critical value of the weight attached to the targeting objective above which the bank will choose to reveal its target. However, it will never find it advantageous to reveal the scale of its intervention activity. Thus we are able to provide a rationale for two distinct forms of "policy secrecy", an issue on which the Federal Reserve has received considerable criticism in the past.6

Our approach to modelling intervention bears some similarity to models of asset prices with a strategic informed insider (for example, Kyle (1985), Laffont and Maskin (1990), Ausubel (1990) and Bhattacharya and Spiegel (1991)). But there are important differences. First, and most obviously, the central bank is not motivated directly by the profitability of its trades. Rather, it seeks to balance expected losses on currency transactions against its success in achieving its targeting objective. Second, we allow for the fact that the "outsiders" (speculators) have their own private information. In this regard, we are combining a feature of the original competitive models of differential information developed by Grossman (1976, 1977) with the later generation of monopolistic models.

Stein (1989) also addresses the question of central bank secrecy and exchange rate targets. His argument is in some respects complementary to ours. He is concerned with the ability of the central bank to transmit credible signals to the market about future changes in monetary policy which will influence the exchange rate. A precise announcement of a target in his framework will not be credible, because if its announcement is believed, the central bank will find that it is in its interest to deviate from the policy implied by the target. But Stein

6Stein (1989) cites a bill put before Congress in 1984 advocating "prompt disclosure of certain decisions of the Open Market Committee of the Federal Reserve System." See also Mussa (1981), who argues for published targets as a precommitment device to guarantee central bank credibility. Williamson (1985, p. 68) points to a "deep gulf on this issue between official and academic thinking."
shows that an imprecise announcement of the target may be a credible signal. In other words, it may be used to signal the central bank’s plans for future changes in domestic money supply. One way of effecting such a change would be by means of unsterilized intervention in the foreign exchange market.

Our concern here is explicitly with the role of sterilized intervention which, by definition, is an instrument that is quite independent of domestic monetary policy. Further, while Stein’s argument hinges on the assumption that the central bank can mislead the market with impunity, and is therefore unconcerned with establishing a reputation for releasing accurate information, we demonstrate that even if a precise announcement to the market were viewed as completely credible, it will sometimes be in the central bank’s own interest not to make such an announcement.

The paper is organized as follows. In Section I the model is presented and the strategic interaction between the central bank and speculators is described. In Section II we solve for equilibrium with central bank intervention. Certain important features of this equilibrium are characterized in more detail in Section III. The factors influencing the informativeness of exchange rates are discussed in Section IV. In Section V we describe conditions under which some uncertainty in the market about the location of the target is essential for intervention to be effective. In Section VI we discuss the results and conclude.

I. The Model

We consider a one-period model in which the objective of the central bank is to limit the variability of the exchange rate around some target value $T$. It does this by intervening in the forward market for foreign exchange. This intervention carries with it the risk of losses on forward transactions, and so the central bank has to trade off its desire to target the exchange

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7Given the link which covered interest rate parity provides between the spot and the forward rate, it is immaterial whether we consider intervention as taking place in spot or forward markets or, indeed, a combination of the two.
rate against the expected cost. We assume that it is risk neutral in its attitude to the costs of intervention.

The problem it solves can be stated as follows:

$$\text{Max } E_t(Q^B(\tilde{P}_1 - F_0) - w(P_0 - T)^2)$$

$$Q^B$$

$Q^B$ = forward purchase or sale of foreign currency by central bank (+ denotes purchase)

$P_0$ = spot exchange rate at $t = 0$

($ price of foreign currency)

$\tilde{P}_1$ = spot exchange rate at $t = 1$

$F_0$ = forward rate at $t = 0$

$$= P_0 \frac{1+r}{1+r^*}$$

$r$ ($r^*$) = domestic (foreign) short-term nominal interest rate per period

$T$ = the current exchange rate target

$w$ = preference weight placed by the central bank on its targeting objective; $w \in [0, \infty)$

$E_t$ = expectation at time $t$.

Random variables are identified by means of a tilde. The first term in (1) measures the capital gain or loss on the bank's position in the forward market. The second term captures the bank's concern for stabilizing the spot exchange rate around a target. Since this act carries with it the risk of losses on forward transactions, the central bank has to trade off its desire to target the exchange rate against the expected cost. The current spot and forward rates are linked by the covered interest parity condition.

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8This treats the targeting objective in a manner similar to Stein (1989).
In order to focus attention exclusively on the effects of information asymmetries, we treat the interest differential \( r - r^* \) as given, and for simplicity set it equal to zero. This makes the forward rate equal to the spot rate. This assumption requires some justification, because a portfolio balance approach would suggest that, even though sterilized intervention will have no direct impact upon interest rates, since by definition the monetary base is unchanged, nevertheless there may be (small) indirect effects. However, since the econometric evidence fails to support the hypothesis that supplies of government debt have any effect upon risk premia in the foreign exchange market (see, for example, Hodrick (1987, pp.119-128), we take this as at least indirect evidence that such effects will be insignificant.

The next period spot exchange rate is given by

\[
\tilde{P}_1 = \tilde{\mu} + \tilde{\epsilon}_p
\]  

The realization of the random component \( \tilde{\mu} \) is assumed to be observed by all market participants before any trade takes place. The component \( \tilde{\epsilon}_p \) is not directly observed. The component \( \tilde{\mu} \) is normally distributed with mean \( \mu \) and precision \( \tau_\mu \). The central bank has a prior that \( \tilde{\epsilon}_p \) is normally distributed with mean zero and precision \( \tau_p \).

The central bank trades in the forward market with foreign exchange speculators. These speculators are atomistic price takers, uniformly distributed on the unit interval \([0,1]\), and they choose their demands to maximize von Neumann-Morgenstern expected utility functions. For reasons of tractability we assume that these utility functions display constant absolute risk aversion; the speculators have negative exponential utility functions with identical risk aversion coefficient \( \theta \). We also assume that these speculators derive their utility from their dollar wealth. The traders have the same prior as the central bank on the

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\(^9\)The precision of a distribution with variance \( \sigma^2 \) is \( (\sigma^2)^{-1} \).
distribution of \( \tilde{\varepsilon}_p \). In addition, speculator \( i \in [0,1] \) receives a private signal \( \tilde{S}_i \) which conveys information about the unobserved component of the fundamental, \( \tilde{\varepsilon}_p \).

