A Conflict Theory of Voting*

Jeremy Petranka

Created: September 2008, Latest Revision: March 2010

Abstract

Research in the behavioral psychology of voting has found that voters tend to be poorly informed, highly responsive to candidate personality, and follow a “fast and frugal” heuristic. This paper analyzes optimal candidate strategies in a two-party election in which voters are assumed to behave according to these traits. Under this assumption, candidates face a trade-off between appealing to a broader base and being overly ambiguous in their policy stances. A decrease in the cost of ambiguity within this model offers a parsimonious justification for the increase in voter independence, candidate ambiguity, and party politics that empirical studies have revealed over the last five decades. I additionally argue a decrease in the cost of ambiguity is a natural result of the primary system, campaign finance reform, and changing media environment.

This paper analyzes the outcome of a two-party election in which voters utilize a voting heuristic based on the findings of behavioral psychologists. My proposed voting behavior is similar in nature to Aragones and Postlewaite (2002), in which voters have an “intensity of preferences” such that a voter displays risk-loving behavior when his ideal issue point is offered as part of a campaign platform. This desire to hear a particular issue stance introduces a role for ambiguity I incorporate within my model. This paper relaxes the assumptions of rationality and instead models voters behaviorally according to a decision process based on the findings of Lau and Redlawsk (2006).1 Each voter is assumed to have extreme intensity concerning his ideal issue point, requiring a candidate to offer his ideal issue, even if the candidate is highly ambiguous in his campaign platform. If both candidates offer the ideal issue, then the voter probabilistically makes his decision utilizing a contest success function based on the candidates’ ambiguity levels and personality traits. This tradeoff between appealing to a broader base and appearing disingenuous is the core tension in the model.

I present results relating three recent trends in American presidential politics to a single parameter in the model. In particular, as ambiguity becomes less costly for a candidate, the model predicts an increase in the number of independent voters, an increase in the equilibrium level of candidate ambiguity, and an increase in the partisanship of political parties. All three trends have been observed in recent decades, and I argue that the primary system, campaign finance reform, and changing media environment have revealed over the last five decades. 1 Additionally argue a decrease in the cost of ambiguity is a natural result of the primary system, campaign finance reform, and changing media environment.

---

*I thank Gary Biglaiser, Helen Tauchen, Peter Norman, Sérgio Parreiras, and Matthew Baker for their valuable comments and suggestions.

1It should be noted that I am agnostic concerning voter rationality. If the proposed decision process reduces information search costs and mental conflict, then use of such a heuristic can be consistent with rational optimizing behavior.
media environment are consistent with this phenomenon.

Existing literature offers potential justifications for each of these trends. Rabinowitz and Macdonald (1989), for instance, predicts that candidates will become more partisan based on voters having a preference for issue intensity as well as direction. Candidate ambiguity can be explained with probabilistic voting models such as those detailed in Coughlin (1992). In a deterministic framework, Alesina and Cukierman (1990) and Aragones and Neeman (2000) have shown that candidates will choose to be ambiguous if they care about more than merely winning the election. Callander and Wilson (2008) have shown that candidates will respond to context-dependent voters by giving ambiguous policy stances. This paper aims to add to the literature by showing that all three political trends can be explained simultaneously following a decrease in the cost of ambiguity. In addition, I offer an explanation for the specific timing of these trends.

This paper is organized as follows. In Section 1, I offer evidence indicating that voters do not systematically behave rationally. In particular, I discuss the use of mental heuristics and emotional cues in the formation of voting decisions. In Section 2, I describe specific voting characteristics determined by Lau and Redlawsk (2006) and posit a formal behavioral model designed to incorporate these findings. In Section 3, I incorporate this model into a modified spatial game of campaign strategy between two candidates. In Section 4, I solve for the optimal strategies of the candidates and in Section 5, I show how these strategies can help explain recent trends in American presidential elections. Section 6 offers a measurement of the efficacy of the proposed model and compares it to the party heuristic. Section 7 discusses the multidimensional formulation of the game and Section 8 concludes.

