Strategic Overbidding for Toehold in Dynamic Auctions:  
Structutal Estimation of the Synergy Effect

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Abstract

In this paper, we formulate and estimate a dynamic auction game where bidder asymmetry due to synergy is endogenous. The seller sells multiple goods via a sequence of first price auctions. While bidders are ex-ante symmetric, the first period winner has a valuation advantage due to synergy in the second period bidding game and becomes a strong bidder. Being a strong bidder gives advantage in bidding in the second stage. This endogenous synergy creation leads to overbidding in the first period auction relative to a static bidding game. We characterize the equilibrium in terms of the observed bid distribution. We suggest a two step estimation procedure to estimate such a dynamic game of toehold creation. The OCS oil tract auctions exhibit one such phenomenon. We apply our method to data on OCS oil tract auctions. We find that the federal government is only recovering 52% of the ‘strong’ buyers’ willingness to pay in the second period. Bidders perceive the value of synergy to be about 5% of their first period’s informational rent. A new semiparametric structural test cannot reject the hypothesis of the strong bidder’s valuation superiority in the second period and sets it at 45% relative to the weak bidder. We use the estimates to design alternate mechanisms and empirically show that government’s revenue could have increased by about 27%.

Keywords: Endogenous Toeholds, Dynamic Auction Estimation, Informational Asymmetry, Value of Information, Test of Asymmetry, Copulas, Synergy Creation.

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1 Introduction

Creation and exploitation of synergy is at the heart of any production and corporate strategy. The impact of an existing synergy in productions and operations strategy is well understood. In a procurement auction setting, existence of such a synergy and their impact is well documented in Kim et al (2014), Elmaghraby and Keskinocak (2004), Sandholm et al (2006) and Hohner et al (2003) describing applications in school meal auction, Home Depot, Proctor & Gamble and Mars Inc. However little is known about how such a synergy is actually created. What is the value of a potential synergy? How much extra a bidder is willing to pay for a potential future synergy effect? If the bidders are willing to pay extra for this synergy effect; can the seller devise a better strategy to increase its revenue? In this paper, we address these questions in the context of a dynamic auction model of heterogenous divisible goods where asymmetry of bidders due to synergy effect is endogenous. A seller sells multiple goods via a sequence of first price auctions. Bidders privately receive signals about the value of the first object from the same distribution. Hence they are symmetric ex ante. However, the winner of the first period auction gains a synergistic valuation advantage in the following periods’ auctioned objects relative to other bidders and receive their valuation signals from a different distribution. Hence bidders are asymmetric in the second period. The first period outcome generates asymmetry in the second period distribution of bidders’valuations leading to endogenous asymmetry. The synergy may arise due to the economies of scale due to some cost advantage or due to the economies of density: a co-location advantage. Many real life examples like the OCS oil tracts auctions, defence procurements, takeovers battles etc. are examples of this kind of auction.

The fundamental quest for the optimal selling strategies in this dynamic framework can be analyzed empirically by using the estimates of the distributions of ex-ante valuations of bidders in both periods, the degree of asymmetry in the second period and the potential synergy effect of valuation in the second period as perceived in the first period. These are typically unobserved to the researcher. In this paper, we estimate these from data on observable bids using a two step procedure. Specifically, we formulate a dynamic auction model where bidder asymmetry is endogenous. We characterize the equilibrium in terms of the observed bid distribution. We propose a two step procedure to estimate the marginal distributions of ex-ante valuations in this dynamic model of bidding by using this characterization and data on bids. We apply the model to data from oil tract auctions where the winner of the wildcat tract (first period auction) has synergy in valuations for the drainage tracts (second period auction). The estimation results and counterfactual experiments suggest that ignoring the dynamic bidding behavior and inherent asymmetry can significantly affect seller’s revenue. Our estimation procedure is inspired by the work of Jofre-Bonet and Pesendorfer’s (2003) work on dynamic auctions.

As an empirical application, we apply the model and methodology developed in this paper to data from oil tract auctions off the coast of Louisiana and Texas. Examples of similar endogenous informational asymmetry also include spectrum auctions, treasury auctions, toeholds and takeovers, defence procurement auctions etc.
goods depending on their privately observed signals about the profit from selling the oil. There are two major kinds of oil and gas lease sales. A wildcat sale covers tracts whose geology is not well known and on which exploration involves searching for a new deposit. Firms can get pre bidding seismic informations but no on site drilling is permitted. A drainage sale consists of tracts in areas surrounding wildcat tracts where a deposit has already been discovered. On-site drilling is not permitted but firms owning adjacent tracts can conduct off-site drilling, which may be informative about the oil deposited in the tracts to be auctioned. Thus bidders who already have won adjacent tracts may have advantages over other bidders for drainage tracts. Wildcat owners may also have cost advantages in drilling in the drainage tracts as they already have expensive resources in place. They may also have synergies in transporting oil from the same location and hence better revenue from oil sell and distribution. Being able to do pre-auction off site drilling also gives them better understanding of their private valuations and they can plan better in terms of timing of the drill and required resources. The off-site drilling may also be informative about the (private) values and wildcat winners may better estimate their private valuation and use of resources. Note that this effect of information advantage for wildcat winner in drainage tracts is purely private value in nature; i.e., they are not correlated to the valuations of other bidders. This creates a clear asymmetry among drainage tract bidders, separating them in two categories: strong and weak bidders. Strong bidders are those who own adjacent tracts. All other bidders are weak. Thus in the oil tract auctions, asymmetry is characterized by the location of the bidders in the second period. This location is determined by the outcome of the first period wildcat bidding, where bidders are symmetric in the sense that they have similar source of signal about the value of the wildcat tract. Hence bidders, while bidding for the wildcat tracts, do not only take their evaluations of the oil storage in the wildcat tracts, but also keep in mind the value advantages they would get in subsequent auctions in that area if they win the current auction. The latter effect generates dynamics in the bidding behavior.

We model the wildcat auction as a common value auction. The winner of the wildcat tract gets a location advantage in the drainage tract. We conduct a semi-parametric test based on Haile Hong and Shum (2006) and cannot accept the assumption of common value in drainage tract (details in the appendix). We therefore treat the drainage tract as a private value auction. Previous authors (Li, Perrigne, Vuong (2002)) have also treated oil tracts as private value auction.

Asymmetry due to synergy in the second period has an important role to play in the bidding behavior. We refer the first period winner as a 'strong' bidder in the second period (drainage) auction and everybody else as weak bidder. Here strong bidder may be characterized as holding a "toehold" due to synergy. However note that unlike the standard takeover battles the actual amount of the toehold is not observed by its weak rival; although who is a strong bidder is common knowledge. A weak bidder may have to bid more aggressively against a strong bidder to win the object (see Maskin & Riley (2000a)).

2If the information advantage was also correlated with the evaluations of other bidders that would have also affected the winner’s curse effect. The winner’s curse of weak bidders in drainage tracts are more severe than that of the strong bidders. This will accentuate the advantage of winning in the wildcat tracts. Since we cannot accept the common value assumption in the drainage tract (test in the appendix) we ignore this possibility.

3In the appendix we test whether the second period drainage auction is of common value type. The test results favors that the drainage tract auction is of private value type.
Since asymmetry due to synergy plays a big role in the expected profitability and bidding behavior of bidders in the second period, ex-ante value of being strong plays an important role in the bidding behavior in the first period symmetric environment too. The asymmetry in the second period is determined by the first period outcome. The first period bidding game is like an investment game whose returns are twofold. One part is aimed at winning the first period auction and the second part is aimed at acquiring the synergy toehold in the second period. The relative advantage of having a toehold due to synergy in the second period guides the relative strength of investing in the first period. Examples of such real option like investments is prevalent in financial and capital investments, production under uncertainty about marginal cost or demand elasticity, auctions etc. In this paper, we separately identify and estimate the real option investment from the first period bidding behavior. Our estimation methodology recovers the underlying fundamentals like the unobserved distribution of signals, degree of asymmetry. The new equilibrium under a different selling strategy would still be functions of the same fundamentals. We therefore construct counterfactual selling strategies and use the estimated fundamentals to compare expected revenues from different selling strategies. We describe which selling strategies could have earned more revenue for the seller under such a general setting.

Bidding in wildcat involves a substantial investment as a sunk cost \(^4\) in conducting and analyzing the seismic survey. In the wildcat sales bidders receive privately observed seismic signals from the same distribution. A puzzle so far has been, whether there has been some kind of overbidding as 64% of wildcat tracts turn out to be dry \(^5\). If the fault lay with the accuracy of the seismic surveys, one would have expected them to be carried out less and less as these are quite costly to carry out. However, that has not been seen. One possible explanation of why people bid high when they apparently should not is because bidders rationally take into account dynamic considerations. Owning an oil tract, even if it has a high chance to turn out dry, gives the owner an advantage in the bidding game for possible future drainage sales. Thus, bidders rationally calculate the advantages of bidding in future drainage auction in deciding on optimal bidding strategies in a wildcat auction. These dynamics lower the threshold level of the signal for bidding for wildcat tracts and overstates the expected valuation of the tract. This leads to excessive bidding in the wildcat sales relative to the static analysis of wildcat auctions. Reduced form analysis suggests that ex-post drainage tract values conditional on ex-post wildcat tract values and competition have a significant positive impact on bidding decisions for wildcat tracts.

A major source of the valuation advantage comes from the ex-ante informational advantage the strong bidder enjoys about his private value relative to the weak bidder. In this paper, we also propose a new semi-parametric technique to estimate and test the degree of ex-ante informational advantage based on copula techniques \(^6\). From a seller’s perspective it may not be enough to know who the strong or weak bidders are. It may be necessary to know ‘how much stronger’ the strong bidder is to design a better selling strategy. The estimated degree of informational advantage is used to conduct counterfactual experiments of alternate selling strategies.

The structural parameters of the model are the distributions of unobserved pseudo values. We suggest

\(^4\) The cost of seismic survey runs in millions of US dollars.
\(^5\) Descriptive statistics as reported by Hendricks and Porter (’89).
\(^6\) Copula is a distribution function which uniquely couples two univariate marginals to their joint distribution function. The copula parameter measures the degree of dependence between two random variable. Details are given later.
a two step estimation procedure based on the identification results to recover the structural parameters from the data. The estimation involves ‘backwards induction’. Specifically, the first order conditions of bidding decisions provide a unique mapping between the distribution of pseudo values and a function of the distribution of observed bids. We first estimate the distribution of pseudo values in the drainage auctions using the equilibrium conditions of drainage bidding. Based on these estimated pseudo values and observed bids, we estimate the expected maximized profit from the drainage auctions. The pseudo values in wildcat auction is characterized as a function of the distribution of wildcat bids and expected maximum profits from the drainage auction. In the second stage, we plug in the estimated maximized profit from drainage auction, estimate the remaining part of this equation from the distribution of observed wildcat bids and get back an estimate of the pseudo values for the wildcat auction.

The estimates indicate that the federal government is only recovering 52% of the ‘strong’ buyers’ willingness to pay in the drainage sales. In the wildcat sales, We find that the bidders are willing to pay up to 5% more to account for possible future information advantage. The degree of asymmetry estimated using copula techniques turn out to be approximately 45%. Asymmetry also has efficiency implications as about 20% of the tracts were own by bidders who did not have the highest value. We use these estimated unobserved ex-ante valuations to conduct counterfactual experiments of different selling strategies for selling such goods and compare the government’s revenue. We find that government’s revenue could have gone up by as much as 27% if a combination of asymmetric sealed bid and open auction format was employed. Interestingly we find that the asymmetry has an impact in the revenue ranking of sealed bid vs open auctions.

The rest of the paper is organized as follows: section 2 reviews related literature, section 3 first describes the model and information structure for a simple case of two bidders and characterizes the equilibrium, identification of ex-ante valuations from observed bids are then established for the general model, section 4 describes the estimation strategy and provides some simulation results, section 5 describes the data and summary statistics, section 6 presents the structural estimation results, section 7 describes the semi-parametric test of asymmetry based on copula, counter-factual experiments are conducted in section 8, section 9 concludes. All proofs, estimation details and a brief review on copulas are relegated to the appendix.

2 Related Literature

Our work is related to three broad strands of literature on empirical auctions: operations management, empirical industrial organization and finance. Elmaghraby (2005) theoretically looked at the impact of the economies of scale and bidder heterogeneity in production capacity in bidding behavior and hence on auction performance. The presence of synergy due to presence of multiple units often led to use of package bidding via combinatorial auction. Kim et al (2014) estimate a structural model of bidding behavior in a combinatorial auctions of Chilean school meals and found significant influence of cost synergies in bidding behavior in package bids. Cantilos et al (2006) and Oliveros (2012) showed that bidders can engage in strategic bidding in a combinatorial auction and inflate their bids even when there is no cost synergy. None of this paper looked at the possibility of dynamics in the bidding behavior induced by the sequential nature of auction format where bidder heterogeneity or synergy creation is endogenous. Our paper showed that the presence
of synergies can generate dynamics and rational overbidding in sequential auctions and structurally estimate the synergy effect.

Auctions with toehold (pre-offer stake in a target) under private and common value settings and in first and second price auctions has been analyzed in Bulow, Huang & Klemperer (’99). Betton and Eckbo (’00) finds that rivals are less likely to bid if the initial bidder has a toehold. Betton, Eckbo and Thorburn (’08) analyze toehold acquiring strategy via an auction where negotiation afterwards happens under the shadow of an auction. Their theoretical analysis and reduced form empirical evidence suggests the importance of toehold bidding in equilibrium. Our paper extends this analysis by making toehold acquisition endogenous and estimating the underlying distribution of signals and counterfactual experiments of optimal selling strategies.