We assume the following structure for this private signal:

\[
\tilde{S}_i = \tilde{\varepsilon}_p + \tilde{\gamma}_i
\]  

(3)

where \( \tilde{\gamma}_i \) is i.i.d., and normally distributed with mean zero and precision \( \tau_\gamma \). This implies that, ex ante, all speculators will receive equally informative signals about the fundamental. However, the actual information transmitted will differ across speculators, although the messages they receive will be positively correlated. The precision \( \tau_\gamma \) is a measure of the similarity of the messages, since when \( \tau_\gamma = 0 \), the correlation between messages is zero, and as \( \tau_\gamma \) becomes large, the correlation approaches unity.

The speculators are also uncertain about the central bank's exchange rate target. They view it as a random variable, \( \tilde{T} \), described by

\[
\tilde{T} = \tilde{\mu} + \tilde{\varepsilon}_T
\]  

(4)

Their prior on \( \tilde{\varepsilon}_T \) is that it is normally distributed with mean \( \mu_T^S \) and precision \( \tau_T^S \). This specification captures the view that although the target may be correlated with currency 'fundamentals (through the component \( \tilde{\mu} \)), the two may diverge from each other in the short run. For the moment we take the mean \( \mu_T^S \) and the precision \( \tau_T^S \) as given. Later we will consider the implications of allowing the bank to manipulate them by releasing information about its target. The central bank, too, faces ex ante uncertainty about the realization of \( \tilde{\varepsilon}_T \), but this uncertainty is resolved in advance of its intervention activity. This ex ante uncertainty is

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\(^{10}\)We could explicitly incorporate the possibility that the central bank has better information about some of the factors influencing fundamentals, such as future monetary policy, by writing \( \tilde{\varepsilon}_p \) as the sum of two components, one of which is observed without error by the bank.
measured by $\tau_T$, which is the bank's prior precision of its target. The random variables $\tilde{\mu}$, $\tilde{\varepsilon}_p$, and $\tilde{\varepsilon}_T$ are assumed to be independent.

There are two alternative interpretations which can be given to the model, corresponding to different views about the objectives of exchange rate targeting. One may think of $\tilde{\varepsilon}_p$ as an unobserved fundamental component of the next period exchange rate, in which case $\tilde{\varepsilon}_T$ is a change in the target which is unrelated to fundamentals. This captures the interpretation that the central bank sometimes sets target levels that are inconsistent with fundamentals in the short run.\(^{11}\) Or $\tilde{\varepsilon}_p$ may be interpreted as "noise" and $\tilde{\varepsilon}_T$ as a component of (long run) fundamentals. This formalizes the view that central banks intervene to prevent misalignments. This particular formalization makes the strong assumption that the "noise" is fixed in the short run, but we would argue that this is not too misleading when examining the effects of intervention. For the sake of clarity we will focus on the first interpretation, and will refer to $\tilde{\varepsilon}_p$ as the fundamental.

A complete description of the sequence of events is given below:

\begin{itemize}
  \item \(t = -2\) Speculators' prior beliefs about the distributions of their private information, the prevailing target exchange rate, and the 'fundamentals' determining exchange rates next period are summarized, respectively, by the precisions $\tau_\gamma$, $\tau_S$, $\tau_\mu$ and $\tau_P$. The central bank's prior precision for its target is $\tau_T$, and for the two components of fundamentals is $\tau_\mu$ and $\tau_P$.
  \item \(t = -1\) Speculator \(i\) observes $\mu$ and $S_i$. The central bank observes $\mu$ and $\varepsilon_T$.
  \item \(t = 0\) Trades and intervention occur. The forward market clears.
\end{itemize}

\(^{11}\)A recent episode provides a clear illustration of the fact that sterilized intervention can be used to establish a balance between internal and external policy objectives (which may be inconsistent in the short run) rather than to signal future changes in monetary policy. In early 1992 there was pressure on the Bank of Japan to reduce interest rates in order to stimulate the domestic economy. But this was perceived as undesirable from an international policy perspective, because it would cause the yen to weaken against the dollar and so exacerbate the problem of the Japanese trade surplus with the US. Thus in March 1992 the central bank was intervening to support the yen. Soon afterwards, however, the official discount rate was cut from 4.5% to 3.75%. The signalling hypothesis would have suggested just the opposite: the Bank of Japan would have bought yen to signal an intention to tighten monetary policy in the future.
All uncertainty is resolved.

II. Equilibrium with Central Bank Intervention

In order to characterize equilibrium in the forward market we must impose a market clearing condition. If the demand of speculator $i$ is written as $Q^S(P_0, i)$, then aggregate speculative demand, $Q^S(P_0)$, is given by

$$Q^S(P_0) = \int_0^1 Q^S(P_0, i)di$$  \hspace{1cm} (5)

So the market clearing condition takes the simple form

$$Q^B + Q^S(P_0) = 0$$  \hspace{1cm} (6)

We confine our attention to linear equilibria, and conjecture that the aggregate speculative demand function takes the form

$$-Q^S = a_1 + a_2P_0 + a_3\varepsilon_p$$  \hspace{1cm} (7)

We will show that this demand function is consistent with a rational expectations equilibrium in which all agents are differentially informed. Notice first that if speculative demand takes the form given in (7), the realization of $\varepsilon_p$ is fully revealed to the central bank. This follows because the market clearing condition (6) ensures that the central bank can observe $Q^S$. The form of the conjecture then enables it to infer $\varepsilon_p$ from $Q^S$ and $P_0$.\textsuperscript{12}