1 Rationality and the American Voter

1.1 Heuristics

Considerable amounts of research by Converse (1964), Bennett (1996), Neuman (1986) and others have shown that the American voter’s political knowledge is, on average, extremely low. Baum (2005), for instance, finds that according to the 2000 American National Election Study (ANES), 60% of respondents who indicated that they follow government and public affairs “hardly at all” or “only now and then” claimed to have voted. In an aggregation of 2000 survey questions asked over the last 50 years concerning questions one might expect an informed citizen to know, Carpini and Keeter (1996) find only 40% of the questions for which over half the population can answer correctly.\(^2\) As a stylized example, Carpini (1999) cites a 1992 report by the Center for the Study of Communication at the University of Massachusetts that found while 86% of a random sample of likely voters knew the Bush family dog was named Millie and 89% knew that Murphy Brown was the TV character criticized by Dan Quayle, only 15% knew that both candidates favored the death penalty and only 5% knew that both had proposed cuts in the capital gains tax.

\(^2\)Carpini (1999) notes that of the questions that cannot be answered by over half the population are “definitions of key terms such as liberal, conservative, primary elections, or the bill of rights; knowledge of many individual and collective rights guaranteed by the Constitution;...candidate and party stands on many important issues of the day; key social conditions such as the unemployment rate or the percentage of the public living in poverty or without health insurance; how much of the federal budget is spent on defense, foreign aid, or social welfare; and so on”.


This trend indicates that voters might deviate from fully informed issue-based rational behavior. One of the accepted tenants of behavioral psychology regarding deviations from rationality is that individuals are systematically unable to perform exceedingly complex mental calculations. As such, in “hard” choice environments, such as the comparison of ideological issues present in national elections, humans use “fast and frugal” mental heuristics. As demonstrated by Gigerenzer et al. (2000), these heuristics allow individuals to make a choice with minimal information requirements that can approach, or even exceed, the accuracy of modern computational techniques. Baron (1990), Hogarth (1987), and Payne et al. (1993) have all shown that when making a choice, individuals try to avoid trade-offs and instead focus on a single good reason to select an option. Despite being seemingly uninformed, research has shown that citizens generally make reasonable policy decisions when using heuristics. In a study of 45 policy issues, Althaus (1998) estimates that 80% of the sampled collective preferences were identical to those made by a highly-informed populace.

To further illustrate the use of noncompensatory heuristics, a study published in the Journal of Cognitive Neuroscience, Westen et al. (2004), performed MRI brain-imaging on “committed” Republicans and Democrats. During the study, each subject was exposed to a series of slides that demonstrated inconsistent statements made by his candidate of choice as well as the opposing candidate. When exposed to contradictory statements by his candidate, brain regions were activated which were involved in implicit emotion regulation and the elicitation of negative emotion. These areas were distinct from the areas of the brain involved in cold-reasoning and conscious emotion regulation. In other words, when exposed to contrary information, brain regions linked to “rational thought” were not used, preventing the information from entering the subject’s decision calculus. This is consistent with the use of heuristics, in which information received after a choice is made is ignored or rationalized.

This nonrational treatment of information can also be seen in the American National Election Studies data. Since 1976, all surveyed individuals who could correctly identify the Democratic candidate as weakly more liberal than the Republican candidate were asked to place each candidate on a Liberal/Conservative scale of 1-7. Individuals who voted for a candidate placed that candidate .05 closer to themselves than the average surveyed individual placed the candidate. More importantly, they placed the opposing candidate .91 points farther from themselves. This is in contract to nonvoters, who placed the Democratic candidate .32 points farther from themselves and the Republican candidate .28 points farther. As compared to nonvoters, voters appear to be processing information differently as predicted with the use of a fast and frugal heuristic.