Guerre, Perrigne & Vuong (’00) (GPV) made an important breakthrough in structural non-parametric analysis of auctions. They used the simple and powerful observation that at equilibrium, each bidder is acting optimally against the distribution of his opponents. When bids are observable then both the distribution of the opponent’s behavior and the equilibrium (optimal) choice of each bidder is observable. The necessary conditions for monotonic equilibrium in first price auctions characterizes the bidder’s unobserved signals in terms of a function of the distribution of observable bids. Under monotonicity these distributions can be inverted to get back the distribution of ‘pseudo values’ which under assumptions is the distribution of unobserved signals. A non-parametric estimate of this function thus is an estimate of the distribution of unobserved signals or ‘pseudo values’. GPV and their coauthors (Li, Perrigne & Vuong ’02) (LPV) in a series of papers established consistency and other properties of this estimate. The semi-parametric analysis in this paper uses their idea and extends it to a dynamic auction setting.

The presence of possible asymmetry in drainage tract sales has been widely documented by Hendricks & Porter (’88,’89,’95) (HP) among others. In their seminal paper (HP’89), they empirically analyzed the case where the strong bidder has perfect information about the value of oil stored in drainage tracts while the weak bidder only receives a public signal. Wilson (’69), Milgrom & Weber (’83) and HP showed that in such a framework, in equilibrium, the strong bidder plays a pure strategy and makes positive profit while the weak bidder randomizes and makes zero profit. HP’s static parametric maximum likelihood analysis does not recover the distributions of unobserved ex-ante values of bidders. The possibility of dynamics in the bidding behavior was also ignored in HP. Clearly, who is strong or weak in drainage auction is actually determined by the outcome of the wildcat auction and rational bidders take this into account while bidding for wildcat tracts. In analyzing the drainage tracts auctions, this paper builds on HP’s work and extends it to semi-parametrically identify and estimate the distributions of pseudo values of strong and weak bidders. In this paper, both types of bidders in drainage auction receive privately observed signals, but the strong bidder is expected to receive valuation signal from a ‘better’ distribution\(^7\). Since the government knows who the strong and weak bidders are, it may adopt an asymmetric auction mechanism\(^8\).

\(^7\)Although this assumption will not be imposed while estimating the distributions and would be tested. All we assume is that bidders, whether strong or weak, have some private information which guarantees pure strategy equilibrium.

\(^8\)One such mechanism is giving the weak bidder some advantage in bidding, like if they win they pay only a fraction of their bids.
The counterfactual experiments described in this paper takes this asymmetry into account. Maskin & Riley (’00a), McAdams (’07a) characterized equilibria in asymmetric first price auctions. Under the priority tie breaking rule McAdams established that all equilibria are monotone.

Athey, Levin and Seira (2010) compares the open and sealed bid auction using data from Timber auctions and analyzes the role of asymmetry in determining optimal auction design. Our paper endogenizes the asymmetry and hence generates dynamics in bidding behavior. Like their paper, we also find asymmetry having important implications in auction design. The degree of asymmetry as estimated in our paper via copula also turns out to be crucial in alternative auction formats.

Jofre-Bonet & Pesendorfer (’03) (JBP) empirically analyzed and estimates dynamics in the bidding behavior originating from capacity constraints. In their paper, previously won uncompleted contracts (inventory) reduce the probability of winning further contracts. They quantified the efficiency loss due to these intertemporal effects. Our paper endogenize the asymmetry due to synergy effect originating from location choice. Unlike their paper bidders in our paper faces a possible fixed time horizon in each location. We also device a new test of asymmetry based on copula and analyze its implication on the information structure.

3 Model

3.1 Description and Information Structure

In this section we describe the bidding model. We delineate different stages of the game, information structure and relevant assumptions.

This is a multi-stage game of incomplete information. A non-strategic seller (government) has two indivisible units of the good and sells off each unit of the good in sequential first price auctions. Potential buyers bids for each unit sequentially. All players are risk neutral. The first period auction is called wildcat auction and the second period is called drainage auctions for reasons described earlier. Bidders receive private signals $s^w$ and $s^d$ about their valuations of wildcat and drainage auction and bid $b^w(s^w)$ and $b^d(s^d)$ respectively. The signals $s^w$ and $s^d$ are drawn from distributions $F^w(s^w)$ and $F^d(s^d)$ respectively. Throughout this paper, we shall restrict attention to monotonic bidding equilibrium in their signals. The inverse of the equilibrium bidding strategies will be denoted by $b^{-1}(b(s))$ where we suppressed the superscript for auction type.

As has been shown by McAdams(07) and others, under assumptions monotonic equilibrium will be unique. The random variables $Y_i$ and $B_i$ will denote the maximum of bidder $i$’s rival’s signal and bid.

In an interdependent valuation model, as other bidder may posses information that would, if known to a particular bidder, affect the value he assigns to the tract. When bidder’s valuation only depend on his signal, $E(u_i|s_i^1, s_i^2) = E(u_i|s_i^1)$, then it is called a pure private value model. On the other hand, if $E(u_i|s_i^1, s_j^2)$ is increasing in $s_j$ for all $j \neq i$, then it is an interdependent value model. The knowledge of opponent’s signal

\footnote{As has been shown by McAdams(07) and others, under assumptions monotonic equilibrium will be unique.}
affects the expected valuation of the bidder in the interdependent valuation setting whereas the expected valuation remains unchanged in a private value setting even if bidder has private information about his opponent’s signal. This effect is generally termed the ‘winner’s curse’. Rational bidders anticipate this when forming expectation about the value conditional on winning. When the ex-post value is identical for all the bidders then it is called a pure common value model. Since wildcat tract is the first tract to be sold in a specific location, bidders are assumed to have a pure common value for the wildcat tract. However, in drainage tract sales, there is already an asymmetry amongst bidders induced by the location advantage of the wildcat winner (strong bidder). The location presence by the strong bidder in the neighborhood wildcat tract generates enough private variation in the cost of drilling hence drainage tracts are assumed to have independent private values. In the appendix, we test whether the drainage tracts are of common value nature and reject the hypothesis of common value drainage tract.

3.1.1 Information in the Wildcat Auction (First Period)

Let the value of oil stored in wildcat tract, drawn from \( F_u \), be denoted by \( u \). Bidders are symmetric for wildcat auctions in the sense that they privately observe independent signals \( s^w \) about \( u \) from the same distribution \( F_W(s^w) \). To economize on notations, let the expected common value of oil stored in a wildcat auction when bidder \( i \) with a signal \( s^w_i \), bidding \( b^w_i(s^w_i) \) and maximum of rival’s bid \( b^w_{-i} \), be given by, \( E[u_i|S^w_i = s^w_i, b^w_{-i} \leq b^w_i] \) where the expectation is taken over the distribution of \( u \). Let \( E[u_i|S^w_i = s^w_i, b^w_{-i} = b^w_i(\tilde{s}^w_i)] \) be the ‘pivotal’ expected value in the sense that a small change of bidder \( i \)’s bid will change the winner of the game. Since bidding function is monotonic, it can be rewritten as,

\[
E[u_i|S^w_i = s^w_i, b^w_{-i} = b^w_i(\tilde{s}^w_i)] = U^w_i(s^w_i, \tilde{s}^w_i) = U_i(s^w_i, \tilde{s}^w_i)
\]

The winner of the wildcat auction becomes a strong bidder in the drainage auction. The relative "strongness" of bidders' are determined by his location state variable. The location of bidder \( i \) evolves according to

\[
D_i = \begin{cases} 
   i \text{ is strong, if } i \text{ won the wildcat auction in the same location} \\
   i \text{ is weak, if } i \text{ did not win wildcat auction in the same location} 
\end{cases}
\]

The dynamics of the model is represented by the evolution of the state variable \( D \). When bidders are bidding for wildcat tracts they are also choosing their future locations \( D \), which affects their marginal distribution of valuations for drainage tracts.

3.1.2 Information in the Drainage Auction (Second Period)

Bidders in drainage auctions are divided into two groups: strong bidder (denoted by 1); who has won the wildcat tract in the first period, and weak bidders (denoted by 0); who did not win in the first period. They independently draw their private signals about the value of the drainage tract, \( V_i \) from different marginal

\footnote{Thus, each of the random variables \( s^w \) are affiliated with the common component \( u \), but conditioned on the common component \( u \) they are independent.}
distributions, $F_1(s^d)$ and $F_0(s^d)$ respectively. Thus $s^d_1$ is the realization of the random variable $s^d$ drawn from the distribution $F_1(.)$. Signals $s^d_1 \in \mathbb{R}$, about the unknown value of oil stored in drainage tracts $V_i$. We assume that all the distributions are continuous. The ex-ante value of the strong bidder is thus given by $E[V_1|s_1] = v_1(s_1)$, and that of the weak bidder be $v_0(s_0)$.

An example of asymmetry could be the case where strong bidders receive their signal from a distribution which dominates the weak bidder’s distribution of signal in the hazard rate order\(^\text{11}\) as in Maskin & Riley (2000a) and in Athey et al (2009). That is for all $s$, $f_1(s)/F_1(s) \geq f_0(s)/F_0(s)$.

### 3.2 Timeline of Events

The sequence of moves, for each location, are as follows:

1) Number of potential bidders is common knowledge. Bidders first receive some information; $s^w_i$, about the value of oil stored in the tract, $u$, $s^w_i \sim F^w(.)$ of the wildcat tract from a seismic survey.

2) When bidders submit bids, they do not know their rival’s signal $s^w_j$. Any bidder submitting a bid $b^w_i$, wins the auction with probability $\Pr(b^w_i, M^w)$, where $M^w$ is the maximum of his rival’s bid.

3) In the following period, bidders decide how much to bid for the neighboring drainage tracts $b^d_i$. Bidders draw their signals about their valuations of the drainage tract $v_i(s^d_i)$, from continuous asymmetric distributions $F_i(s^d)$. They win the drainage tract with a probability $\Pr(b^d_i, M^d_i, D)$, where $D$ is the state (location) variable of bidder and $M^d_i$ being the maximum of rivals’ bid.

We assume that the auctioneer (government), does not act strategically and always sells the objects to the highest bidder via two independent first price auctions. We also abstract away from reserve price issues\(^\text{12}\).

In this game bidders have two kinds of choices: a continuous choice of how much to bid for the wildcat tract and a second continuous choice of how much to bid for the drainage tract. All the potential bidders bid for both wildcat and drainage tracts. Thus there is no endogeneity of number of bidders. Some of them may bid zero though depending on their signals received.

### 3.3 Value Function

The value of the wildcat tract has two components to a bidder: one part comprised of winning the wildcat tract and the second component of having extra advantages in drainage tract bidding by winning the wildcat tract. The value of the wildcat tract, to the $i^{th}$ bidder, $V_i$, can therefore be written as follows:

$$V_i(s^w_i) = \max_{b^w_i \geq 0} \{ (U(s^w_i, \bar{s}^w_i) - b^w_i) \times \Pr(b^w_i < b^w_i|s^w_i) \\ + \beta \sum_{j=1}^{N} \Pr(j \text{ wins } WA) \int T_i(D) dF(s^d_i, D) \}$$

\(^{11}\)We however do not impose this assumption in our empirical section. We do not impose any assumption on the relative ranking of the distributions while deriving the equilibrium first order conditions which will be used for estimation. All we assume is that each bidder has some private informations for the equilibrium to be in pure strategies. The relative ranking of the distributions shall be tested using data on ex-post value of the tract and the estimated ex-ante pseudo values.

\(^{12}\)For the application of OCS auctions in this paper, the average reserve price was US$15 per acre the average bid being around US$600 per acre, the reserve price being not binding; it may be safe to assume that there was no reserve price.
where \((U(.) - b_i^w) \times \Pr(b_i^w < b_i^w|s_i^w)\) represents bidder \(i\)'s expected value from bidding and winning the wildcat tract, and \(T_i(D)\) is his expected value from bidding the drainage tract discounted by \(\beta\), \(\int T_i(D) dF(s_i^d|D)\) is the ex-ante value from the drainage auction, before bidders receive their signals about the drainage tracts\(^{13}\). The later term is weighted by the probability of who wins the wildcat tract as that determines who would be the strong bidder in the drainage tract.

The \(i\)th bidder received a signal \(s_i^w\); the probability distribution function that bidder \(i\) assigns that his rival got a signal \(s_i^w\) given that the \(i\)th bidder received a signal \(s_i^w\) be given by \(F(s_i^w|s_i^w)\). The bidder, conditional on receiving a signals \(s_i^w\), bids \(b_i^w(s_i^w)\) to solve the above maximization problem.

The expected value from the drainage tract can be written as

\[
T_i(D) = \max_{b_i^d \geq 0}\{(v_i(s_i^d) - b_i^d) \times \Pr(b_i^d < b_i^d|s_i^d, D)\}
\]

From the bidder’s perspective, the drainage tract being auctioned next period. Hence its expected value depends on the next period’s state \(D\). This in turn depends on bidders choices and outcomes in the wildcat auction this period.

### 3.4 Equilibrium

We solve for symmetric perfect Bayesian equilibrium in monotone strategies for each stage game. Solution method involves backward induction. We first solve the drainage tract auction as a function of bidders state variable next period \(D\). This solution is substituted in the wildcat bidding problem which solves the wildcat auction and determines \(D\). This endogenously creates the heterogeneity amongst bidders.