\textsuperscript{12}The fact that the conjectured equilibrium is fully revealing to the central bank is crucial to our solution procedure. We have found that including a noise term in the market clearing condition (6) makes the model, to us at least, intractable. But we argue below that our results should be robust to the introduction of a small amount of noise trading.
Hence, so long as $a_2 \neq 0$, the optimal intervention can be treated as equivalent to the optimal choice of the current spot exchange rate, $P_0$. Substituting the relevant values from (2), (4), (6) and (7) in (1), the problem solved by the central bank reduces to:

$$\max \{ (a_1 + a_2 P_0 + a_3 \varepsilon_p)(\mu + \varepsilon_p - P_0) - w(P_0 - \mu - \varepsilon_T)^2 \}$$

$$P_0$$

Notice that the problem is a deterministic one, reflecting the conjectured feature of equilibrium, namely that it is fully revealing to the central bank. The first order condition for the optimal choice of $P_0$ can be written in the following way:

$$P_0 = a_4 + a_5 \varepsilon_p + a_6 \varepsilon_T \quad (8)$$

where

$$a_4 = \frac{a_2 \mu - a_1 + 2w\mu}{2(a_2 + w)}$$

$$a_5 = \frac{a_2 - a_3}{2(a_2 + w)}$$

$$a_6 = \frac{w}{a_2 + w}$$

The relationship in (8) is important because it describes the information that speculators are able to extract from observing the equilibrium spot exchange rate. They come to the market with priors on $\tilde{\varepsilon}_T$ and $\tilde{\varepsilon}_p$. Their priors on $\tilde{\varepsilon}_p$ have already been updated as a consequence of observing the signal $S_t$, and so will differ. Knowledge of the objective of the central bank enables speculators to infer further information about $\tilde{\varepsilon}_T$ and $\tilde{\varepsilon}_p$ from (8).

Analyzing how these different sources of information interact in determining individual
speculative demand for forward contracts is the key to solving for equilibrium values of $a_1$, $a_2$, and $a_3$.

We derive the demand function of speculator $i$ in a series of steps. Our assumptions on preferences and on the distributions of $\tilde{\varepsilon}_T$ and $\tilde{\varepsilon}_P$ imply that speculators maximize a simple function of mean and variance of speculative profit:

$$\max \{E_0(P_1 | I_i) - P_0|Q^S_i - \theta \var 0([P_1 - P_0]Q^S_i | I_i)\}$$

(9)

$Q^S_i$

Both mean and variance are evaluated conditional upon the private information of speculator $i$, $I_i$. The demand function, obtained from the first order condition of the above optimization problem, takes the form:

$$Q^S(P_0, i) = \frac{E_0(P_1 | I_i) - P_0}{\theta \var 0(P_1 | I_i)}$$

(10)

where

$\theta = \text{coefficient of absolute risk aversion of the speculators}$

$I_i = \text{information set of speculator } i \text{ (containing } S_i \text{ and } P_0)$.

We need to calculate the conditional expectation and variance which appear in (10). Using the rules for Bayesian updating, we know that after observing $S_i = \varepsilon_p + \gamma_i$ at time $t = -1$, the posterior precision of speculator $i$ on $\tilde{\varepsilon}_P$ is

$$\tau_{-1}(\tilde{\varepsilon}_P | S_i) = \tau_p + \tau_\gamma$$

(11)

and the posterior expectation is


Speculator i will update precision and expectation a second time on observing the spot exchange rate $P_0$ at time $t = 0$. These will constitute the values which, when substituted into (9), will give us equilibrium demands.

To determine how the expressions in (11) and (12) are updated, we note first that we can rewrite (8) as

$$
\frac{P_0 - a_4}{a_5} = \varepsilon_p + \frac{a_6}{a_5} \varepsilon_T
$$

We have explicitly identified $\varepsilon_p$ and $\varepsilon_T$ as random variables to emphasize the fact that this relationship is now viewed from the point of view of the speculator. Defining $\varepsilon_T^* = \varepsilon_T - \mu_T$, and noting that $E(\varepsilon_T^*) = 0$, we obtain

$$
\frac{P_0 - a_4 - a_6 \mu_T}{a_5} = \varepsilon_p + \frac{a_6}{a_5} \varepsilon_T^*
$$

Since the parameters $a_4$, $a_5$ and $a_6$ will be common knowledge in equilibrium, observing $P_0$ is equivalent to observing the expression on the right-hand side of (14). So the precision and the expectation of $\varepsilon_p$ are now calculated conditional upon the realization of $\varepsilon_p + \frac{a_6}{a_5} \varepsilon_T^*$ at time $t = 0$. A Bayesian update gives us:

$$
\tau(\varepsilon_p \mid \varepsilon_p + (a_6/a_5)\varepsilon_T^*) = \tau_p + \tau_T + \tau_S \left( \frac{a_2 - a_3}{2w} \right)^2
$$

and
Substituting these expressions into (10), we find that

$$Q^S(P_0,i) = \left( \frac{a_2 - a_3}{2w} \right)^2 \left( \frac{P_0 - a_4 - a_6 \mu_T}{a_5} \right) \left( \mu - P_0 \right) \left( \frac{\tau_p + \tau_\gamma + \tau_T}{2w} \right)^2 \theta^{-1} \quad (17)$$

Since we have assumed that $\tilde{\gamma}_i$ are i.i.d. random variables, when we aggregate demands, the influence of the individual realizations of $\tilde{\gamma}_i$ disappears. Hence, our conjecture that the aggregate speculative demand would fully reveal $\epsilon_p$ to the central bank is confirmed.

The final step in solving the model involves substituting the expression we have obtained above for the aggregate speculative demand into equation (7), and using the method of undetermined coefficients to solve for $a_1$, $a_2$, and $a_3$. This is done in Section I of the Appendix. We present the results in the next section.

**III. Characterization of Equilibrium**

The most succinct description of the equilibrium of the model is contained in equation (7), which gives the equilibrium demand function of the speculators. But to be able to interpret this, we first need to understand how the coefficients in the demand function are determined.

The derivation of the coefficient $a_2$ is given in Section I of the appendix. It is shown there that:

$$a_2 = -w \left[ 1 + \frac{20w}{\tau_T} \right] + \left[ w(1 + \frac{20w}{\tau_T}) \right]^2 - \frac{2w \tau_\gamma}{\theta} + 4w^2 \left[ \frac{\tau_p + \tau_\gamma}{\theta} \right] + \left[ \frac{\tau_\gamma}{\theta} \right]^2 \right]^{1/2} \quad (18)$$

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13 This is strictly speaking an approximate result. In other words, $\int_{\gamma_1}^{1} d\gamma_1 = 0$. See Judd (1985) for a formal discussion.
In Figure I we illustrate $a_2$ as a function of $\tau^S_T$, the speculators’ precision on the distribution of the unobserved component of the target exchange rate. The features to note are as follows. First, as the precision approaches zero, $a_2$ approaches a positive limiting value. Second, $a_2$ is monotonically declining in $\tau^S_T$. Third, as $\tau^S_T$ approaches infinity, $a_2$ approaches a limiting value which may be positive or negative, depending upon parameter values.