1.2 Candidate Personality

Another deviation from fully informed issue-based rationality is the role of candidate personality on voter affect. Affect is defined as the emotional “feeling” an individual develops toward a specific choice. It is distinctly separate from rational intelligence, both conceptually and neurobiologically. In the political realm, familiarity, perceived truthworthiness, overall image, enthusiasm, and other “non-issue” traits have all been found to play an important role in voter behavior through their manipulation of voter emotions. Kinder and Abelson (1981), Marcus et al. (2000), Rosenberg and McCafferty (1987), and Rosenberg et al. (1986) all show that candidate image influences voter choice. In presidential elections, the effect is even more pronounced, with the ANES finding that
the candidate who rates higher in the public’s “Average Feeling Thermometer”\(^3\) has not lost a presidential election since the question was created in 1968. Miller et al. (1986) even finds that voter perception of candidates relies more on personality than issue concerns. In addition, these perceptions are not superficial, but reflect performance-based criteria such as integrity and reliability.

2 A Dynamic Voting Heuristic

In an effort to determine what heuristics voters use, Richard R. Lau and David P. Redlawsk (Lau and Redlawsk, 2006) created an extensive experimental environment that mimicked the dynamic nature of presidential politics. In particular, individuals were faced with general information cues (such as “Candidate A’s Stand on Taxes/Tax Reform”) that they could select to learn the specific information contained within. As with a real election, spending the time to focus on a specific cue meant possibly ignoring another piece of information. Through this process and extensive pre/post-trial surveys, they were able to make conclusions as to how voters make their decisions. Among other findings, they determined:

**Finding 1:** In selecting the voter’s preferred candidate, memory and affective perception both play a role. In other words, specific information cues as well as overall perception are used as a basis for choice. In selecting the voter’s rejected candidate, however, affective memory does not play a significant role. The voter seemingly uses specific information to reject the candidate, and does not retain an “overall feeling”.

**Finding 2:** Voters recall more total information concerning their preferred candidate.

**Finding 3:** Of the information recalled, voters recall a higher percentage of positive information for their preferred candidate.

In addition, the study was able to offer a metric for the number of voters using various decision strategies. The authors found that approximately 25% of voters were using a party-heuristic and 15% of voters were using a rational decision strategy. Over 50% of voters were using a single-issue or noncompensatory heuristic, which are analytically equivalent within the model utilized in this paper.

2.1 “Hear What You Want To Hear” Voter Heuristic

I propose a voter heuristic inspired by Lau and Redlawsk’s findings. Like Downs, I assume each voter has an ideal point at which they would like a candidate to locate.\(^4\) Unlike Downs, I assume a voter uses the following heuristic in choosing his preferred candidate:

---

\(^3\)Question VCF0201 in the Cumulative Data File. 1976 question text: “We’d also like to get your feelings about some groups in American society. When I read the name of a group, we’d like you to rate it with what we call a feeling thermometer. Ratings between 50 degrees-100 degrees mean that you feel favorably and warm toward the group; ratings between 0 and 50 degrees mean that you don’t feel favorably toward the group and that you don’t care too much for that group.”

\(^4\)Alternatively, I could assume each voter has an interval for which he would consider a candidate acceptable. The main results of the paper do not change.
1. A voter receives enough information to determine which candidates put positive probability on his ideal issue point.

2. If neither candidate puts positive probability on the voter's ideal issue point, he does not vote.

3. If only one of the candidates puts positive probability on the voter's ideal issue point, he votes for that candidate.

4. If both candidates put positive probability on the voter's ideal issue point, he votes for each candidate as probabilistically determined by a function incorporating each candidate's ambiguity and personality characteristics.

Note that under this heuristic, the voter is displaying a form of lexicographic preferences over his ideal issue point. Particularly, regardless of any other criteria, a candidate will be selected only if his issue platform contains the voter's ideal point. This coincides with Luce et al. (2001), who find "the major form of emotion-focused coping relevant to decision processing is a desire to avoid particularly distressing explicit tradeoffs between attributes. That is, if tradeoff difficulty is elicited by the perception that valued goals must be given up, then the decision maker should try...to avoid these sacrifices altogether."