#### 3.4.1 Last Stage Decision: Analysis of Drainage Tracts

This section considers a private value framework in the drainage auction. The distribution of valuations of strong and weak bidders are given by \(F_1(s_1)\) and \(F_0(s_0)\) respectively. As we cannot accept the hypothesis of common value (in the appendix) we assume that the bidder have private values in the drainage auction. Let \(v_i(s_i^d) = E[V_i|S_i^d = s_i^d]\) be the expected private value of the tract to the strong bidder, when he received a signal \(s_i^d\), \(v_0(s_i^d)\) is defined analogously. Bidder’s valuations are private. i.e., \(v_i(s_i^d)\) is non-decreasing in \(s_i^d\) and does not depend on \(s_j^d\), \(\{i, j\} \in \{0, 1\}\). Moreover we assume \(v_i\) is strictly increasing in \(s_i^d\) and \(v_i(0) = 0\). The distribution of bidders’ signals are independent.

Let the bids by two types of bidders be \(b_i^d\) and \(b_0^d\) respectively. By construction there can be only one strong bidder: the winner of the nearest wildcat tract. By definition everybody else is a weak bidder. Therefore there are multiple potential weak bidders bidding for a drainage tract. Let each bidder adopt a monotone bidding strategy \(b_i^d(s_i)\) with an inverse \(\phi_i(b_i^d) = (b_i^d)^{-1}(b_i^d(s_i))\). Thus by bidding \(b\), the strong bidder wins the auction with probability \(\Pr(b_i^d \leq b) = G_0^d(b) = \Pr(\phi_0(b_i^d) \leq \phi_0(b)) = F_0(\phi_0(b))\), similarly the weak bidder wins the auction with probability \(G_1^d(b)\). Let the common support of the distributions of signals be \([s, \bar{s}]\).

\(^{13}\)Note that, by an abuse of notation we represent \(F(s_i^d|D = i \text{ is strong}) = F_1(.)\), and \(F(s_i^d|D = i \text{ is weak}) = F_0(.)\)
Proposition 1 a) With zero probability of ties, there exists a pure strategy equilibrium of the drainage auction in monotonic strategies. It is characterized by the following conditions

\[
\frac{F'_0}{F_0} \phi'_0(b_i^d) = \frac{1}{v_1(s_i^d) - b_i^d}
\]

(2)

and

\[
\frac{F'_1}{F_1} \phi'_1(b_i^d) = \frac{1}{v_0(s_i^d) - b_i^d}
\]

(3)
in the common support of signals, i.e., for all \(s_0, s_1 \in [s, \bar{s}]\), satisfying the boundary conditions, \(b_0(\bar{s}) = b_1(\bar{s}) = \bar{b}^d\), \(b_0(s_0) = b_1(s_1) = 0\). The equilibrium pair of inverse bid function is given by, \(\phi_i(b_i^d) = b_i^{-1}(b_i^d)\), \(i \in \{0, 1\}\), where \(s_i^d = \phi_i(b)\), is the inverse bid function. \(\theta_i(s_i^d) = \phi_j(b_i^d(s_i^d))\), \(\{i, j\} \in \{0, 1\}, i \neq j\), is a monotonic function of \(s_i^d\).

Proof. Details are given in the appendix. Here is a sketch, first note that the utility function satisfies the single crossing property and independence of signals guarantees that the distribution of valuations is log-supermodular. Moreover, it is assumed that there are zero probabilities of ties. Hence the assumptions of McAdams (’07) is satisfied and a pure strategy equilibrium in monotone strategies exists. The exact characterization of the equilibrium in terms of the differential equations given above then can be found by taking the first order conditions with appropriate boundary conditions\(^{14}\). ■

We call \(v_1(s_i^d)\) as the pseudo value of bidder 1. Let the underlying distribution of \(v_1(s_i^d)\) be \(F_i^d\). Since the parameter of interest is the distribution, henceforth with an abuse of notation. Similarly let the underlying distribution of \(v_0(s_i^d)\) be \(F_0^d\). Under the assumption of hazard rate dominance (for details see Maskin & Riley (2000a)), equilibrium bidding strategies represent several key properties. First, bid strategies are type symmetric, i.e. bidder of the same type (strong or weak) follow symmetric strategies within their group. Second, strong bidders submit higher bids: \(G_1(b; n) \leq G_0(b; n)\) for all \(b\). Third, the strong bidder shades his bid further below his valuation than the weak bidder. Moreover, if a strong bidder faces a weak bidder rather than a strong bidder he will bid less aggressively. Symmetrically a weak bidder will bid more aggressively facing a strong bidder. A corollary of these results is that strong bidder makes more ex ante equilibrium surplus (profit) than the weak bidder. Hence having a synergy leads to higher profit for strong bidders in the drainage auction.

Note that in equilibrium, the probability that bidder \(i\) wins with bid \(b_i\) can be written as (suppressing the number of bidders \(n\))

\[
\Pr(b_i \geq \max_{j \neq i} \phi_j(s_j)) = G_{M_i}(b_i) = \Pi_{j \neq i} G_j(b_i)
\]

(4)

where \(G_j\) is the cumulative distribution of \(b_j\). The probability that bidder \(j \neq i\) wins when bidder \(i\) bids \(b_i\) is

\[
\int_{b_i}^{\bar{s}_j} \left(\Pi_{k \neq i, j} G_k(b_j)\right) g_j(b_j) db_j
\]

Finally using 4 we can write

\[
\frac{G_{M_i}(b_i)}{g_{M_i}(b_i)} = \frac{1}{\sum_{j \neq i} \frac{g_j(b_i)}{G_j(b_i)}}
\]

\(^{14}\)It is easy to verify that second order condition is also satisfied.
Hence the first order conditions for strong and weak bidders in the drainage auctions (subscripted as 1 and 0 respectively) can be written as

\[ v_{1i} = b_{1i} + \frac{1}{\sum_{j \neq i} g_{j}(b_{ij})} = \xi_{1}(b_{1i}, G) \]  

(5)

\[ v_{0i} = b_{0i} + \frac{1}{\sum_{j \neq i} g_{j}(b_{ij})} = \xi_{0}(b_{0i}, G) \]  

(6)

The first order conditions, together with the boundary condition \( b_{i}(r; n) = r \) for all \( i \), uniquely characterizes optimal bidding strategy (Guerre, Perrigne and Vuong, 2000).

### 3.4.2 First Stage Decision: Analysis of Wildcat Auction

Let \( Y^{w} \) be the signal of the maximum rivals’ bid in the wildcat auction, then the probability of bidder \( i \) wins the auction is the probability that his bid is higher than \( M^{w} \). Let each bidder adopts the monotone bidding strategy \( b(s^{w}) \) with an inverse \( \phi(b^{w}) \).

Then,

\[ \text{Pr}(i \text{ wins } W_{A}) = G_{M^{w}|b^{w}}(y|b^{w}) = \text{Pr}(b_{i}^{w} > M_{i}^{w} = \max_{j \neq i} b_{j}^{w}, \text{ for } j \neq i) \]

\[ = \text{Pr}(Y_{i}^{w} \leq \phi(b_{i}^{w}), \text{ for } j \neq i) \]

\[ = F^{w}(\phi(b_{i}^{w})) \]

where the first line states that the bidder wins the auction if his bid is higher than the maximum of his rivals’ bid \( (M^{w}) \). The second line uses monotonicity of the bidding strategy of bidders to express distribution of valuations signals in terms of equilibrium bid distribution of the rival bidder \( (G_{M^{w}|b^{w}}(b_{i}^{w})) \). Let \( g^{w} \) be the density function associated with \( G^{w} \).

**Lemma 2** The equilibrium bidding rule of the wildcat bidding game can be characterized by the following equations:

\[ b_{i}^{w} = U(s_{i}^{w}, s_{i}^{w}) - F^{w}(\phi(b_{i}^{w})) \times \frac{b'(s_{i}^{w})}{f_{Y^{w}|s^{w}}(s_{i}^{w}|s_{i}^{w})} + \beta \int [T(1 \text{ is strong}) - T(1 \text{ is weak})]dF(s^{d}, D) \]  

(7)

with the terminal condition, \( b_{i}^{w}(s_{i}^{w}) = 0 \), and writing in terms of the distribution of bids,

\[ b_{i}^{w} = U(s_{i}^{w}, s_{i}^{w}) - \frac{G_{M^{w}|s^{w}}(b_{i}^{w}|b_{i}^{w})}{g_{M^{w}|s^{w}}(b_{i}^{w}|b_{i}^{w})} + \beta \int [T(1 \text{ is strong}) - T(1 \text{ is weak})]dF(s^{d}, D) \]  

(8)

**Proof.** In the appendix. ■

The first order condition states that bid equals expected valuation plus a mark down and plus a markup. The markdown accounts for the level of competition in the wildcat sale. The mark up accounts for the discounted future rent from winning the wildcat today, expressed as the relative advantage of being a strong bidder. We refer this as the perceived value of synergy of being a strong bidder in the drainage tract. Note as shown in Maskin-Riley (2000), under the assumption of hazard rate dominance, strong bidder will make more expected profit in equilibrium than the weak bidder. Hence the third term is expected to be positive. This leads to rational overbidding in equilibrium in the wildcat tract. This overbidding stems from the fact that all bidders want to pay for the expected extra benefit from having a synergy in the drainage tract.
### 3.5 Identification

#### 3.5.1 Identification of Valuations in the Drainage Auctions

In this section we establish the identification of the distribution of signals from the observed bidding behavior of bidders for the drainage and wildcat auctions respectively. We are interested in identifying the following primitives of our model: the distribution of signals in the drainage tracts and the distribution of signals in the wildcat tracts. We (the econometrician) observe the distribution of all the bids in the wildcat and drainage auctions as well as we know the identity of the bidders. Under the assumption that the discount factor $\beta$ is fixed, the broad principle of identification is based on Guerre, Perrigne and Vuong (2000). Specifically we identify the distribution of signals in the drainage tracts under the assumptions of monotonic bidding using the equilibrium first order conditions 2 and 3. Given the distribution of signals from drainage tracts and a fixed $\beta$ we can identify the distribution of 'pseudo values' from the distribution of observed bids again under the assumption of monotonic bidding from the equilibrium first order conditions in 7.

We assume that there are two types of bidders in the drainage auctions. There is 1 strong bidder\(^{15}\) (type 1) and $n_{0d}$ weak bidders (type 0). Note that the set $n_{0d}$ is endogenous. It is determined by who did not win in the wildcat auction. However, before bidding for the drainage auctions, bidders (and the econometrician) can observe who are strong and weak bidders\(^{16}\).

Let

$$v_1(s^d_1, n) = E[V^d_1|S^d_1 = s^d_1, n = 1 + n_{0d}]$$

be the expected value to bidder $i$ of type 1 when he received a signal $s^d_1$, where the expectation is taken over the distribution of $V^d_1$.

Similarly, let

$$v_0(s^d_{0i}, n) = E[V^d_{0i}|S^d_{0i} = s^d_{0i}, n]$$

be the expected value to bidder $i$ of type 0 when he received a signal $s^d_{0i}$.

There is only one strong bidder in this model. However the number of weak bidders can be more than one depending on the set of potential bidders of the wildcat auction. Note that bidders are asymmetric across the groups but symmetric within each group. Hence $v_0(s^d_{0i}, n) = v_0(s^d_{0i}, n)$, for all $i \in$ Type 0. We assume $v_k(s^d_{ik}, n)$ is non-decreasing in $s^d_{ik}$, for all $i$. Moreover we assume that $v_k$ is strictly increasing in $s^d_{ik}$ and $v_k(0, .) = 0$. For notational simplicity we will suppress the argument $n$ below. The ‘strong’ bidder receives a private signal $s^d_{1i}$ about his unknown valuation $v^d_1$ and chooses $b^d_{1i}$ to maximize $E[(v^d_1 - b^d_{1i})I(B^d_{1i} \leq b^d_{1i})|s^d_{1i}]$.

---

\(^{15}\)Note that, by our formulation there is only one strong bidder in any drainage tract. However it may be possible to have more than one strong bidder. For example, in general there are eight drainage tracts, and if the winner of the first and second drainage tract auctions are different then in the third drainage tract auction there are 2 strong bidders. However, for this paper by assumption all the drainage tract are sold on the same date which rules out such a possibility. Selling of all drainage tracts on the same date does not literally means they are sold on the same day. All it requires that there is not enough time in between two drainage sales for information transmission to the winner of the earlier sale.

