Multidimensional Cheap Talk with Transparent Motives

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Multidimensional Cheap Talk

- When can make credible “tradeoffs” across dimensions?
- Do asymmetries across dimensions make tradeoffs harder?
- When does sender benefit from such tradeoffs?
- Does transparency of sender’s preferences make tradeoffs easier?
Crawford-Sobel model for one-dimensional cheap talk

- State of world is $\theta \in \Theta$
- Receiver’s ideal action $a = \theta$, e.g., $u^R = -(a - \theta)^2$
- Sender’s ideal action $a = \theta + b$, e.g., $u^S = -(a - (\theta + b))^2$

- Communicate by costless, non-verifiable messages
- Revealing $\theta$ exactly is not credible
- For $b$ sufficiently small exists informative equilibrium where Sender reveals partition of $\Theta$
- No informative cheap talk equilibrium for sufficiently large $b$
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Uncertainty along multiple dimensions?

- Sender knows $(\theta_1, \theta_2, \ldots, \theta_N) \in \Theta \subset \mathbb{R}^N$
- Receiver takes action $a_i$ on each issue

- Stock analyst - multiple stocks
- Lobbyist - multiple projects
- Salesperson - multiple goods
- News network - multiple issues
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Suppose Sender and Receiver payoffs additively separable and supermodular in \((a, \theta)\)

And environment (nearly) symmetric

Then “comparative cheap talk” is credible, informative even if strong incentive to exaggerate within each dimension

Trades off higher action on one dimension for lower action on another dimension

Supermodularity, separability, symmetry also used in multidimensional bargaining (Chakraborty and Harbaugh, Econ. Letters, 2003) and multi-object auctions (Chakraborty, Gupta, and Harbaugh, RAND, 2006)
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Existence with large asymmetries?

- The sender might care about one issue more than another.
- Or the distributions of $\theta_i$ might not be symmetric.
- Chakraborty and Harbaugh (2007) show that results are robust to small asymmetries.
- And provide some particular cases where large asymmetries are ok, e.g., $U = \theta_1 a_1 + \theta_2 a_2$.
- But multidimensional cheap talk with asymmetries mostly open question.
- Levy and Razin (Econometrica, 2007) show that asymmetries can be problem.
Sender preferences are state-independent

- Sender has private information about state $\theta$
- Receiver’s ideal action $a$ depends on $\theta$
- Sender’s preferences do not depend directly on $\theta$ – only on Receiver’s actions $a$
- Full transparency: Sender’s preferences over actions (“motives”) are common knowledge (“transparent”)
Model assumptions

- Sender knows $\theta = (\theta_1, \ldots, \theta_N) \in \Theta$ where $\Theta$ is compact, convex subset of $\mathbb{R}^N$
- Distribution $F$ with density $f$ has full support on $\Theta$
- Sender sends message $m \in M$
- Receiver action $a = (a_1, \ldots, a_N) = (E[\theta_1|m], \ldots, E[\theta_N|m])$
- Sender’s preferences $U(a)$ continuous in $a_i$

- Note that sender does not care directly about $\theta$
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Equilibrium

- A hyperplane $h$ divides $\Theta$ into halfspaces
- Message $m$ indicates in which halfspace of $\Theta$ is $\theta$
- PBE if $h$ is such that sender has no incentive to misreport the halfspace
- Note equilibrium is a convex partition of $\Theta$
Example: Media Bias

- Seriousness of two scandals $\theta_1$ and $\theta_2$, uniform i.i.d. on $[0, 1]$
- Network wants to push scandals to increase viewership
- $U = a_1 + a_2$ so biased within but not across dimensions
- Comparative cheap talk equilibrium
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  \[ U = 4a_1 + a_2 \]

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- Actions continuously move around point
- Double back on each other when orientation of line is reversed
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\[ h \]
\[ a^- \]
\[ a^+ \]
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- Two actions not the same - equilibrium is influential
- Each action taken with positive probability - equilibrium is informative
- If \( c = E[\theta] \) then for any distribution each action taken with probability 1/3 or higher
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Borsuk-Ulam Theorem

- For every continuous, odd mapping $G : S^{N-1} \to \mathbb{R}^{N-1}$ there exists a point $s \in S^{N-1}$ satisfying $G(s) = 0$.
- [Odd mapping: $G(s) = -G(-s)$]

- Let $N = 2$
- $s \in S$ is orientation of line $h$ dividing plane into two regions, $R^+$ and $R^-$
- $a^+(s) = (E[\theta_1|\theta \in R^+(s)], E[\theta_2|\theta \in R^+(s)])$
- $a^-(s) = (E[\theta_1|\theta \in R^-(s)], E[\theta_2|\theta \in R^-(s)])$
- $G(s) = U(a^+(s)) - U(a^-(s))$
- $G$ continuous since expectations for regions continuous
- $G$ odd since
  $U(a^+(s)) - U(a^-(s)) = U(a^-(s)) - U(a^+(s))$
- So there exists $s$ such that $G(s) = 0$
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Existence of Informative Cheap Talk

Theorem

An influential cheap talk equilibrium exists for all continuous $U$.