Also under this heuristic, if a candidate is rejected in Step 2, a particular issue stance will cause the rejection. The voter will not make an emotional judgment, in that once the decision is reached, no additional information is required. If both candidates continue to Step 3, we expect the voter to make a more affective decision, in that non-issue related cues are helping to form the vote. Note that memory will still play a role, as expected, since each candidate can only reach Step 3 if their memory-related issue cues are in line with the voter. These predictions are consistent with Lau and Redlawsk's Finding 1.

In addition, because this heuristic is noncompensatory, the voter will be expected to be exposed to contradictory information after his decision is made. As such, we would expect internal justifications to occur of the sort seen by Westen et al. (2004). In particular, "good" information will be retained, while "bad" information will be justified or neglected. This coincides with Lau and Redlawsk's Findings 2 and 3 which state that voters will recall more information concerning the preferred candidate, and the information will be of a more positive nature.

It is important to note that while Step 1 might seem informationally intensive, it is in fact a fast and frugal heuristic. Voters generally have a limited number of key issues on which they base their vote, restricting the number of issues on which they need information. In addition, voters do not need to analyze all policy positions by a candidate to determine if their ideal point is included. Instead, they need only look at the policy stances the candidate has emphatically rejected to determine if their ideal point is excluded. Given that the President's authority exists in his veto power, the only unambiguous statements a candidate can make concerning potential laws involves bills he will NOT enact. For instance, according to his website, John McCain's stance on abortion is:

John McCain believes Roe v. Wade is a flawed decision that must be overturned, and as president he will nominate judges who understand that courts should not be in the business of legislating from the bench. Constitutional balance would be restored by the reversal of Roe v. Wade, returning the abortion question to the individual states. The difficult issue of abortion should not be decided by judicial fiat.
For pro-life voters, it is unclear whether he would support a federal ban on abortion, a federal ban on third-trimester abortions, etc. For pro-choice voters, however, it is unambiguous that he would support the overturn of Roe v. Wade, a critical issue.

It should be noted that this heuristic is essentially a formalization of the observation that “people hear what they want to hear”. It is also important to realize that this heuristic is not proposed as a universal voting heuristic. It is specific to the environment of modern American presidential elections in which “difficult” issues are addressed in a two-party environment. I will later show that the proposed heuristic is ecologically rational\(^5\) in the American presidential election environment, but this might not be the case in a different election format, or even in a different period in American politics. Pre-1960, for instance, an even faster and more frugal heuristic of voting along party lines proved highly accurate. I will later discuss some of the changes that have occurred since 1960 that have led to the partial abandonment of the party heuristic.

3 Candidate Campaign Game

Having established the voter heuristic, I now turn to the actual game played by the candidates, \(D\) and \(R\). These politicians, only concerned with winning, compete with each other in an election. As is common in spatial models, I assume a unidimensional issue space on the real line, \(\mathbb{R}\). As will be discussed later, unidimensionality is not required under the assumption that ambiguity in one issue does not relate to ambiguity in another. A uniformly distributed continuum of voters exists.

The sequence of the game is as follows.

1. Nature selects the most liberal possible position of the Democratic candidate, \(\bar{D}_L\).\(^6\) This can be viewed as an indication of past candidate voting bias, the current ideology of the Democratic party, or the natural checks-and-balances inherent in American politics. Simultaneously, nature selects the most conservative possible position of the Republican candidate, \(\bar{R}_C > \bar{D}_L\). Both values are known by all players.

2. The Democratic candidate selects her most conservative campaign position, \(D_C \in [\bar{D}_L, \bar{R}_C]\). This will establish her level of campaign ambiguity, \(D_C - \bar{D}_L\). Simultaneously, the Republican candidate selects his most liberal campaign position, \(R_L\). This will establish his level of campaign ambiguity, \(\bar{R}_C - R_L\). Note I explicitly assume a candidate’s strategy is a convex interval. This avoids dubious campaign strategies such as simultaneously claiming to be extremely pro-life, extremely pro-choice, but against abortions in the case of rape and incest. I explicitly focus on pure strategies.