\(^{16}\)The econometrician can observe the latitude and longitude of the wildcat and drainage tracts, the sale date of the tracts and the identity of the bidders. A strong bidder is someone who has won the nearest (defined by the combination of latitude and longitude) wildcat tract, sold before the drainage tract.
where $M_i^d$ is the maximum of his rivals’ bid. Thus $M_i^d = \max_i\{b_{0i}^d(s_{0i}), b_{1i}^d(s_{1i})\}$, and $b_{1i}^d(\cdot)$ and $b_{0i}^d(\cdot)$ are the equilibrium strategies of ‘strong’ and ‘weak’ bidders respectively. We restrict our attention to symmetric, strictly increasing and differentiable equilibrium strategies. By ‘symmetry’ we mean symmetry within each sub-group of weak bidders. Let $B_i^d = b_{1i}^d$ and $B_i^d = \max_i b_{0i}^d$, let $G_{B_i^d}B_0^{d_1}\text{d}\_X, Y|X$ and $G_{B_i^d}B_0^{d_1}$ be the distribution of bids. Then, as shown in the previous section, the first order conditions associated with bidder’s optimization problems can be written as; for the strong bidders

$$v_i^d(s_{1i}^d) = b_{1i}^d + \frac{1}{\sum_{j \neq i} g_j(b_{1i}^d)} = \xi_i^d(b_{1i}^d, G)$$

(9)

Similarly for the weak bidders,

$$v_i^d(s_{0i}^d) = b_{0i}^d + \frac{1}{\sum_{j \neq i} g_j(b_{0i}^d)} = \xi_i^d(b_{0i}^d, G)$$

(10)

with the boundary conditions $b_{0i}^d(s_{0i}^d) = b_{1i}^d(s_{1i}^d) = 0$.

The following lemma establishes the identification of distribution of interdependent values $v_i^1$ and $v_i^0$, $F^d(v_1, v_0)$ from bid distribution.

**Lemma 3** The distribution of signals for the drainage auction are non-parametrically identified from the observed bids.

**Proof.** In the Appendix.

### 3.5.2 Identification of Valuations in the Wildcat Auctions

The first order conditions associated with the Bayesian equilibrium strategies for the bidders who have already entered to bid in the wildcat auction is given by

$$U(s_i^w, s_i^w) = b_i^w + \frac{G_{B_i^w}s_i^w(b_{1i}^w|b_{1i}^w)}{g_{B_i^w}s_i^w(b_{1i}^w|b_{1i}^w)} - \beta \int [T(1 \text{ is strong}) - T(1 \text{ is weak})] dF^d(s^d, D)$$

(11)

where $U(s_i^w, s_i^w)$ is the expected common value of the tract to bidder $i$ when he received a signal $s_i^w$, and the maximum of his rival’s signal$^{18}$ is also $s_i^w$.

**Lemma 4** Given $\beta$, the distribution of $U$ from the common value model for wildcat auction is identified for the bidders who submitted bids, from the observed distribution of bids and the data on actual and potential number of bidders.

**Proof.** In the Appendix.

---

$^{17}$The existence and uniqueness issue of such equilibrium has been discussed in McAdams (’04) and Lizzeri & Persico (’00) and Maskin and Tirole (2000b).

$^{18}$Since there could be more than two bidders, the maximum of the rival’s signal is what is important to the bidder to account for the winner’s curse in equilibrium.
4 Estimation Strategy

The structural parameters of interest are the distributions of ex-ante values in the wildcat auction $F^w$, and that of strong bidder $F^d_1$ and weak bidders $F^d_0$ respectively in drainage auctions. The observable of the model are the distribution of bids of wildcat and drainage auctions and the identity of bidders. The estimation of our model will follow the two stage backward induction procedure used to establish the equilibrium. In the first stage, we use a parametric procedure and the first order conditions and the distribution of bids from the drainage auctions, we estimate $F^d_1$, $F^d_0$. In the second stage, using the estimated distributions in the first stage and first order conditions and the distribution of bids from the wildcat auctions we estimate $F^w$.

First Stage: In the first step, we estimate the bid distributions from the drainage auctions for a particular location and recover the ‘pseudo’ values using the equations (9) and (10). Following JBP we adopt a parametric approach in estimating these distributions. We estimate the conditional (conditioned on the number of actual bidders) bid distribution functions $G^d_1(.|X,N)$ and $G^d_0(.|X,N)$ via Weibul bid distribution, where $X$ is corresponding wildcat sale characteristics (like wildcat winning bids, no of wildcat bidders) and $N$ is the number of participating bidders in the drainage tract. We specifically assume the bid density function to be a monotone bid transformation of the Weibul distribution as in Jofre Bonet and Pesendorfer(2003) and Athey et al (2010). The transformation involve taking the logarithm of one plus the bid, $\ln(b+1)$. The lower endpoint of the Weibul density function is $\ln(\theta_3 + 1)$. The parameters of the bid distribution function is assumed to be a function of the number of actual bidders in each auction.

$$g(b|N, \theta_1, \theta_2, \theta_3) = \frac{1}{b+1} \left[ \frac{\theta_1 [\ln(b+1) - \ln(\theta_3 + 1)]^{\theta_1 - 1}}{\theta_2} \right] \times \exp(-\frac{\ln(b+1) - \ln(\theta_3 + 1)}{\theta_2})$$

Note that the right hand side of the first order conditions can be written as

$$v_i = b_i + \frac{1}{\sum_{j \in n \setminus i} g_j(b_i|N)} \frac{g_j(b_i|N)}{G_j(b_i|N)}$$

Plugging in these values in the objective function we get upper bound of the expected maximum values of strong and weak bidders $T_1$ and $T_0$ respectively. (Details are given in the appendix.) The integration is evaluated numerically over a grid point.

Second Stage

In the second stage we first parametrically estimate the bid distributions for each wildcat sales $G^w(.|N)$ and $g^w(.|N)$ as a Weibul distribution. These and estimated values of $T_1 - T_0$ are then plugged in the following first order equation characterizing the equilibrium, to get the distribution of ‘pseudo’ values of the wildcat tracts,

$$\hat{U}(s^w_i, s^d_i) = b^w_i + \frac{G^w(b^w_i|s^w_i)}{g^w(b^w_i|s^d_i)} - \beta \int [T(D = 1) - T(D = 0)]dF(s^d, D)$$

\(^{(12)}\)

19In a previous version of this paper we used fully non parametric estimation. A fully nonparametric approach also gave similar results. We adopt the parametric approach here to overcome the data limitation.
5 Data and Summary Statistics

We apply the model to data on sales of wildcat and drainage tracts off the coasts of Texas and Louisiana held since 1954. The government sells off these tracts to the highest bidder via a sealed bid auction and charges his bid (first price auction). Bidder for these tracts are oil companies. Before firms conduct tract specific seismic surveys, a set of tracts are nominated based on some area wide pre-sale exploration. Government constructs a final list based on the nominations. Many more tracts are nominated than receive bids, and the nomination process probably contains no information. Nominated tracts are then sold in a first price sealed bid auction. The winning bidder has five years to explore a tract. If no exploration is done in 5 years then the lease reverts back to the government and the tract may subsequently be reoffered. If oil or gas is discovered in sufficient quantities then the lease is automatically renewed as long as production occurs.

For each tract the data set contains the date of sale; acreage; location (Latitude and Longitude); the identity of all bidders and the amounts they bid; whether the government accepted the high bid; the number, date and depth of any wells that were drilled; monthly production of oil, condensate, natural gas and other hydrocarbons through 1991. The dataset also have information on drilling costs of wildcat and production wells obtained from annual surveys by the American Petroleum Institute. Typically an wildcat tract consists of 5000 to 5760 acres and covers on an average 0.0463 degrees of longitude and 0.0405 degrees of latitude. There are generally eight drainage tracts surrounding an wildcat tract and each one covers around 2500 acres. The strong and weak bidders are identified using their firm code and the latitude and longitude information of the tracts. Specifically, a strong bidder for a drainage tract is a bidder who has owned the nearest wildcat tract in the neighborhood before the drainage tract is up for sale.

A detailed description of the dataset can be found in Hendricks and Porter (‘89) and Porter (‘95). We present here only salient features of the data relevant for our model. A descriptive statistics of the tracts offered for sales are presented below, where bids are in millions of nominal US $.

Table 1: Descriptive Statistics of Tracts Offered for Sale: 1954 – 1990

<table>
<thead>
<tr>
<th>Period</th>
<th>Tracts Receiving Bids</th>
<th>Bids Per Tract</th>
<th>Sold</th>
</tr>
</thead>
<tbody>
<tr>
<td>1954 – 1960</td>
<td>424</td>
<td>3.05</td>
<td>422</td>
</tr>
<tr>
<td>1968 – 1974</td>
<td>1117</td>
<td>4.12</td>
<td>1065</td>
</tr>
<tr>
<td>1975 – 1982</td>
<td>1276</td>
<td>2.65</td>
<td>850</td>
</tr>
<tr>
<td>1983 – 1990</td>
<td>5505</td>
<td>1.45</td>
<td>3606</td>
</tr>
</tbody>
</table>

Data of oil tracts auctioned between 1954 and 1990 are used for the estimation. Possibility of joint bidding is not modelled in this paper and was not prevalent in 1954 – 69. Hence, we ignore joint bidding and work with the subset of the data with no joint bidding. The maximum number of bidders in any tract was 18, the median being 4. The reserve price of oil field tracts were US $15 per acre which is much lower than the average bid. Also there is less than 0.25% of the bids in the interval [15, 20]. We therefore assume, as in the theoretical model that the reserve price is non-binding. Sometimes the federal government rejected

\[I am grateful to Prof. Ken Hendricks and Prof. Joris Pinkse and Prof Rob Porter for sharing the data.\]
bids above the reserve price too. This accounted for around 2% of the bids. Hence the random reserve price issue is also ignored. Also we regress, as in Porter (’95), log of tract bids on possible tract specific dummies to analyze possible heterogeneity issue. The F-test rejects the tract heterogeneity when controlled for the number of bidders. Since our econometric methodology is for a given number of bidders we ignore tract heterogeneity issues.

Selected Statistics on wildcat and drainage tracts for 1954-1982 are given below\textsuperscript{21}

\textbf{Table 2:} Summary Statistics on Wildcat and Drainage Tracts: 1954-1982

<table>
<thead>
<tr>
<th></th>
<th>Wildcat</th>
<th>Drainage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Tracts</td>
<td>2510</td>
<td>339</td>
</tr>
<tr>
<td>Number of Tracts Drilled</td>
<td>1793</td>
<td>244</td>
</tr>
<tr>
<td>Number of Productive Tracts</td>
<td>880</td>
<td>171</td>
</tr>
<tr>
<td>Average Winning Bid per acre ($’82 Prices)</td>
<td>2464</td>
<td>5421</td>
</tr>
<tr>
<td>Average Net Profits per acre ($’82 Prices)</td>
<td>–329</td>
<td>3342</td>
</tr>
<tr>
<td>Average Tract Value per acre ($’82 Prices)</td>
<td>2135</td>
<td>8763</td>
</tr>
<tr>
<td>Average Number of Bidders</td>
<td>3.52</td>
<td>2.46</td>
</tr>
</tbody>
</table>

The following table describes the major features of the drainage tracts sold for the period\textsuperscript{22} 1954 – 1982.

\textbf{Table 3:} Role of Information in Drainage Tracts

<table>
<thead>
<tr>
<th></th>
<th>Wins by Neighbor Firms</th>
<th>Wins by Non-Neighbor Firms</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Tracts</td>
<td>52</td>
<td>287</td>
</tr>
<tr>
<td>Number of Tracts Drilled</td>
<td>35</td>
<td>212</td>
</tr>
<tr>
<td>Number of Productive Tracts</td>
<td>32</td>
<td>140</td>
</tr>
<tr>
<td>Average Winning Bid per acre ($’82 Prices)</td>
<td>5887</td>
<td>5336</td>
</tr>
<tr>
<td>Average Gross Profits per acre ($’82 Prices)</td>
<td>14000</td>
<td>7813</td>
</tr>
<tr>
<td>Average Net Profits per acre ($’82 Prices)</td>
<td>8112</td>
<td>2477</td>
</tr>
</tbody>
</table>

The gross profit was calculated as the value of oil recovered minus the drilling cost. Net profit was measured as ex post revenue minus the drilling costs minus the bid.

The above tables suggest the following major points:

1) **Strong bidder has valuation advantage:** Both social rents and net profits are much higher on tracts won by a strong bidder. Discounted social value as measured by ex post revenue minus drilling costs was on an average US $28.44 million for tracts won by strong bidders and US $18.69 million for tracts won by weak bidders. Net profit measured as ex post revenue minus the drilling costs minus the bid was on an average US$ 13.3 million for tracts won by strong bidders and only US $2.93 million for tracts won by weak bidders. This suggests that information however noisy has some role to play in deciding how much to bid for the drainage tracts.

\textsuperscript{21} Dollar Figures are in millions of 1982 US$.

\textsuperscript{22} Dollar Figures are in millions of 1982 US$.
2) Higher net profit per acre from drainage tracts than wildcat tracts: Average gross profit per acre measured as ex post revenue minus cost for wildcat tracts was US$14000 and that for drainage tracts was US$7813.

3) ‘Overbidding’ in wildcat tracts: Out of the wildcat tracts sold oil was found only in 35% of them whereas in more than 51% of the cases oil was found in drainage tracts.

5.1 Reduced Form Prediction

In this section, we analyze whether the bidders were also taking the potential profitability of the drainage tracts into account while deciding to bid for the wildcat tracts. A researcher only observes the decision to bid and the bid amount for the wildcat tracts. However the data set also have ex-post informations on tracts’ value of oil, drilling cost, acreage of the drainage tracts and whether the tracts were dry or not. Although these ex-post information are also unavailable to the firm while entering and bidding for the wildcat tracts, we use them as a reasonable proxy about bidder’s information level.

In the theoretical model it was argued that bidders also bid higher in the wildcat auction depending on the synergy advantage he will foresee as a strong bidder over the weak bidder. We take ex-post gross profit of the drainage tracts as a proxy for the profitability of the drainage tracts. Below we explore whether these predictions were true at the reduced form level for bidding stage of the wildcat tracts.