If $a_2$ is positive, then the speculative demand curve is downward sloping, and we have the “normal” case in which increasing purchases of the domestic currency by the central bank are associated with a currency appreciation. We see from Figure I that if either $\tau^S_T$ or $w$ is sufficiently small, this will guarantee that $a_2 > 0$. We will focus our discussion on three particular cases:

(i) $a_2$ is “close” to zero
(ii) $a_2 < 0$
(iii) $a_2 = 0$

If $a_2$ is close to zero, the speculative demand function will be very inelastic and the spot rate will be very sensitive to the size of intervention. We see this by referring back to equation (7). This result can be understood by noting that an increase in the current spot rate influences the level of speculative demand through two channels, which can be identified with the two terms in the numerator of the expression in (10). An increase in the spot rate implies an increase in the forward rate, and this unambiguously decreases speculative demand for the forward currency. However, an increase in the spot rate also induces the speculators to revise upwards their expectations of the future spot rate. To see why, notice that from (16) the response of the conditional expectation $E_0(P_{t+1} \mid I_t)$ to $P_0$ is:

$$\frac{\partial E(\epsilon_p | [\epsilon_p + (a_0/a_3)\epsilon_p])}{\partial P_0} = \frac{\tau^S_T [a_2 - a_3]^2}{2w} \frac{\tau_1 [a_2 - a_3]^2}{2w}$$

(19)

In our model they move one for one, because we have assumed a zero interest differential. In reality, the spot rate and short-term forward rates are highly correlated, but not perfectly so.
As \( a_5 \) is non-negative (see the Appendix, Section III), the demand response produced by this expectation effect is positive. This can be understood by noting that \( a_5 > 0 \) implies that the spot exchange rate is positively correlated with the fundamental variable \( \varepsilon_p \) in equilibrium (see (8)).

When the term in (19) is close to unity, the two effects mentioned above almost offset each other, and we have a situation in which elastic expectations generate very inelastic demand. In other words, the spot rate becomes very sensitive to the size of intervention - the case where \( a_2 \) is close to zero. This addresses a persistent criticism of the explanation for the effectiveness of intervention provided by traditional models of rational speculative trade in the forward market. For sterilized intervention to be effective in these models, a large change in relative asset stocks is needed to induce a significant change in relative asset prices: in practice, intervention operations are not carried out on the sort of scale that would be required. Dominguez (1992a) reports that the average daily coordinated intervention in the late 1980s was about $350 million, which is small compared either to the daily volume of trade or to the value of total foreign exchange assets outstanding.

This discussion also provides us with a simple explanation for why \( a_2 \) can be negative, implying that purchases of domestic currency are accompanied by depreciation of the currency, conditional on the realization of next-period fundamental. If the upward revision of the conditional expectation consequent upon an increase in \( P_0 \) more than offsets the increase in the forward rate, then purchases of the domestic currency by the central bank are associated with depreciation and vice versa.

The conditions which are necessary and sufficient for \( a_2 \) to be negative are (see Appendix Section II):

\[
(i) \quad \frac{s}{c_T} > \frac{4w^2(\tau_p + \tau_\gamma)}{(\tau_\gamma + \theta)(2w - (\tau_\gamma + \theta))}
\]

(20)
(ii) \( w > \frac{\tau_t}{20} \).

So speculative demand responds positively to movements in the spot rate if (i) \( \tau_t^S \) is large (speculators' information about the target is quite accurate) (ii) \( \tau_t \) is small (speculators' prior information about fundamentals is imprecise).

These conditions can be understood with reference to the earlier discussion of speculators' conditional expectations of the future fundamental. If speculators have a lot to learn about the fundamental, i.e., \( \tau_p \) is small, a good knowledge of the target makes the conditional expectation so sensitive to movements in the spot rate that it more than offsets the simultaneous increase in the forward rate. As we show in Section IV, this actually translates into a reduction in the informativeness of the spot rate, as an equilibrium response to this heightened sensitivity.

The implication that \( a_2 \) - the market response to central bank intervention - can have either sign is a unique feature of our model. Other theories which seek to explain the effectiveness of sterilized intervention - like the "signalling hypothesis" or the simple portfolio rebalancing hypothesis - predict unidirectional responses.

The results of an econometric analysis of the effectiveness of intervention reported by Dominguez and Frankel (1993) can be given a plausible explanation in the context of our model. They find that during the period after the Louvre Accord in February 1987 intervention by the Fed was consistently associated with a perverse response of the exchange rate \( (a_2 < 0) \). This is inconsistent with the signalling model. But that was a time when quite a tight target "band" was set for the exchange rate (± 2.5% around current levels according to Funabashi (1988)). Although this band was never publicly announced, it is likely that the market came to have a better idea of the target than in earlier periods. In other words, the precision \( \tau_t^S \) was relatively high during this period. It was also a time of considerable currency volatility. These
are the conditions which from (20) (i) and (20) (ii) increase the probability that $a_2$ will be
negative.

It is important to emphasize that, despite the apparently perverse response to
intervention, the central bank still achieves targeting benefits in this situation. It is perfectly
well aware that "support" intervention will tend to produce currency depreciation because of
the very high elasticity of expectations in the market. This again is consistent with the evidence.
The Fed successfully engineered a "soft landing" for the dollar in part by its consistent
purchases of dollars throughout 1987 as the currency continued its trade-weighted
depreciation.