- Equilibrium is partitional
  - Partitions $\Theta$ into two convex regions so influential

- Equilibrium is informative
  - If $c = E[\theta]$, then each message sent with probability at least $1/(N+1)$
  - If density $f$ logconcave then probability at least $1/e$
  - If $N > 2$ can make each message probability $1/2$
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Sender and Receiver payoffs

- Receiver always better off from communication
- Sender better off if utility function quasiconvex
- And worse off if utility function quasiconcave
Example: Private Value Auction

- Four buyers:
  - $v_1 = a_1$
  - $v_2 = (1/4)a_1 + (3/4)a_2$
  - $v_3 = (3/4)a_1 + (1/4)a_2$
  - $v_4 = a_2$

- $U = \text{2nd max}\{v_j\}$
- $U$ quasiconvex so cheap talk helps seller
- True generally for $N \geq 4$ bidders
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Example: Voting

- Defense lawyer
- Two blocs of jurors
- $a_i$ is probability of juror bloc $i$ voting acquittal
- $U = p(a)$ is probability of acquittal
- Need both blocs: $p(a) = a_1a_2$ (quasiconcave)
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Example: negative advertising

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- And again, and again....
- So can reveal arbitrarily fine slice of plane
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Very Informative Cheap Talk with Linear Preferences

\[ U(a; t) = \lambda_1(t)a_1 + \ldots + \lambda_N(t)a_N \]

**Theorem**

A \( k \)-message cheap talk equilibrium exists for all linear \( U \) and all \( k \). As \( k \) becomes large, the sender can reveal almost all information in \( N - 1 \) dimensions.

- Sender and receiver have no conflict on \( N - 1 \) dimensions
- With linear preferences can credibly reveal all this info
- With nonlinear preferences can use mixed messages
- Also \( k \)-message equilibrium, also more informative as \( k \) increases, but noisy so not all info revealed
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Robustness

- Suppose Sender has $T > 1$ different types (preference orderings). Results still hold if $N > T$.
- Suppose some small uncertainty about Sender’s type - still holds with mild conditions
- Suppose Sender has Euclidean preferences (not transparent!) - still holds for $\varepsilon > 0$ cost of lying if biases large enough
Euclidean Preferences

\[ U(a; \theta) = -d(a, \tau(\theta)) = -\left( \sum_{i=1}^{N} (a_i - (\theta_i + b_i))^2 \right)^{1/2} \]

- \( d(\cdot, \cdot) \) is the Euclidean distance function
- \( b = (b_1, ..., b_N) \in \mathbb{R}^N \) is bias in each dimension
- \( \tau(\theta) = (\theta_1 + b_1, ..., \theta_N + b_N) \) is sender's ideal point
- Interested in arbitrarily large biases: \( b = (\rho_1 B, ..., \rho_N B) \) for \( B \geq 0 \) and \( \rho = (\rho_1, ..., \rho_N) \neq 0 \)
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Convergence to Linear Preferences

- For small $B$ indifference curves very curved
- As $B$ increases become straighter
- And become more similar for each $\theta$
- Converge uniformly to linear preferences as $B \to \infty$
- So always cheap talk equilibrium in limit

$$U = - ((a_1 - (\theta_1 + A B))^2 + (a_2 - (\theta_2 + B))^2)^{1/2}$$
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Convergence to Linear Preferences

- For small $B$ indifference curves very curved
- As $B$ increases become straighter
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Epsilon Cheap Talk

- Suppose there is some $\varepsilon > 0$ cost to lying
- Define “$\varepsilon$—cheap talk equilibrium for large biases” as case where for any $\varepsilon > 0$ there is some $\bar{B}$ such that for all $B > \bar{B}$ the gain from deviating from candidate equilibrium is less than $\varepsilon$

**Theorem**

An announcement strategy is a $k$-message $\varepsilon$—cheap talk equilibrium for Euclidean $U$ with large $B$ if and only if it is a cheap talk equilibrium for the limiting linear $U$ with $\rho = \lambda$. 
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**Theorem**

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Large biases help or hurt communication?

- Levy and Razin (2007) find asymmetries imply no influential cheap talk equilibrium exists for sufficiently large biases.
- How reconcile with our result that such equilibria always exist in limit?
- Asymmetries imply that there is some incentive to lie for sufficiently large biases.
- But any such incentive to lie goes to zero as biases become larger.
Other transparency models

- We model transparency as common knowledge of sender’s preferences over actions.
- Not same as just knowing sender’s biases.
- In canonical CS model still uncertainty over sender’s preferences if receiver knows $b$ because preferences are state-dependent.
- When transparency is modeled as receiver knowing $b$, transparency helps or hurts depending on size of $b$ that is revealed (Morgan and Stocken, 2003; Dimitrakas and Sarafidas, 2004).
- Li and Madaras (2006) find transparency can hurt when uncertainty over sign of large $b$. 
Conclusion

- Sufficient “transparency” implies existence of cheap talk equilibria
- Asymmetries not a barrier when preferences are transparent
- Simple conditions for when gain/lose from communication
- Seems applicable to many environments
- Borsuk-Ulam Theorem can be useful in economics - only previous use by Simmons and Su (2003) on “fair division”?