5.1.1 Bidding in Wildcat Auction

The estimated elasticity on bidding decision \( (\text{wbid}) \) based on OLS regression of bidding of wildcat tracts is regressed on ex-post wildcat \( (w\pi) \) and drainage values \( (D\pi) \).

\[
\text{wbid} = \alpha_0 w\pi + \alpha_1 w\pi^2 + \alpha_2 D\pi + \alpha_3 D\pi^2
\]

*Table 4: Elasticity Calculation Based on Median Value*

<table>
<thead>
<tr>
<th>Independent Variable</th>
<th>Elasticity</th>
<th>Standard Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w\pi )</td>
<td>1.28</td>
<td>0.30</td>
</tr>
<tr>
<td>( D\pi )</td>
<td>0.29</td>
<td>0.15</td>
</tr>
</tbody>
</table>

Dependent Variable log of winning bid

Thus all the reduced form variables have expected signs with significant positive effect. Since bids are in millions of dollars hence their economic effects are big too. Therefore bidding in wildcat auctions are affected by drainage tract values. There are of course significant limitations of the above reduced form analysis. A major problem would be selection bias: strong bidders may be bidding and winning those drainage tracts for which they have higher valuation. We cannot separately identify the impact of the drainage tracts on wildcat tracts based on reduced form analysis and hence resort to structural estimation.
6 Structural Estimation Results

In this section, we present the structural estimation results. Major structural elements of our model are the distributions of pre-bidding pseudo values of the drainage tracts to the strong and weak bidders and the distribution of pseudo values of the wildcat tracts.

6.1 Estimating the Bid Distribution

Given the assumptions of the theoretical model, we assume that the econometrician as well as the bidders of the drainage auction knows the nearest wildcat sale characteristics (number of bidders in the wildcat auction, winning bid) as well as the number of participating bidders for the drainage auction. The equilibrium bid distribution can therefore be written as $G(\cdot|X,N)$ where $X$ is the wildcat sale characteristics and $N$ is the participating bidders in the drainage tract. As in Athey et al (2010) and JBP we estimate the bid distribution via a parametric Weibul distribution.

$$g(b|N, \theta_1, \theta_2, \theta_3) = \frac{1}{b+1} \left( \frac{\theta_1 \ln(b+1) - \ln(\theta_3+1)^{\theta_1-1}}{\theta_2^{\theta_1}} \right) \times \exp\left(-\frac{(\ln(b+1) - \ln(\theta_3+1))^{\theta_1}}{\theta_2}\right)$$

where $\theta_1$ is the shape, $\theta_2$ is the scale and $\theta_3$ is the location parameter. The wildcat equilibrium bid distribution is estimated as $G(\cdot|N)$. We report the estimated coefficients and the bootstrap standard (based on a 10000 bootstrap sample) errors below. The Intercept $\theta_k$ and Coeff $\theta_k$ are the intercept and coefficient when $\theta_k; k = \{1,2,3\}$ are expressed as $\theta_k = \text{Intercept}_\theta + \text{Coeff}_\theta N$, where $N$ is the number of bidders.

<table>
<thead>
<tr>
<th>WildCat Auction</th>
<th>Intercept $\theta_1$</th>
<th>Coeff $\theta_1$</th>
<th>Intercept $\theta_2$</th>
<th>Coeff $\theta_2$</th>
<th>Intercept $\theta_3$</th>
<th>Coeff $\theta_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original Data</td>
<td>1.41</td>
<td>0.022</td>
<td>1.81</td>
<td>0.021</td>
<td>15.72</td>
<td>3.27</td>
</tr>
<tr>
<td>Bootstrap Sample Mean</td>
<td>1.52</td>
<td>0.014</td>
<td>1.8</td>
<td>0.012</td>
<td>14.13</td>
<td>2.42</td>
</tr>
<tr>
<td>Bootstrap Standard Dev.</td>
<td>0.022</td>
<td>0.0035</td>
<td>0.0043</td>
<td>0.0005</td>
<td>1.18</td>
<td>0.72</td>
</tr>
<tr>
<td>Drainage Auction: Strong Bidder</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Original Data</td>
<td>0.89</td>
<td>0.061</td>
<td>1.60</td>
<td>-0.027</td>
<td>3.37</td>
<td>-0.95</td>
</tr>
<tr>
<td>Bootstrap Sample Mean</td>
<td>1.03</td>
<td>0.053</td>
<td>1.41</td>
<td>-0.17</td>
<td>2.76</td>
<td>-0.73</td>
</tr>
<tr>
<td>Bootstrap Standard Dev.</td>
<td>0.23</td>
<td>0.056</td>
<td>0.124</td>
<td>0.024</td>
<td>1.13</td>
<td>0.38</td>
</tr>
<tr>
<td>Drainage Auction: Weak Bidder</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Original Data</td>
<td>1.07</td>
<td>0.035</td>
<td>1.42</td>
<td>-0.0012</td>
<td>2.18</td>
<td>-0.89</td>
</tr>
<tr>
<td>Bootstrap Sample Mean</td>
<td>1.43</td>
<td>0.022</td>
<td>1.45</td>
<td>-0.0081</td>
<td>2.38</td>
<td>-0.52</td>
</tr>
<tr>
<td>Drainage Auction: Weak Bidder</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Original Data</td>
<td>0.11</td>
<td>0.017</td>
<td>0.07</td>
<td>0.0071</td>
<td>0.55</td>
<td>0.17</td>
</tr>
</tbody>
</table>

To address the concern of whether our parametric model is too restrictive, following Athey et al (2010) we implement a specification test due to Andrews (1997). The null hypothesis of the conditional Kolmogorov specification test is that conditional on a set of exogenous covariates, a set of endogenous variables is generated by a particular parametric distribution. In our case, for the drainage auctions, the endogenous variables are the bids and the exogenous covariates are the nearest wildcat sale characteristics: $(X: \text{ number of bidders}, \text{ winning bid})$ and $(N: \text{ number of drainage tract bidders})$. The parametric model is the Weibul
distribution function. In Andrews test one draws repeated bootstrap samples from the estimated distribution and compares these simulated data with the observed bids. We perform this test and compare the true distribution of bids and the simulated distribution of bids. We cannot reject the hypothesis that the two distribution are same. The corresponding Kolmogorov Smirnov has a $p-$ value of 0.65 and we cannot reject the null of equal distribution of estimated and actual bid distribution.

### 6.2 Signal Distribution

We first report the estimates from the drainage auctions in table 5. The strong bidder on an average has higher valuation for the drainage tracts. The difference is particularly striking in the high end of the valuations. The strong bidder’s high end valuation is much higher than that of the weak bidder. This may be influenced by the fact that the strong bidder: having resources next door will have a cost advantage in drilling the drainage tracts. In figure 1 we represent the estimated functions of the bid distribution of the drainage tracts. The estimated densities of $\tilde{v}_1$ and $\tilde{v}_0$ are depicted in figures 2 and 3. The mean, median and variance of the strong and weak bidders’ estimated valuations are reported in the table (all values are in US$1982 \$ in millions) below.

| Table 5: Summary Statistics of Estimated Valuation (in $ per acre) |
|------------------------|------------------------|
|                       | Strong Bidder | Weak Bidder |
| 1st Quintile          | 1237          | 1658        |
| Mean                  | 140400        | 24780       |
| Median                | 2596          | 3411        |
| 3rd Quintile          | 8653          | 7169        |

It appears that the density of the strong bidder has lower mean, median and variance than that of the weak bidders. The smoothed densities of the strong and weak bidders are plotted in figures 2 and 3. It is apparent that the weak bidder’s distribution of pseudo values are lower than that of the strong bidder. This is consistent with the valuation superiority and synergy in drainage tracts for the strong bidder. Note that the higher valuation is really prevalent in the right tail of the distribution (high end). This may reflect the fact that net of the drilling cost the strong bidder really wanted to win those tracts which has higher values. A Kolmogorv-Smirnov test and a bootstrap test based on 10000 bootstrap samples cannot reject the hypothesis of different distributions of pseudo values for strong and weak bidders. The test statistics are reported in the following table.

<table>
<thead>
<tr>
<th>Table 6 : Test of Equality of Distributions of Pseudo Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Null Hypothesis : Strong and Weak bidder’s Distributions are Equal</td>
</tr>
<tr>
<td>Kolmogorov-Smirnov Test (H$_0$:Strong Signal Dist$\neq$Weak Signal Distrbn)</td>
</tr>
<tr>
<td>Bootstrap Test (Sample size 10000) (H$_0$:Strong Signal$&lt;$Weak Signal)</td>
</tr>
<tr>
<td>Test of Equal Density(Sample size 10000) (H$_0$:Strong Signal Dist$=$Weak Signal Distrbn)</td>
</tr>
</tbody>
</table>
We therefore cannot reject the hypothesis that the strong bidder had better pseudo valuations of the tract than the weak bidder. A formal test of whether the strong bidder was more informed ex ante about his private valuation or not is given in next section. A bootstrap test also cannot accept the hypothesis whether the densities are equal ($p-value = 0.02$).

In figure 4 we depict $\hat{U}$ as estimated by equation (12) which also takes into account the possible dynamics.

An important measure of the mark-up is given by the information rent as measured by $\frac{\hat{v} - \mu}{\mu}$, where $\hat{v}$ is the estimated pseudo values. We report below the bidder’s information rents for drainage and wildcat auctions in the following table.

| Table 7 : Summary Statistics of Bidder’s Information Rent |
|-----------------|-----------------|-----------------|-----------------|
| Auction Type    | 1st Quintile    | Mean            | Median          | 3rd Quintile    |
| Wildcat Auction (no Dynamics) | 0.24 | 0.41 | 0.37 | 0.57 |
| Wildcat Auction (with Dynamics) | 0.17 | 0.37 | 0.32 | 0.54 |
| Drainage Auction: Strong Bidder | 0.28 | 0.48 | 0.47 | 0.63 |
| Drainage Auction: Weak Bidder | 0.16 | 0.36 | 0.31 | 0.58 |

Thus on an average the information rent is 48% for strong bidders and 36% for weak bidders in the drainage auctions. Thus on an average the government is capturing only 64% from the weak bidders willingness to pay and 52% from the strong bidder’s willingness to pay. The information rent in the wildcat auctions are quite high too. In the wildcat auctions, the bidders are willing to pay about 5% more on an average to take possible future information advantage. This is the value of information bidders perceive about the drainage tract while bidding for the wildcat tract.

There are some possible limitations. For example, we ignored the issue of joint bidding and only analyzed the cases for which there were no joint bidding for our sample. Joint bidding although very low for the sample period of the data used\textsuperscript{23}, the absence of joint bidding, however, could be an equilibrium phenomenon. Allowing for joint bidding, in a dynamic setting, would complicate the analysis by introducing dynamic information sharing, punishments etc.

7 Semi-parametric Test of Asymmetry

A major feature of the data is that bidders in the drainage auctions can be divided into two categories: strong and weak. A strong bidder is the bidder who receives a ‘better’ signal of his private value of oil stored relative to the weak bidder. Note that this was not imposed as an assumption while estimating the distributions of equilibrium ex ante values. It was only assumed that both bidders receive private signals and they are asymmetric. We present below a semi-parametric test that strong bidder knew more about his ex post valuation while bidding based on copula methods. The basic idea of the test is as follows: we

\textsuperscript{23}Joint bidding was very low and affected only less than 20% of the data for the sample period.
estimate the ‘pseudo values’ for different bidders $\tilde{v}_i^d$ and $\tilde{v}_0^d$, for the drainage auction model. We also have ex-post value of oil using the production and drilling cost data $\tilde{V}_1^d$ and $\tilde{V}_0^d$ for the strong and weak bidder respectively. Let the underlying joint distribution of the values with the signals be $F_1(v_1^d, V_1)$ and $F_0(v_0^d, V_0)$ for the strong and weak bidders respectively. A simple test of ‘strongness’ would be that strong bidder’s ex-ante signal $v_1$ is more ‘related’ with ex-post realization $V_1$ relative to that of the weak bidder’s $v_0$ with $V_0$. One test can be a test of simple correlation, however correlation is only a linear concept and may not capture the possible non-linearity of the dependence structure. We present below a new test procedure using copula methods. The details about copula can be found in the appendix.

### 7.1 Copulas

In this section, we briefly review the concept of copula originated in the statistics literature. The word ‘copula’ came from the word couple. Copulas are functions that ‘couples’ or joins multivariate distribution function to their one dimensional marginal distribution functions in such a way that it captures the entire dependence structure in the multivariate distribution.

Specifically, let $X$ and $Y$ be two random variables with joint distribution $F(X, Y)$ and continuous marginal distribution functions $F_X$ and $F_Y$ respectively. Then according to Sklar’s theorem there exists a unique copula function $C(v_1, v_2)$ such that $C(v_1, v_2) = F(F_X^{-1}(v_1), F_Y^{-1}(v_2))$ that connects $F(X, Y)$ to $F_X$ and $F_Y$ respectively. More discussion and specific examples on copulas can be found in Joe (’97) and Nelson (’99).

It is easy to see that copula is a map from $[0,1]^n$ to $[0,1]$. $C$ can be parametrized by a parameter $\alpha$, to have a specific functional form and be denoted by $C(v_1, v_2; \alpha)$. The most simple copula is the independent copula, given by $C(v_1, v_2) = v_1v_2 = F_1 \times F_2$. Thus if bidders’ ex-ante valuations are independent then the probability that strong and weak bidder’s ex-ante valuations are below their 50th percentile be given by, $C(v_1, v_0) = 0.5 \times 0.5 = 0.25$. A popular measure of dependence is simple correlation which may fail to capture the inherent non-linearity in the relationship.