If $a_2 = 0$, then speculative demand is completely unresponsive to the spot exchange
rate, and so to the intervention activity of the central bank. A given increase in the forward rate
is now precisely offset by an equal upward revision in the conditional expectation of next
period's spot exchange rate. Individual speculative demand is influenced only by the private
signal $S_i$, and optimal intervention is still characterized by (7). The bank simply accommodates
speculative demand, since any attempt to manipulate the exchange rate would lead to very
large fluctuations, thereby exposing the bank to huge potential losses. The equilibrium spot
exchange rate is determined by (8) as before, and remains a function of both fundamental and
target, since neither $a_5$ nor $a_6$ are zero when $a_2$ is zero. This reveals a curious paradox –
intervention may appear to be unrelated to movements in the spot exchange rate, and yet still
generate significant targeting benefits. One must be careful in one's interpretation of the
comparative statics of this singular case.\footnote{We are grateful to a referee for forcing us to clarify our thoughts on this issue.}

Because both the scale of intervention and the spot
rate are endogenously determined, it is misleading to talk of the response of the spot rate to
intervention, as if there were a unidirectional causal link. For the same reason, when $a_2 = 0$, it
is misleading to describe this as a situation in which the spot rate is "infinitely sensitive" to
intervention. Rather, the scale of intervention is independent of the spot rate. However, the
spot rate remains positively correlated with the target, because \(a_6\) is positive in (8) (See Appendix, Section III).

Returning to equation (7), we see that the influence of fundamentals on speculative demand is captured by the coefficient \(a_3\), which is given by

\[ a_3 = -\frac{\tau_p}{\theta}. \]  

(See Appendix, Section I, equation (A.2)). So speculative demand becomes more sensitive to movements in fundamentals if (i) the informativeness of speculators' private signals increase, or (ii) speculators become less risk averse. Note that this sensitivity is independent of the speculators' priors about the target. It is a simple consequence of aggregating individual responses to the private signals about fundamentals.

The coefficient \(a_1\) in (7) is given by

\[ a_1 = -\mu a_2 + k \mu_T^S \]  

where

\[ k = \frac{2w(\tau_p + \tau_p - \theta a_3)}{\tau_p + \theta w} > 0. \]  

The speculative demand function can be written as

\[ Q^S = -k \mu_T^S + a_2(\mu - P_0) - a_3 \epsilon_p. \]

Thus speculative demand depends upon the magnitude of the deviation of the current spot rate from \(\mu\), the realization of the common knowledge component of the future spot rate.

Another point worth remarking on has to do with the motive for trade in our model. We know from Milgrom and Stokey (1982) that there will be no purely speculative trade in an
economy where some agents are better informed than others. The reason why trade occurs within our framework is because the central bank is not behaving like a profit maximizing insider, but rather wishes to achieve its targeting objective; as a consequence, it expects on average to incur losses during interventions, and speculators will make positive expected profits at the bank's expense.

In order to demonstrate this we observe that the conditional expected profit for speculator i is

\[ Q^S(P_0, i) [E_0(P_1 | I_i) - F_0] = \frac{[E_0(P_1 | I_i) - F_0]^2}{\text{var}_0(P_1 | I_i)} \geq 0 \]

(25)

where the inequality is strict if \( Q^S(P_0, i) \neq 0 \). It is immediately evident that the unconditional expectation of the profit must be positive for all i. This illustrates, incidentally, that there could not be a signalling equilibrium as it is commonly understood in the model we describe. If the bank signals a future change in monetary policy by its current intervention, then for the signal to be credible, the bank must consistently make profits at the expense of speculators. But rational speculators would refuse to trade with a central bank on these terms. The feature of our model described above reflects the frequently stated view that the central bank, in targeting the exchange rate, trades against the "smart money" and must expect, on average, to make losses. Some recent empirical support for this position has appeared in a paper by LeBaron (1995), who looks at the performance of technical trading rules in the foreign exchange market and finds that the only times when they appear to be consistently profitable is during periods of central bank intervention.

To conclude this section, we consider the robustness of our results to the introduction of liquidity trading. If we were to introduce an exogenously specified random variable representing the volume of liquidity trade into the market clearing condition in (6), we would
certainly expect the equilibrium of the extended model to converge in a well-behaved manner to the one we have described as the variance of liquidity trading tended to zero. So we would argue that our results are robust to the introduction of small amounts of liquidity trade. A harder question, which is beyond the scope of the present study, is what would happen in the presence of significant hedging demand.

IV. The Informativeness of the Exchange Rate

We have already demonstrated that the central bank, by the very act of intervening, both acquires information about fundamentals and transmits some of it to the market. It is natural then to investigate the factors which determine how much information is transmitted to the market by intervention.

We need first to construct a measure of information transmission. If all the information of the central bank is revealed, then \( \text{var}_0(\tilde{P}_1 \mid P_0) = 0 \). If none of the bank’s information is revealed, \( \text{var}_0(\tilde{P}_1 \mid P_0) = (\tau_p + \tau_s)^{-1} \). In other words, observing \( P_0 \) in this case is completely uninformative, and the updated variance depends only upon the informativeness of the speculator’s private signal \( S_t \). This suggests a natural measure of information transmission:

\[
\psi = 1 - [\tau_p + \tau_s] \text{var}_0(\tilde{P}_1 \mid P_0) \tag{26}
\]

which ranges from zero, when no information is revealed, to unity, when everything is revealed.

Substituting the expression for \( a_3 \) from (21) into (14), (26) reduces to:

\[
\psi = \frac{1}{1 + \frac{\tau_p + \tau_s}{\tau_T} \left[ \frac{2w0}{a_3\theta + \tau_s} \right]^2} \tag{27}
\]
This measure has some interesting properties. If \( w < \frac{\tau_r}{\theta} \), then \( \psi \) approaches one (zero) as \( \tau_r \) approaches infinity (zero). Thus the exchange rate becomes more informative as the market’s prior on the target becomes more concentrated, so long as the targeting objective is not "too important".