3. Non-strategic voters select their candidate of choice using the proposed voter heuristic. In the scenario where both candidates offer voters their ideal points, support will be divided according to a voter contest success function.

4. The winner will be probabilistically selected according to an increasing, twice-continuously differentiable function mapping the percentage of voter support to the probability of winning.

\(^5\)Ecological rationality is defined by Gigerenzer et al. (2000) as “the structure of environments, the structure of heuristics, and the match between them”.

\(^6\)I use the labels “Democratic” and “Republican” for ease of exposition. No connotations should be inferred.
\( \nu : [0, 1] \rightarrow [0, 1], \nu(\cdot)' > 0 \). This function incorporates the uncertainty inherent to voter turnout, the idiosyncrasies of the electoral college, etc.\(^7\)

Since the function mapping voter support to probability of winning is increasing, I will use the terms “support” and “votes” interchangeably.

### 3.0.1 Voter Contest Success Function

To determine each candidate’s percentage of voter support when policies overlap, I utilize the concept of a contest success function as detailed in Hirshleifer (2005) and is common in the rent-seeking literature.\(^8\) In particular, I assume that each candidate, \( i \), with personal affect, \( \alpha_i \), and ambiguity, \( A_i \), wins the following share of contested voters:

\[
s_i(A_i) = \frac{\alpha_i}{\sum_i \alpha_i} \tag{1}
\]

Making the bipartisan share of contested voter support in the model:

\[
s_D \equiv \frac{\alpha_D}{\alpha_D + \alpha_R \left( \frac{D_C - D_L}{R_C - R_L} \right)^m} \tag{2}
\]

\[
s_R \equiv \frac{\alpha_R}{\alpha_R + \alpha_D \left( \frac{R_C - R_L}{D_C - D_L} \right)^m} \tag{3}
\]

with

\( \alpha_i > 0 \): The affect potential of candidate \( i \). This variable indicates the ability of the candidate to express confidence, competency, positive emotional appeal, and to garner a sense of trust in the voters.

\( m > 0 \): The mass effect parameter. As \( m \) decreases, the less detrimental ambiguity becomes.

Intuitively, \( \alpha_D \) and \( \alpha_R \) can be described as follows. Assume each candidate expresses an identical level of ambiguity. \( \frac{\alpha_D}{\alpha_D + \alpha_R} \) will represent the share of the contested votes won by Candidate \( D \). Note that if \( \alpha_D > \alpha_R \), Candidate \( D \) will win more than 50% of the contested votes on strength of personality. If \( \alpha_D = 2 \) and \( \alpha_R = 1 \), for instance, Candidate \( D \) will win \( 2/3 \) of the contested votes.

The mass effect parameter, \( m \), can be interpreted as the cost of being more ambiguous than the other candidate. To illustrate this intuition, assume the two candidates are competing for voters whose ideologies range from 1 to 7, as is standard in the American National Election Survey and is

\(^7\)\( \nu \) is included in the model for two reasons. First, it removes a discontinuity in the candidates’ best response functions. In particular, if candidates care only about winning and voter support has a one-to-one mapping with total votes, then a discontinuity will exist at the ambiguity level that ensures 50% of the vote. This will result in strategies in which the winning candidate is not concerned with maximizing her support, and the losing candidate will behave entirely arbitrarily. Neither seems to accurately represent American presidential politics. Second, the 2000 presidential elections in which the more popular candidate lost implies a probabilistic function is a reasonable assumption.

\(^8\)For a microfoundational treatment of this specific functional form, please refer to Petranka (2010).
demonstrated in Figure 1(a). In addition, assume these candidates have identical affect potentials \((\alpha_D = \alpha_R)\). Lastly, assume Candidate \(D\) has selected an ambiguity of 3. This would be the case, for instance, if she claimed to be a 2-4 on the ANES scale. For values of \(m\) ranging from 2 to 5, Figure 1(b) shows the share of the contested votes candidate \(R\) will receive as his level of ambiguity changes. Note that lower values of \(m\) allow Candidate \(R\) to be more successful when he is more ambiguous than Candidate \(D\). Likewise, lower values of \(m\) allow candidate \(D\) to be more successful when she is more ambiguous than Candidate \(R\).