Gaussian copula, Frank copula, Gumbel copula, Clayton copula are examples of parametric families of copulas. The parameter $\alpha$ measures the degree of dependence between the random variables $v_1$ and $v_2$.

Now by definition copula is a unique distribution function $C_1(F_1(s_i^d), F_V(V_1), \alpha_1)$ such that $F_1(s_i^d, V) = C_1(F_1(s_i^d), F_V(V_1), \alpha_1)$, similarly $C_0(F_0(v_0^d), F_V(V_0), \alpha_0) = F_0(v_0^d, V_0)$, where $\alpha_1$ is the copula parameters measuring the dependence (concordance) between $v_1^d$ and $V_1$, $\alpha_0$ is defined similarly. A test that $s_i^d$ is more related with $V_1$ than relative to $v_0^d$ with $V_0$ would be equivalent to test that $\alpha_1 > \alpha_0$. To test this we need to estimate $\alpha$’s. Estimation of $\alpha$ follows the following two step simple procedure.

**Step 1:** Estimate the marginal distribution of $\tilde{v}_i^d$, $i \in \{1, 0\}$ and that of $\tilde{V}_1$ non-parametrically.

**Step 2:** Using the estimated $\tilde{v}_0$, and $\tilde{v}_1$, estimate the copula parameter $\alpha_i$ by maximizing the likelihood function

$$
\max_{\alpha} L(\alpha) = \sum_{k=1}^{T} \log[c(\tilde{F}_i(\tilde{v}_i), \tilde{F}_V(\tilde{V}_i), \alpha_i)], \quad i = \{1, 2\}
$$

---

24 see appendix for details.

25 Some details about the properties of copulas are given in the appendix.

26 There are many well known copula functions to chose from, like Frank copula, Clayton copula, Gumbel copula etc.
The likelihood function can be formed using the respective density function of the underlying copula. For example the Frank copula has a distribution function given by,
\[
C(v_1, v_2) = \log_\alpha \left[ 1 + \frac{(\alpha^{v_1} - 1)(\alpha^{v_2} - 1)}{(\alpha - 1)} \right]
\]
with an associated density function
\[
c(v_1, v_2) = \log(\alpha^{-1}) \frac{\alpha^{v_1} \alpha^{v_2}}{1 - \alpha} \left[ 1 - \frac{(1 - \alpha^{v_1})(1 - \alpha^{v_2})}{(1 - \alpha)} \right]^{-2}, \text{ if } \alpha > 0, \alpha \neq 1
\]
\[= 1 \text{ if } \alpha = 1\]

More details about different forms of copulas and their properties can be found in the appendix.

7.2 Copula Based Asymmetry and Informativeness

In this subsection we explore the implication of the copula parameter on the asymmetry. We define below an informativeness structure which is consistent with the copula formulation of asymmetry.

**Definition 5** For two random variables, \(v_1' \geq v_1, v_2' \geq v_2\), informativeness is defined as to \(f(v_1', v_2)f(v_1, v_2') \leq f(v_1, v_2)f(v_1', v_2')\). Standard manipulations yield,
\[
\frac{f(v_2'|v_1')}{f(v_2'|v_1)} \leq \frac{f(v_2'|v_1)}{f(v_2'|v_1')}
\]
i.e., \(\frac{f(v_2'|v_1')}{f(v_2'|v_1)}\) is increasing in \(v_1\) : which is referred to as the monotone likelihood ratio property. Thus higher values of \(v_1\) implies higher values of \(v_1'\) more likely. When the density function \(f : V \rightarrow \mathbb{R}_+\) is strictly positive in the interior of \(V\) and twice continuously differentiable then it is easy to verify that \(f\) represent an informative structure defined above if and only if, for all \(i \neq j\),
\[
\frac{\partial^2}{\partial v_i \partial v_j} \ln f \geq 0
\]
i.e., \(f(.)\) is log-supermodular\(^{27}\).

Note the informativeness structure defined here is similar to the concept of affiliation defined in Milgrom & Weber (’82). It is easy to verify that for the Archimedean class of copulas\(^{28}\), \(\frac{\partial^2}{\partial v_i \partial v_j} \ln c \geq 0\) hence they represents the monotone likelihood ratio type of informative structure. Thus a higher copula parameter between the ex ante and ex post value means higher ex ante information about the ex post value. Frank, Clayton, Gumbel are different forms of Archimedean copulas.

\(^{27}\)A function \(g\) is supermodular if, \(g(v' \vee v'') + g(v' \wedge v'') \geq g(v') + g(v'')\). Hence, according to the definition of affiliation, \(f\) is affiliated if and only if \(\ln f\) is supermodular or \(f\) is log-supermodular. For more details see Milgrom & Weber (1982) and Krishna (2002)

\(^{28}\)Details on Archimedean copulas are in the appendix.
7.3 Copula Based Estimation: Results

Based on the estimation procedure described above we report the copula estimates ($\alpha_i$), its bootstrap standard errors and confidence intervals below.

Table 8: Test of Asymmetry Based on Copula

<table>
<thead>
<tr>
<th>Copula</th>
<th>$\alpha_i$</th>
<th>Bootstrap $t$-statistic</th>
<th>95% Confidence Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strong Bidder</td>
<td>0.39</td>
<td>2.5</td>
<td>[0.08, 0.70]</td>
</tr>
<tr>
<td>Weak Bidder</td>
<td>0.21</td>
<td>3.5</td>
<td>[0.093, 0.33]</td>
</tr>
</tbody>
</table>

Both the strong and weak bidders’ ex-ante pseudo values are significantly related to their ex-post values. This supports the assumption that both bidders’ signals are related to their ex-post valuation. Note the underlying pseudo values are estimated using equilibrium conditions and not observed. Hence standard asymptotic distributions of copula estimation procedure are not valid. We therefore use a dependent bootstrap procedure to evaluate the standard errors of our estimates. Based on the bootstrap $t$-statistic of 1.98, we also cannot reject the hypothesis that strong bidder’s ex-ante pseudo value is better associated to its ex-post values relative to the weak bidder.

8 Counterfactual Experiments of Choosing Different Mechanisms

In this section we perform counterfactual policy experiment of adopting different selling mechanisms and compare government’s revenue. We have estimated the distributions of ex-ante valuations of bidders in wildcat and drainage auctions. We use them to compute the bidding strategies under different selling strategies. We do not know what the optimal mechanism is in selling such kind of auctions. In the following subsections we only explore what revenues some other popular mechanisms would have yielded.

8.1 Experiment 1: Asymmetric Drainage Auctions

Note that the we know that bidders in drainage auctions are asymmetric. In previous section we used copula methods to quantify the degree of asymmetry in terms of the copula parameters. The weak bidder is obviously in a disadvantageous position, which is common knowledge before bidding and this may reduce competition. There are many mechanisms proposed in the theoretical literature to promote competition in such an environment. One such is called bidding credits. In such mechanisms weak bidder must pay only a fraction of his bids if he wins. Now what fractions should be a pertinent question. We use the degree of asymmetry measured as the copula parameters as the fraction. Hence here are the mechanisms: Let $\alpha = \frac{\alpha_0}{\alpha_1}$ be the degree of asymmetry where $\alpha_1$ is the degree of association of strong bidder’s ex-ante and ex-post valuations and $\alpha_0$ be the same for the weak bidder: arguably $\alpha < 1$, we consider the following two mechanisms of bidding credits.

**Mechanism 1**: Strong bidder bids $b_1$, weak bidder bids $b_0$;

If $b_1 > \frac{\alpha_0}{\alpha_1}b_0$ ⇒ Strong bidder wins and pays $b_1$ else weak bidder wins and pays $b_0$.

**Mechanism 2**: Strong bidder bids $b_1$, weak bidder bids $b_0$;
If $b_1 > b_0 \Rightarrow$ Strong bidder wins and pays $b_1$ else weak bidder wins and pays $\alpha b_0$.

**Mechanism 3:** Strong bidder bids $b_1$, weak bidder bids $b_0$;

If $b_1 > \frac{1}{\alpha} b_0 \Rightarrow$ Strong bidder wins and pays $b_1$ else weak bidder wins and pays $\alpha b_0$.

It is easy to see that the equilibrium bidding strategies under two mechanisms are given by,

**Table 9:** Counterfactual Experiments: Equilibrium Strategies

<table>
<thead>
<tr>
<th>Mechanisms</th>
<th>Strong Bidder</th>
<th>Weak Bidder</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mechanism 1</td>
<td>$b_1 = v_1 - \frac{1}{\alpha} \times haz_1,\alpha$</td>
<td>$b_0 = v_0 - \alpha \times haz_0,\alpha$</td>
</tr>
<tr>
<td>Mechanism 2</td>
<td>$b_1 = v_1 - haz_1$</td>
<td>$b_0 = \frac{1}{\alpha} \times (v_0 - \alpha \times haz_0)$</td>
</tr>
<tr>
<td>Mechanism 3</td>
<td>$b_1 = v_1 - \frac{1}{\alpha} haz_1,\alpha$</td>
<td>$b_0 = \frac{1}{\alpha} \times (v_0 - \alpha \times haz_0,\alpha)$</td>
</tr>
</tbody>
</table>

where $haz_i,\alpha$ is the inverse of the hazard of bid distribution faced by the $i^{th}$ bidder when bidder 1 bids $b_1$ and bidder 0 bids $\alpha b_0$. $haz_i$ is inverse of the hazard of bid distribution faced by the $i^{th}$ bidder when bidder 1 bids $b_1$ and bidder 0 bids $b_0$. This counterfactual experiment led to the weak bidder winning more of the auctions and also generating more revenue than the current format. Below we present the summary statistics of bidders’ information rent for these mechanisms.

**Table 10:** Comparisons of information Rents in Counterfactual Experiments

<table>
<thead>
<tr>
<th>Mechanisms</th>
<th>Strong Bidder</th>
<th>Weak Bidder</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1stQ</td>
<td>Median</td>
</tr>
<tr>
<td>Mechanism 1</td>
<td>0.23</td>
<td>0.43</td>
</tr>
<tr>
<td>Mechanism 2</td>
<td>0.28</td>
<td>0.49</td>
</tr>
<tr>
<td>Mechanism 3</td>
<td>0.24</td>
<td>0.43</td>
</tr>
<tr>
<td>Current Auction</td>
<td>0.28</td>
<td>0.47</td>
</tr>
</tbody>
</table>

Note that the information rents go down for strong bidders and it is higher for the weak bidder. Also the weak bidder is winning more auctions.

The above analysis only considers the drainage auctions and ignores the impact of drainage sales mechanism on wildcat auctions. Intuitively if a wildcat bidder knows that he is going to be treated differently in drainage auctions depending on whether he is strong or weak, the option value of being a strong bidder may go down. This would lead to reduction of the overbidding and excessive entry component in the wildcat sales. We consider the impact of different selling strategies of drainage tracts on wildcat bidding behaviors below.

### 8.2 Impact of Asymmetric Drainage Auction on Wildcat Bidding

Asymmetric Drainage auction like bidding credit as described above is aimed at giving the weak bidder some advantage and reduce the degree of asymmetry. This reduces the advantage of being a strong bidder and the attractiveness of being a strong goes down relatively. Looking at the equilibrium bidding equation in 11 this reduces the future rent or the value of information by reducing the gap between the strong and weak bidders.
bidders expected profits from drainage auction. This in turn reduces the overbidding amount in the wildcat tracts. Thus although government’s revenue goes up in the drainage auction, it goes down in the wildcat bidding and the overall impact on government’s revenue depends on the relative strengths of the two. We present below the impact of asymmetric drainage bidding on wildcat bidding.

Table 11: Impact of Asymmetric Drainage Auction on Wildcat Bidding

<table>
<thead>
<tr>
<th>Mechanisms</th>
<th>Winner’s information Rent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mechanism 1</td>
<td>0.55</td>
</tr>
<tr>
<td>Mechanism 2</td>
<td>0.57</td>
</tr>
<tr>
<td>Mechanism 3</td>
<td>0.54</td>
</tr>
<tr>
<td>Current Auction Format</td>
<td>0.57</td>
</tr>
</tbody>
</table>

However the overall impact was still positive and preliminary numbers suggests that the overall revenue of the government could go up by about 15% (see table 12 below) more if they had followed the asymmetric drainage auction. The wildcat auction is assumed to follow the current sealed bid format. Note that this computation depends crucially on the estimated degree of asymmetry. A higher (lower) \( \alpha \) may increase (decrease) the overall revenue via its impact on the value of information in the wildcat bidding.