If \( w > \frac{\tau_r}{\theta} \), \( \psi \) first increases with \( \tau_r \) and then decreases. In stark contrast to the previous case, \( \psi \) now approaches zero as \( \tau_r \) approaches infinity. In order to understand the very different effects on the informativeness of the exchange rate in these two cases one needs to go back to equation (14), which represents clearly the way in which information about next-period fundamentals is extracted from the current exchange rate. The term \( \frac{a_6}{a_5} \) is the error or noise in the message transmitted by the exchange rate. The variance of this term is affected in two opposing ways by an increase in \( \tau_r \). First, by definition it reduces the variance of \( \tilde{\epsilon}_T \). Second, it increases the value of \( \frac{a_6}{a_5} \) (\( = \frac{2w}{a_2+ \tau_r/\theta} \)). But this term approaches a finite positive limit so long as \( w < \frac{\tau_r}{\theta} \) (See Appendix, Section II, (A.9)). So in this case an increasingly high prior precision on the target translates into an increasingly accurate inference about next-period fundamentals. But when \( w > \frac{\tau_r}{\theta} \), \( a_6/a_5 \) becomes unbounded in the limit, swamping the effects of the increased precision of \( \tilde{\epsilon}_T \). Our result indicates that the combined effect is to increase the variance of \( \frac{a_6}{a_5} \tilde{\epsilon}_T \) without bound, making the spot exchange rate progressively less informative as \( \tau_r \) increases.

V. When Will the Bank Show Its Hand?

We have provided a characterization of equilibrium in our model under the assumption that the central bank does not reveal any information to the market about its target, other than that transmitted by the act of intervention. Now we consider the question of whether the bank would ever find it advantageous to release information about its intentions. We suppose that the bank can credibly commit to transmitting a public message \( \tilde{m} \) about the unobserved component of its target:
\[
\tilde{m} = \tilde{\varepsilon}_T + \tilde{\eta} \quad (28)
\]

where \( \tilde{\eta} \) is normally distributed with mean zero and precision \( \tau_{\eta} \). The precision is a choice variable for the bank. The decision on \( \tau_{\eta} \) is assumed to be taken at \( t = -2 \), and the message is transmitted at \( t = -1 \), when the true target is known with certainty by the bank. The message can be viewed as a means of manipulating the market’s prior on the target. Speculators are now assumed to have the same initial prior at \( t = -2 \) on the target as the bank, i.e. \( \tilde{\varepsilon}_T \) is normally distributed with mean 0 and precision \( \tau_T \). They update both mean and precision according to the standard Bayesian rules:

\[
\tau_T^S = \tau_T + \tau_{\eta} \quad (29)
\]

\[
\mu_T^S = \frac{\tau_{\eta}}{\tau_T} \tilde{m} \quad (30)
\]

The central bank decides how much information it is willing to commit itself to revealing by choosing \( \tau_{\eta} \) to maximize its ex ante expected utility, \( V(\tau_{\eta}) \) at \( t = -2 \).

\[
V(\tau_{\eta}) = E_{-2}[\{a_{1} + a_{2}P_{0} + a_{3}\tilde{\varepsilon}_P(\mu + \tilde{\varepsilon}_P - P_{0}) - w(P_{0} - \mu - \tilde{\varepsilon}_T)^2\}]
\]

(31)

Using (22), (23), and (30) we obtain:

\[
\tilde{a}_1 = -\tilde{\mu} a_2 + \frac{2w(\tau_T + \tau_{\eta} - \tilde{\varepsilon}_T)}{\tau_T + 20w} \frac{\tau_{\eta}}{\tau_T + \tau_{\eta}} \tilde{m} \quad (32)
\]

Note that the above term \( a_1 \), which appears in the expression for optimal intervention is now a random variable that depends both on the publicly observed component of the target \( \tilde{\mu} \) and on
the message $\hat{m}$. If $\tau_{\eta}$ is zero, the message $m$ is completely uninformative and speculators' priors on target mean and precision are unchanged.

In order to determine whether any release of information is desirable, we consider first the case where $\tau_\gamma/\theta > w$. Then it is possible to show that (see Appendix, Section IV):

$$\lim_{\tau_{\eta} \not\to \infty} P_0 = \mu + \varepsilon_p$$  (33)

The market learns more about the target as $m$ becomes more informative, and through the relationship in (14) is able to infer the value of next-period fundamentals more precisely. In the limit it predicts them perfectly. The central bank loses all ability to target the exchange rate. The limiting value of ex ante expected utility is $-w \left(\frac{1}{\tau_T} + \frac{1}{\tau_p}\right)$ which is lower than $-\left(\frac{w}{\tau_T}\right)$, the value associated with no intervention. In this case, therefore, we can conclude that the bank will not commit to revealing its target accurately. We have been unable to provide a general demonstration that it will never choose to release any information, but numerical calculations for a wide variety of parameter values suggest that this is the case. In all instances we find that expected utility is declining in $\tau_{\eta}$.

The second case to consider is where $\tau_\gamma/\theta < w$. Then we find that (see Appendix Section IV):

$$\lim_{\tau_{\eta} \not\to \infty} P_0 = \mu + \frac{20w}{\tau_p + 20w} \varepsilon_T$$  (34)

Now intervention is structured so as to become progressively less informative about fundamentals as knowledge about the target increases. This is just a restatement of the result derived in the previous section. If the market is informed of the exact realization of $\varepsilon_T$, this is of no use in predicting next period's exchange rate. Again, although we have not been able to provide a general proof, for a wide variety of parameter values we find that ex ante utility now
increases with $\tau_\eta$. The bank would choose to commit to revealing its target to the market.

However, the bank will still engage in intervention activity and speculators will still be prepared to trade, because their individual expectations of the rate next period will diverge from the current spot rate. The effectiveness of targeting now depends on the amount of prior information about fundamentals available to the market. With relatively little information (as measured by $\tau_p$), (34) tells us that the spot rate will track the target closely. As $\tau_p$ rises, the effectiveness of targeting declines monotonically.

It is clear that there is a close link between the central bank’s decision on whether or not to reveal its target and the informativeness of the exchange rate (which was analyzed in the last section). We find that the central bank would like to reveal its target only when optimal intervention is structured so that the conditional correlation between spot and next-period exchange rate is zero. This will occur only when the weight on the targeting objective is large ($w > \tau_f/\theta$). In such a situation knowledge of the target carries no additional information about fundamentals. Interestingly, if we relate this to (20), we find that if the target is revealed ($\tau^S_T \to \infty$) the conditions for $a_2 < 0$ are satisfied and the response of the exchange rate to intervention becomes perverse.