Table 12: Impact of Counterfactuals on Overall Revenue

<table>
<thead>
<tr>
<th>Mechanisms</th>
<th>Increase in $ Billion</th>
<th>Increase as % of Current Revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mechanism 1</td>
<td>0.3</td>
<td>10.4%</td>
</tr>
<tr>
<td>Mechanism 2</td>
<td>-0.15</td>
<td>-5.19%</td>
</tr>
<tr>
<td>Mechanism 3</td>
<td>0.44</td>
<td>15.11%</td>
</tr>
</tbody>
</table>

8.3 Comparing Open vs. Sealed Bid Auction

One of the central issue in auction design is whether to use open vs sealed bid format. As shown in Maskin-Riley under asymmetry of bidders one format may dominate the other in terms of expected revenue depending on the type of asymmetry. If the asymmetry is defined in terms of distribution of stretches; sealed bidding tends to favor weaker bidder. In a sealed bid auction, strong bidders have greater incentive to shade their bids below their true valuation, so a weak bidder may win despite not having the highest valuation. Hence a sealed bid auction is better in terms of expected revenue if asymmetry is defined in terms of distribution shifts or stretches. (Proposition 4.3 & 4.4: Maskin-Riley(2000)). However open auction is superior in terms of revenue for shifts of probability mass points to the lower end of the distribution (Proposition 4.5 of Maskin-Riley(2000)). Note that we did not impose any of the three types of the asymmetry as considered by Maskin–Riley during estimation. The asymmetry amongst bidders in the drainage tract auctions are probably a mix of all these: strong bidder may have a stretched distribution as his valuation may be higher due to lower drilling costs, whereas weak bidder may have more mass points in the lower end points as it did not have access to as much information about the amount of oil stored as the strong bidder. Given the
dynamics the expected asymmetry and different auction format for the drainage auction may also have an impact on the revenues from the wildcat auction.

Under the assumption of independent private value drainage auctions, it is a weakly dominated strategy to bid their own signal and pay the second highest bid in an open auction. Given the dynamic nature of the game, we have couple of possible choices to compare: a sealed bid wildcat auction and an open drainage auction (alternative 1), an open wildcat and an open drainage auction (alternative 2), an open wildcat coupled with mechanism 1 as described above (alternative 3), an open wildcat coupled with mechanism 3 as described above (alternative 4) and an open wildcat coupled with mechanism 3 as described above (alternative 5). We report below the results based on the counterfactual experiments:

*Table 13: Open and Sealed Auction and Counterfactual Increase Revenue*

<table>
<thead>
<tr>
<th>Alternative Mechanisms</th>
<th>Drainage Auction Revenue</th>
<th>Wildcat Auction Revenue</th>
<th>Overall Revenue</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>% increase</td>
<td>Increase $Bn</td>
<td>% increase</td>
</tr>
<tr>
<td>Alternative 1</td>
<td>7%</td>
<td>$0.14Bn</td>
<td>0.16%</td>
</tr>
<tr>
<td>Alternative 2</td>
<td>7%</td>
<td>$0.14Bn</td>
<td>40%</td>
</tr>
<tr>
<td>Alternative 3</td>
<td>16%</td>
<td>$0.32Bn</td>
<td>35%</td>
</tr>
<tr>
<td>Alternative 4</td>
<td>-7%</td>
<td>- $0.14Bn</td>
<td>39%</td>
</tr>
<tr>
<td>Alternative 5</td>
<td>22%</td>
<td>$0.45Bn</td>
<td>38%</td>
</tr>
</tbody>
</table>

As evident from the above table, the open auction does not unequivocally dominate the expected revenue. Asymmetry plays a huge role here. For example, drainage auction under alternative 3 which is an asymmetric sealed bid auction generates more revenue than an open drainage auction (alternative 2). The highest possible revenue would come from alternative 5 which is an asymmetric sealed bid drainage auction and an open wildcat auction.

9 Conclusion

In this paper, we have formulated and estimated a dynamic auction model where asymmetry due to synergy is endogenous. The winner from the symmetric bidders in the first period auction gains a synergy toehold for the second period auction, making the second period a contest among asymmetric bidders. This makes the toehold asymmetry for the second unit endogenous and the first unit more valuable reflecting the option value of having a synergy for the second unit. Bidders therefore overbid for the first unit for this option value. We separately identified the premium paid for this option value of this information from the willingness to pay for the first unit. We structurally estimated the unobserved distribution of valuations for both the auctions, the degree of information asymmetry and the option value of acquiring costly information. We used copula techniques from statistics literature to quantify and semi-parametrically test the degree of asymmetry amongst bidders. Finally we conduct counterfactual experiment to find better selling strategies.
We apply this model to data from OCS oil tract auctions, where the first period (wildcat tract auction) winner has a synergy in valuation in the tract being sold next period (drainage tract auction) and becomes a strong bidder. The possibility of future asymmetry affects bidders’ bidding behavior in the first period. We estimate this dynamic model using a two step procedure. The estimates indicate that the federal government is only recovering 52% of the ‘strong’ buyers’ willingness to pay in the drainage sales. In the wildcat sales, we find that the bidders are willing to pay up to 5% more as an option premium to account for possible future synergy advantage. We use these estimated unobserved ex-ante valuations to conduct counterfactual experiments of different selling strategies for selling such goods and compare the government’s revenue.

We cannot reject the hypothesis that strong bidder in drainage auctions has a synergy effect in valuation than the weak bidder. As a first paper, we quantify this degree of asymmetry as a structural element of the model and found that the strong bidder synergy valuation is about 45% more than the weak bidder. The estimated marginal valuations are then used to first time conduct counterfactual experiment of alternate selling strategies in asymmetric drainage auctions. Since bidders are asymmetric, an asymmetric auction may yield higher revenue. We use the estimated degree of asymmetry to treat strong and weak bidders and compare three alternate mechanisms of ‘bidding credits’ where weak bidder is given some advantage in bidding according to this degree of asymmetry. The results suggests that the government could have earned on an average 22% more in drainage auctions following one of these mechanisms. Asymmetric treatments of bidders in drainage auctions should affect the bidding behavior in wildcat auctions too as being a strong bidder may not be as attractive as before. When the asymmetric drainage auctions are taken into account in the bidding behavior in wildcat auctions the total revenue earned goes down but is still higher than the current mechanism. Overall revenue could have been increase by approximately 27% if a combination of open wildcat auction was coupled with an asymmetric sealed bid auction and would have been worth about a $1 billion more. Notably open auction need not give higher revenue in an asymmetric drainage auction. The empirical methodology developed in this paper is easily applicable to analyze many other hitherto unexplored dynamic auction settings like defence procurements, optimal toeholds and takeovers etc. where information asymmetry is endogenous.
10 Appendix A: Testing for Common Values in the Drainage Auction

In this subsection we follow Haile Hong and Shum (2006) and Hendricks Pinkse and Porter (2003) to test whether the drainage auction was of common value type. Let $U_i$ be the valuation of bidder $i$. Let us define

$$v(x, x', n) = E[U_i | X_i = x, \max_{j \neq i} X_j = x']$$

as the conditional expectation of the bidder $i$’s expectation of his valuation conditional on he receiving a signal $x_i$ where the maximum of his rivals’ signal being $x'$ when there are $n$ bidders. $v(x, x, n)$ therefore is the expectation of a pivotal bidder. Bidders have private values if $E[U_i | X_1, X_2, ... X_n] = E[U_i | X_i]$ and bidders have common values iff $E[U_i | X_1, X_2, ... X_n]$ strictly increases in $X_j$ for $j \neq i$. Haile Hong and Shum has shown that under the assumptions of symmetry, affiliation, nondegenracy and exogenous participation, $v(x, x, n)$ is strictly decreasing in $n$ for all $x$ in a common value model. Intuitively, a rational bidder’s downward adjustment in his bid due to winner’s curse in a common value model depends on his number of opponents. They also noted that this theorem goes through even if bidders are asymmetric as long as at least one bidder participates in auctions with different number of competitors. As we argued earlier; bidders in drainage auction can be classified into two groups strong and weak and hence bidders are asymmetric in nature. (it holds in drainage auction). The principle for testing common value is to test whether $H_n(v) = \Pr(v(X_i, X_i; n \leq \tilde{v})$ is first order stochastically dominated by $H_{n+1}(\tilde{v})$.

Under the null hypothesis of private values, $H_n(v)$ should be invariant to $n$ while under the common values alternative,

$$H_n(v) < H_{n+1}(\tilde{v}), \quad \text{for all } \tilde{v}, n$$

To estimate

$$v(x, x, n) = E[U_i | X_i = \max_{j \neq i} X_j = x, n]$$

we follow Hendricks, Pinkse and Porter (2003) (HHP) to exploit the availability of ex-post values in our dataset. HHP showed that under the assumption of monotonicity of bidding strategies in the underlying signals, we can express

$$v(x, x, n) = E[U_i | X_i = \max_{j \neq i} X_j = x, n] = E[U_0 | B_i = \max_{j \neq i} X_j = b_i, n]$$

(13)

where $U_0$ is the ex-post value of oil for the drainage auctions. Following HHP we estimate the right hand side of the equation above by local linear estimation (see, e.g. Loader (1999)). We then use the empirical distribution of the right hand side above to test of equal distributions against the one sided alternative of stochastic dominance using Kolgomorov Smirnov test statistic. The private value component in the drainage tract can stem from various sources. Since the strong bidder already has resources in the neighborhood it can have lower costs of drilling relative to a weak bidder. The strong bidder may well also have synergy in

\footnote{pg6; Theorem 1, Haile Hong and Shum (2003)}
transporting oil from the same location from both wildcat and drainage tract and hence has a private value component in the revenue of oil. Hence we test for possible common value component in the distribution of the ex-post values of drilling costs, drilling revenue and profit (defined as drilling revenue minus drilling cost). We therefore test for stochastic dominance in all the three series. As discussed above the test involves estimating the ex-post distribution of \( v(x, x, n) \) as given in equation 13 for the tracts for the number of bidders 2, 3, 4 and 5 separately using local linear regression. For example, the estimated distribution of \( v(x, x, 2) \) (with number of bidders in a tract being 2) is given by \( H_2(\bar{v}) \). The test of common value therefore requires testing whether \( H_2(\bar{v}) \) is first order stochastically dominated by the distribution \( H_3(\bar{v}) \) or \( H_4(\bar{v}) \) and so on. We plot the estimated cumulative distribution functions of \( H_n(\bar{v}) \) \( \{ \text{for } n = 2, 3, 4, 5 \} \) for the drilling costs, drilling revenues and profits separately in the following figure. Evidently the distributions do not display an order of first order stochastic dominance in either of the three series rejecting the pure common value assumption for the drainage tracts.

In the table below we report the \( p- \) values of the Kolmogorov-Smirnov test of stochastic dominance where the null is the equality of distribution against the one sided alternative of first order stochastic dominance. Clearly we cannot accept the stochastic dominance of distribution of \( v(x, x, n) \) in increased order of number of bidders \( (n) \).

<table>
<thead>
<tr>
<th>No of Bidders in an Auction</th>
<th>Drilling Costs</th>
<th>Drilling Revenues</th>
<th>Drilling Profits</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 and 3 bidders</td>
<td>0.004</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>3 and 4 bidders</td>
<td>0.24</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>4 and 5 bidders</td>
<td>0.53</td>
<td>0.14</td>
<td>0.08</td>
</tr>
</tbody>
</table>

The graphs from the test of equality of distributions are given below.
Figure: Test of Common Values for Drainage Tracts
11 Appendix B: Proofs of the Theoretical Model

Proof of Lemma 1: First note that the utility function has the single crossing property and affiliation (independence) of signals guarantees that the distribution of valuations is log-supermodular and we assume that the assumptions required by McAdams('07) are satisfied for a pure strategy equilibrium to exist. Note that, bidder $i$ solves the following problem,

$$T_i(D) = \max_{b_i^d \geq 0} \{ (v_i(s_i^d) - b_i^d) \times \Pr(b_j(s_j^d) < b_i(s_i^d)), \ i, j = \{0, 1\}, \ i \neq j \}$$

where subscripts 1 and 0 denotes strong and weak bidder respectively, setting inverse of $b_i^d(s_i)$ to $\phi_i(b_i^d)$,

$$T_i(D) = \max_{b_i^d \geq 0} \{ (v_i(s_i^d) - b_i^d) \times \Pr(\phi_j(b_j^d) < \phi_j(b_i^d(s_i^d))), \ i, j = \{0, 1\}, \ i \neq j \}$$

$$T_i(D) = \max_{b_i^d \geq 0} \{ (v_i(s_i) - b_i^d) \times F_j(s_j < \phi_j(b_i^d(s_i^d))), \ i, j = \{0, 1\}, \ i \neq j \}$$

Taking logarithm and differentiating with respect to $b_i^d$, we get the first order conditions,

$$\frac{F_i'}{F_i} \phi_i'(b_i^d) = \frac{1}{v_i(s_1) - b_i^d}$$

Similarly,

$$\frac{F_i'}{F_i} \phi_i'(b_i^d) = \frac{1}{v_0(s_0) - b_i^d}$$

satisfying the relevant boundary conditions as described in the text. Hence the proof of the lemma.

Proof of Lemma 2: Suppose bidder $i$ decides to bids for the wildcat auction, in the simple case of two bidders he is solving the following

$$V_i(s_i^w) = \max_{b_i^w \geq 0} \{ (U(s_i^w, s_i^w) - b_i^w) \times \Pr(b_i^w < b_i^w|s_i^w) + \beta \sum_{j=1}^2 \Pr(j \text{ wins } W_A) \int T_i(D) dF(s_i^d, D) \}$$

where $T(1)$ is strong, $T(1)$ is weak) are as defined before. Equivalently, using the definitions of $M_{Y^w|s_i^w} = F_i^w(\phi(b_i^w))$, and differentiating with respect to $b_i^w$, we get the first order condition,

$$\{(U - b_i^w) \frac{f_{Y_i|s_i^w}(s_i^w|s_i^w)}{b_i^w} + \beta \frac{f_{Y_i|s_i^w}(s_i^w|s_i^w)}{b_i^w} \int [T(D = 1) dF(s_i^d, D = 1)] - \beta \frac{f_{Y_i|s_i^w}(s_i^w|s_i^w)}{b_i^w} \int [T(D = 0) dF(s_i^d, D = 0)] \} = 0$$

(15)

with the terminal condition, $b_W(s_i^w) = 0$. We have suppressed the arguments of $U(s_i^w, s_i^w).$ Equivalently,

$$(U - b_i^w) \frac{f_{Y_i|s_i^w}(s_i^w|s_i^w)}{b_i^w} + \beta \frac{f_{Y_i|s_i^w}(s_i^w|s_i^w)}{b_i^w} \int [T(D = 1) - T(D = 0)] dF(s_i^d, D)$$

$$= \{(U - b_i^w) + \beta \int [T(D = 1) - T(D = 0)] dF(s_i^d, D) \} \frac{f_{Y_i|s_i^w}(s_i^w|s_i^w)}{b_i^w} = 0$$

(16)

Equivalently,

$$\{(U - b_i^w) + \beta \int [T(D = 1) - T(D = 0)] dF(s_i^d, D) \} \frac{f_{Y_i|s_i^w}(s_i^w|s_i^w)}{b_i^w} = 0$$

$$\Rightarrow \ b_i^w = U - F_{Y_i|s_i^w}(s_i^w|s_i^w) \times \frac{b_i^w(s_i^w)}{m_{Y_i|s_i^w}(s_i^w|s_i^w)} + \beta \int [T(D = 1) - T(D = 0)] dF(s_i^d, D)$$

(17)
Note that, under the assumption of monotonic bidding strategies, we expressed the bid distribution function, 

\[ G_{w}^{*}(y|b_{i}^{w}) = F_{Y_{w}B_{w}}(\phi(B_{w})|\phi(b_{i}^{w})) \]

with the density function,

\[ g_{w}^{*}(y|b_{i}^{w}) = \frac{f(.)}{b'(s_{i}^{w})} \]

Substituting in the first order conditions, we get,

\[ b_{i}^{w} = U - \frac{G_{w}^{*}(b_{i}^{w}|b_{i}^{w})}{g_{w}^{*}(b_{i}^{w}|b_{i}^{w})} + \beta \int [T(1 \text{ is strong}) - T(1 \text{ is weak})]dF(s, D) \]  

(18)

11.1 First Order Conditions in the Wildcat Auction in terms of Distribution of Bids

Let us define \( G_{M}(.) = \Pr[\text{bidder } i \text{ wins by bidding } b_{i}] = G_{M}(b_{i}) = \Pi_{j \neq i} G_{j}(b_{i}) \) (i.e., all other bidder bids less than \( b_{i} \))

Hence the probability that bidder \( j \neq i \) wins when bidder \( i \) bids \( b_{i} \) is

\[ \int_{b_{i}}^{s_{j}} (\Pi_{k \neq i,j} G_{k}(b_{j}))g_{j}(b_{j})db_{j} \]

Finally using 4, differentiating both sides with respect to \( b_{i} \) we can write

\[ \frac{G_{M}(b_{i})}{g_{M}(b_{i})} = \frac{1}{\Sigma_{j \neq i} \frac{g_{j}(b_{j})}{g_{M}(b_{i})}} \]

For the wildcat auction, the bidders want to maximize (writing \( U(s_{i}^{w}, s_{-i}^{w}) = U \))

\[ \max_{b_{i}}(U_{i} - b_{i})G_{M_{i}}(b_{i}) + \beta \Sigma_{j} \Pr(j \text{ wins}|b_{i})T(j \text{ wins}) \]

\[ = \max_{b_{i}}(U_{i} - b_{i})G_{M_{i}}(b_{i}) + \beta T(i \text{ wins}) - \beta \Sigma_{j \neq i} \Pr(j \text{ wins}|b_{i})T(i \text{ wins}) \]

\[ + \beta \Sigma_{j \neq i} \Pr(j \text{ wins}|b_{i})T(j \text{ wins}) \]

since for all \( j \neq i, j \text{ wins} \) is equivalent to \( i \text{ loses} \).

Hence we can write the above equation as

\[ \max_{b_{i}}(U_{i} - b_{i})G_{M_{i}}(b_{i}) + \beta T(i \text{ wins}) - \beta \Sigma_{j \neq i} \Pr(j \text{ wins}|b_{i})[T(i \text{ wins}) - T(i \text{ loses})] \]

\[ = \max_{b_{i}} \left( U_{i} - b_{i} \right) G_{M_{i}}(b_{i}) + (A) \left( B \right) - \beta \Sigma_{j \neq i} \left[ \int_{b_{i}}^{s_{j}} (\Pi_{k \neq i,j} G_{k}(b_{j}))g_{j}(b_{j})db_{j} \right] \times \left( C \right) \]

Differentiating (A) above with respect to \( b_{i} \):

\[ -G_{M_{i}}(b_{i}) + (U_{i} - b_{i})g_{M_{i}}(b_{i}) \]

Differentiating (B) above with respect to \( b_{i} \) we get 0:
Differentiating (C) above with respect to $b_i$:

$$-\beta [T_1 - T_0] \times \left[ \frac{\partial}{\partial b_i} \int_{b_i}^{\bar{b}_i} (\Pi_{k \neq i,j} G_s(b_j)) g_i(b_j) db_j \right]$$

$$= -\beta [T_1 - T_0] \times \left[ -1 \times \Sigma_{j \neq i} \Pi_{k \neq i,j} G_s(b_j) g_i(b_j) \right]$$

$$= -\beta [T_1 - T_0] \times \left[ -1 \times \Sigma_{j \neq i} \Pi_{k \neq i} G_s(b_i) \times \frac{g_i(b_j)}{G_s(b_i)} \right]$$

$$= -\beta [T_1 - T_0] \times \left[ -1 \times \Sigma_{j \neq i} G_M(b_i) \times \frac{g_i(b_j)}{G_s(b_i)} \right] \quad (20)$$

Hence the first order condition can be written as

$$-G_M(b_i) + (U - b_i) g_M(b_i) + \beta [T_1 - T_0] \times [\Sigma_{j \neq i} G_M(b_i) \times \frac{g_i(b_j)}{G_s(b_i)}] = 0 \quad (21)$$

Equivalently

$$U = b_i + \frac{G_M(b_i)}{g_M(b_i)} \times \beta [T_1 - T_0] \times [\Sigma_{j \neq i} g_i(b_j)/G_s(b_i)] \quad (22)$$

Now since

$$\frac{G_M(b_i)}{g_M(b_i)} = \frac{1}{\Sigma_{j \neq i} g_i(b_j)/G_s(b_i)}$$

we have

$$U = b_i + \frac{G_M(b_i)}{g_M(b_i)} - \beta [T_1 - T_0] \quad (23)$$

11.2 Identification Proofs

Proof of Lemma 3

a) The proof is very similar to the proof of Campo, Perrigne & Vuong (‘02) to identify the private values. Let the joint distribution of bids from the asymmetric drainage auction be $G(.)$ with the support $[b, \bar{b}]^n$. Let there be two distributions of private values $F_d(.)$ and $\tilde{F}_d(.)$ leading to the same joint distribution of bids. Let $b^d_i(., F)$ and $\tilde{b}^d_i(., F)$ and $\tilde{b}^d_i(., F)$ be the strictly increasing Bayesian equilibrium strategies corresponding to $F_d(.)$ and $\tilde{F}_d(.)$ respectively. Therefore they satisfy the first order differential equations. Hence

$$F(v^d_1, v^d_0) = \Pr(\xi^d_1(b^d_1, G) \leq v^d_1, \xi^d_0(b^d_0, G) \leq v^d_0) = G(\xi^{d-1}_1(s^d_1, G), \xi^{d-1}_0(s^d_0, G))$$

$$\tilde{F}(v^d_1, v^d_0) = \Pr(\xi^d_1(b^d_1, G) \leq v^d_1, \xi^d_0(b^d_0, G) \leq v^d_0) = G(\xi^{d-1}_1(s^d_1, G), \xi^{d-1}_0(s^d_0, G))$$

Hence $F(v^d_1, v^d_0) = \tilde{F}(v^d_1, v^d_0)$ on their common support $[s^d_1, \bar{s}^d_1] \times [s^d_0, \bar{s}^d_0]$. Hence the asymmetric independent values of the drainage auction is identified.

Proof of Lemma 4

First note that by lemma (b) on the identification of the drainage auction, the third term is identified from data on drainage auctions. Now recall that for the two bidder case, we had,

$$G_{B_2|B_1}^w(y|b^w_1) = F^w(\phi(b^w_1))$$

(24)

Since $G_{B_2|B_1}^w(y|b^w_1)$ is observable from the observed data on bids and entry behavior, $F^w(\phi(b^w_1))$ is identified. Hence the expected common value component $U$ is identified using the first order condition.
12 Appendix C: Copulas

In this subsection we describe different forms of copulas and its implications on informativeness in the drainage auctions.

Definition 6 A copula is the distribution function of a random vector in $\mathbb{R}^n$ with uniform $(0,1)$—marginals. Alternatively a copula is a function $C: [0,1]^n \rightarrow [0,1]$ which has these properties:

1. $C(v_1, v_2, ..., v_n)$ is increasing in each component $v_i$.
2. $C(1, 1, v_i, ..., 1) = v_i$ for all $i \in \{1, ..., n\}$, $v_i \in [0,1]$.

Below, for simplicity, we discuss properties for $n = 2$.

Theorem 7 (Sklar’s) Let $F$ be a joint distribution function with marginals $F_1$ and $F_2$. Then there exists a copula $C$ such that for all $x, y \in \mathbb{R}$,

$$F(v_1, v_2) = C(F_1(v_1), F_2(v_2))$$

If $F_1$ and $F_2$ are continuous then $C$ is unique; otherwise $C$ is uniquely determined on $\text{Ran}F_1 \times \text{Ran}F_2$. Conversely, if $C$ is a copula and $F_1$ and $F_2$ are distribution functions, then the function $F(\cdot, \cdot)$ defined above is a joint distribution function with margins $F_1$ and $F_2$.

Theorem 8 If $(v_1, v_2)$ has copula $C$ and $W_1$ and $W_2$ are increasing continuous functions, then $(W_1(v_1), W_2(v_2))$ also has copula $C$.

Below the order in copulas is defined.

Definition 9 Let $C_1$ and $C_2$ be copulas, we say that $C_1$ is smaller than $C_2$ ($C_2 \succeq C_1$), if $C_1(v_1, v_2) \leq C_2(v_1, v_2)$.

Below we present a special group of copulas called Archimedean copulas. Archimedean copulas are those distribution functions $F(v_1, v_2)$, such that $F(v_1, v_2) = \tau^{-1}[\tau(v_1)+\tau(v_2)]$ for some convex, decreasing function $\tau$.

Copulas can thus be thought of as a form of the utility function. A bivariate copula depicts the relationships between two random variables, same as an utility function expresses the dependence between two goods, parametrically represented by the marginal rate of substitution. For example, consider the following two copulas, $C(v_1, v_2) = v_1 + v_2 - 1 = W(v_1, v_2)$ and $C(v_1, v_2) = \min(v_1, v_2) = M(v_1, v_2)$. It can be shown that for every copula $C$ and every $(v_1, v_2) \in [0,1]^2$; $W(v_1, v_2) \leq C(v_1, v_2) \leq M(v_1, v_2)$.

If we are thinking copulas as utility functions then $W(v_1, v_2)$ represents perfect substitutes and $M(v_1, v_2)$ represents perfect complements and all relationships between two goods falls in between these two. The bounds above is called the Frechet-Hoeffding bounds in copula literature (see Nelson(’99)). Since we have drawn the analogy of copulas to utility functions here is one nice property of Archimedean copulas. The Archimedean copulas also follow the monotone likelihood ratio property as used in the definition of informativeness in the text. In the statistics literature it is referred as total positivity. We give below some Archimedean copulas and their major properties.

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<table>
<thead>
<tr>
<th>Copula</th>
<th>$C(v_1, v_2)$</th>
<th>$\tau(v)$</th>
<th>Range of $\alpha$</th>
<th>Total Positivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frank</td>
<td>$\log_\alpha\left[1 + \frac{(\alpha^{-1} - 1)(\alpha^{-2} - 1)}{(\alpha - 1)}\right]$</td>
<td>$\log_\alpha\left(\frac{1-\alpha}{\alpha}\right)$</td>
<td>$[0, \infty)$</td>
<td>Yes</td>
</tr>
<tr>
<td>Clayton</td>
<td>$\left(v_1^{-\alpha} + v_2^{-\alpha} - 1\right)^{\frac{1}{\alpha}}$</td>
<td>$\frac{v_1^{-\alpha} - 1}{\alpha}$</td>
<td>$[0, \infty)$</td>
<td>Yes</td>
</tr>
<tr>
<td>Gumbel-Hugard</td>
<td>$\exp\left(-\left[(-\log v_1)^\alpha + (-\log v_2)^\alpha\right]\right)^{1/\alpha}$</td>
<td>$[-\log(v)]^\alpha$</td>
<td>$[1, \infty)$</td>
<td>Yes</td>
</tr>
</tbody>
</table>
Graphs

Figure 1: Bid Distribution: Drainage Tracts
Figure 2: Signal Distribution: Strong Bidder
Figure: 3: Signal Distribution: Weak Bidder
Figure: 4 Wildcat Auction Signal Distribution
References


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