In Figure 2 we illustrate the dependence of the central bank’s ex ante expected utility on the parameters $\tau_\eta$ and $w$. The parameter values chosen are: $\tau_T = 0.1$, $\tau_f = 2$, $\tau_p = 5$, $\theta = 1$. For values of $w$ greater than 2 the central bank would commit to revealing all information about its target. For $w$ less than 2 the central bank would choose not to reveal any information. (Note that for the critical set of values satisfying $w = \tau_f/\theta = 2$, the problem is undefined in the limit as $\tau_\eta$ tends to infinity, since the second order condition $a_2 > -w$ is violated.)

The discussion above reveals that there is a subtle distinction to be made between secrecy about the target and secrecy about the scale of intervention. It is always important for the bank to conceal the scale of its intervention activity. If it did not do this, the market’s combined observations of $Q^B$ and $P_0$ would permit $\varepsilon_p$ to be inferred perfectly (see (7)). The market learns too much and, as a consequence, the central bank loses all ability to influence
the spot exchange rate. However, we have shown that there are circumstances in which making the target known to the market will be of advantage. This accords well with the observed facts. Intervention operations are sometimes conducted directly with banks and sometimes through the broker market, but in both cases it is impossible for an individual market participant to infer accurately the exact scale of the intervention. There would be nothing to stop the Fed from publicly announcing the scale of its operations, but neither it nor any other central bank to our knowledge has ever chosen to do this.

VI. Discussion

Our aim has been to provide a somewhat different explanation from those hitherto proposed for the impact of intervention in currency markets. We have characterized the asymmetric information rational expectations equilibrium of a model of the foreign exchange market. In this model the central bank, by virtue of its privileged position in the market, is able to make an accurate inference about speculators' private information related to expectations about the exchange rate. The speculators, by contrast, learn something, but not everything from the actions of the central bank. Since the spot rate conveys information to the market about the future value of the exchange rate, it is possible for significant changes in the spot rate to be associated with small amounts of intervention. This is a phenomenon supported by empirical observation but which cannot be convincingly explained in a standard portfolio balance framework.

Our model is consistent with a number of other observations as well. It provides a rationale for central bank secrecy surrounding the precise scale of intervention operations, and enables us to determine conditions under which secrecy about the exchange rate target will be desirable. It should be emphasized that this secrecy does not imply that the market will be ignorant of the fact that intervention is under way. On the contrary, it is a simple implication of our analysis that the market is able to detect intervention. What is important is that the market be unable to infer exactly the scale of the bank’s activity. If this were known, it would
be equivalent to revealing its private information about the fundamental, and would render intervention completely ineffective.

It has been argued that the central bank, if it intervenes by 'leaning against the wind' to stabilize the exchange rate, may actually destabilize foreign exchange markets by preventing the exchange rate from adjusting to its fundamental value. Our analysis illustrates the sense in which this may be correct. The volatility of the spot rate depends both upon the volatility of fundamentals and of the target, as is revealed in equation (8). There are circumstances in which the influence of the target will outweigh that of the fundamental - for example, when both \( \omega \), the weight on the targeting objective and \( \tau_{ST} \), the measure of the market's prior information on the target, are high.

This kind of stabilization policy is sometimes criticized on the grounds that sophisticated speculators will be able to take advantage of such predictable behavior, profiting from the actions of the central bank. Our analysis confirms that there is indeed a cost to this sort of activity; the central bank, on average, will make losses. But it is able to limit these losses either by concealing information about the target from the market or by designing its intervention to offset the impact of fundamentals on the spot exchange rate. This prediction is supported by the recent findings of LeBaron (1995) that trading rules are only significantly profitable during periods of central bank intervention.

There is evidence also to suggest that intervention policy is sometimes motivated by a desire to control "overshooting". To the extent that such overshooting is identified with the phenomenon described in Dornbusch (1976) - there it is a rational response of markets to unanticipated changes in policy - such attempts would correspond exactly to the scenario in which the bank's short run target is inconsistent with fundamentals.

Our work suggests several avenues for future research on strategic interaction in the foreign exchange market during times of central bank intervention. The result that the central

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16 In the inaugural Roy Bridge Memorial Lecture in January, 1980, the then Governor of the Bank of England (now Lord Richardson), in discussing the role of central bank intervention, explicitly identified one objective as being to counter or mitigate the tendency of the foreign exchange market to "overshoot" (see Chapter 8 of the Croham Report (1986), "The Role of Central Bank Intervention" (John Sangster)).
bank learns everything about the value of the exchange rate next period could be weakened by introducing hedgers or liquidity traders into the forward market. This would open up the possibility that the bank might actually find intervention profitable, if there were a sufficient number of such agents in the market.

It would be interesting to examine the possibility that speculators may be able to coordinate their trading activity in some way. This would require that one drop the assumption of atomistic price-taking traders, and would place the bank at a further disadvantage relative to the market. But perhaps the most important, and certainly the most challenging extension to our work would be to move to a multi-period framework, to avoid the “one-shot” flavor to intervention as we have modelled it.
REFERENCES


APPENDIX

I. Solving for the coefficients in equation (7)

The individual speculative demand function from (16) is:

\[ Q^S(P_0, i) = \left[ \tau_s (e_p + \gamma) + \tau_s \left( \frac{a_2 - a_3}{2w} \right)^2 \left( P_0 - a_4 - a_6 \frac{\mu S^s}{a_5} \right) + (\mu - P_0) (\tau_p + \tau_s + \frac{s a_2 - a_3}{2w}) \right] \theta^{-1} \]

Hence, the aggregate demand function from (5) is:

\[ Q^S(P_0) = \int_0^1 Q^S(P_0, i) di \]

\[ = \left[ \tau_s (e_p + \gamma) + \tau_s \left( \frac{a_2 - a_3}{2w} \right)^2 \left( P_0 - a_4 - a_6 \frac{\mu S^s}{a_5} \right) + (\mu - P_0) (\tau_p + \tau_s + \frac{s a_2 - a_3}{2w}) \right] \theta^{-1} \quad (A.1) \]

Equating the coefficients in (A.1) and (7), we obtain:

Coefficient on \( \epsilon_p = a_3 = - \frac{\tau_s}{\theta} \) \quad (A.2)

Constant term = \( a_1 = k_1 + k_2 a_2 \)

where

\[ k_1 = - \left[ \frac{2w(\tau_p + \tau_s)}{\tau_p + 2\theta w} \right] \frac{S^s}{\mu_s} \quad \text{and} \quad k_2 = - \left[ \frac{\mu S^s + 2\theta w(\mu + S^s)}{\tau_p + 2\theta w} \right] \quad (A.3) \]

Alternatively, we may write:
\[ a_1 = -\mu a_2 + \left[ \frac{2w(\tau_p + \tau - \theta a_2)}{\tau_p + 20w} \right] \mu_T \tag{A.4} \]

The coefficient on \( P_0 \) in (A.1) will give \( a_2 \), which is a root of the following quadratic equation:

\[ Aa_2^2 + Ba_2 + C = 0 \tag{A.5} \]

where

\[ A = \theta^2 \tau_S^s \]
\[ B = 2\theta^2 w (\tau_S^s + 2\theta w) \]
\[ C = \tau_S^s \tau_r (2\theta w - \tau) - 4\theta^2 (\tau_p + \tau_r) \]

The roots of equation (A.5) are:

\[ -w \left[ 1 + \frac{2\theta w}{\tau_S^s} \right] \pm \left[ w(1 + \frac{2\theta w}{\tau_S^s})^2 - \frac{2w\tau_r}{\theta} + 4w^2 \left[ \frac{\tau_p + \tau_r}{\tau_S^s} \right] + \left[ \frac{\tau_r}{\theta} \right]^2 \right]^{1/2} \]

The second-order condition for the maximization problem requires that \( a_2 \geq -w \). This enables us to rule out the smaller root. Hence, the unique solution for \( a_2 \) is:

\[ a_2 = -w \left[ 1 + \frac{2\theta w}{\tau_S^s} \right] + \left[ w(1 + \frac{2\theta w}{\tau_S^s})^2 - \frac{2w\tau_r}{\theta} + 4w^2 \left[ \frac{\tau_p + \tau_r}{\tau_S^s} \right] + \left[ \frac{\tau_r}{\theta} \right]^2 \right]^{1/2} \tag{A.6} \]

II. Properties of \( a_2 \)

Let us first analyze \( a_2 \) as a function of \( \tau_S^s \). By L'Hôpital's Rule, it can be shown from (A.6) that

\[ \lim_{\tau_S^s \to 0} a_2 = \frac{(\tau_p + \tau_r)}{\theta} \tag{A.7} \]
We can also show from (A.6) that

\[
\lim_{\tau_T \to \infty} a_2^S = -w + |w - \frac{\tau_T}{\theta}|
\]  

(A.8)

(A.8) implies that

\[
\lim_{\tau_T \to \infty} a_2^S = \begin{cases} 
\frac{\tau_T}{\theta} - 2w > 0 & \text{if } 2w < \frac{\tau_T}{\theta} \\
(2w - \frac{\tau_T}{\theta}) < 0 & \text{if } w < \frac{\tau_T}{\theta} < 2w \\
\frac{\tau_T}{\theta} < 0 & \text{if } \frac{\tau_T}{\theta} < w
\end{cases}
\]  

(A.9)

Finally, from (A.6) we are able to show that

\[
\frac{\partial a_2^S}{\partial \tau_T} < 0.
\]  

(A.10)

Equating \(a_2^S\) in (A.6) to zero and using (A.7), (A.9) and (A.10), we get the necessary and sufficient conditions for \(a_2^S\) to be negative:

(i) \(\tau_T^S > \frac{4w^2(\tau_T + \tau_f)}{(\tau_T/\theta)(2w-(\tau_T/\theta))}\)

(ii) \(w > \frac{\tau_T}{2\theta}\).

This allows us to depict \(a_2^S\) as a function of \(\tau_T^S\) in Figure 1.

III. Properties of \(a_5^S\) and \(a_6^S\)

(i) We first analyze \(a_5^S\) as a function of \(\tau_T^S\). From (A.7) - (A.10) it follows that
From (8) and (A.2),

\[ a_5 = \frac{a_2 - a_3}{2(a_2 + w)} = \frac{a_2 + \frac{\tau_T}{\theta}}{2(a_2 + w)} \] \hspace{1cm} (A.12)

Hence, it follows from Figure 1 and (A.12) that \( a_5 \geq 0 \). Further, as \( \frac{\partial a_2}{\partial \tau_T} < 0 \), it follows from (A.12) that

\[ \frac{\partial a_5}{\partial \tau_T} \geq 0 \] \hspace{1cm} (A.13)

Finally, the limits in (A.9) enable one to establish:

\[ \lim_{\tau_T \to \infty} a_5 = \begin{cases} 
1 & \text{if } w < \frac{\tau_T}{\theta} \\
0 & \text{if } \frac{\tau_T}{\theta} < w
\end{cases} \] \hspace{1cm} (A.14)

(ii) Next we analyze \( a_6 \) as a function of \( \tau_T \). From (8),

\[ a_6 = \frac{w}{a_2 + w} \] \hspace{1cm} (A.15)

Hence, it follows from the second order condition that \( a_6 > 0 \). Further, as \( \frac{\partial a_2}{\partial \tau_T} < 0 \) from (A.9), it follows from (A.14) that \( \frac{\partial a_6}{\partial \tau_T} > 0 \).
IV. Limiting values of $P_0$

Case I: $w < \frac{\tau y}{\theta}$

Taking the limit of $a_1$ in (A.4) and the limit of $a_2$ in (A.9), we obtain the limit of $a_4$ in (8). The limit of $a_5$ is obtained from (A.14) and the limit of $a_6$ is obtained from (A.16). These three limits are substituted in (8) to get:

$$\lim_{s \to \infty} P_0 = \mu + \varepsilon_p$$

Case II: $w > \frac{\tau y}{\theta}$

Proceed as before, using the different limits for $a_5$ and $a_6$ in (A.14) and (A.16) respectively. We find that:

$$\lim_{s \to \infty} P_0 = \mu + \frac{2\theta w}{\tau_p + 2\theta w} \varepsilon_T$$