Note that the general contest success function expressed in Equation 1 displays the following desirable properties regarding voter behavior.

- As a candidate’s ambiguity increases, his share of the contested votes will decrease.
- As a candidate’s affect potential increases, his share of the contested votes will increase.
- The units of the underlying issue space do not affect the shares of contested votes.
- If all candidates display an equal level of ambiguity, the more emotionally appealing candidate will win a higher share of the contested vote.
- The share of the contested voters sums to 1, and each candidate receives a share of the vote between 0 and 1.
- The choice between two alternatives is independent of a third candidate who receives no share of the vote. i.e. Independence of Irrelevant Alternatives.

---

9This would be the case if she claimed to not be a Strong Democrat, but also was not a Republican. e.g. she campaigned on the platform of being a moderate Democrat.
Clark and Riis (1998) show that if $(\bar{D}_L, \bar{R}_C) = (-\infty, \infty)$, the contest success function must be in the form of Equation 1 for these properties to hold.

3.0.2 Candidate Objective Functions

I assume each candidate achieves a utility of 1 from winning the election and, as such, maximizes his expected utility by maximizing his probability of winning.\(^{10}\) For the contest success function defined above and a given $\{\bar{D}_L, \bar{R}_C\}$, the expected utility for each candidate in the game represented in Figure 2 is therefore:

\[ V_D(D_C) = \nu \left( \frac{1}{R_C - D_L} \left( R_L - D_L + s_D(D_C - R_L) \right) \right) \]

\[ V_R(R_L) = 1 - V_D = 1 - \nu \left( \frac{1}{R_C - D_L} \left( R_L - D_L + s_D(D_C - R_L) \right) \right) \]

\(^{10}\)Instead of maximizing the probability of winning by maximizing vote share, we could also assume there exist $n$ voters with iid uniform distributions over $[\bar{D}_L, \bar{R}_C]$ whose exact locations are unknown to the candidates. Per Ledyard (1984), the difference in probability a voter will vote for Candidate D and the probability a voter will vote for Candidate R is a good approximation for the probability Candidate D wins the election. If we interpret the share of contested voter support as the probability a voter in the contested range will vote for a particular candidate, this approximation can be used. Assuming candidates attempt to maximize their probability of winning under this scenario and using this approximation results in no fundamental changes.
4 Results

Given the complete information environment and the simultaneous moves of the candidates, I use the Nash equilibrium solution concept to determine the optimal strategies. Due to the symmetry of the problem, I assume w.l.o.g. that $\alpha_D > \alpha_R$. All proofs are relegated to the Appendix.

**Proposition 1.** For $m > \frac{\ln[\frac{\alpha_D}{\alpha_R}]}{W(\ln[\frac{\alpha_D}{\alpha_R}])} + 1 \equiv m^*$, the optimal strategies for the candidates are:

\[
D_C^* = \exp\left[\frac{\ln[\frac{\alpha_D}{\alpha_R}]}{m-1}\right] \frac{(m\bar{R}_C - \bar{D}_L) + \bar{D}_L(m - 1)}{\ln[\frac{\alpha_D}{\alpha_R}](m-1)[1 + \exp\left(\frac{\ln[\frac{\alpha_D}{\alpha_R}]}{m-1}\right)]}
\]

\[
R_L^* = \bar{R}_C - \frac{m(\bar{R}_C - \bar{D}_L)}{\ln[\frac{\alpha_D}{\alpha_R}](m-1)[1 + \exp\left(\frac{\ln[\frac{\alpha_D}{\alpha_R}]}{m-1}\right)]}
\]

where $W(y)$ is the real-valued branch of the Lambert W-function (also called the omega function), which is the function satisfying $W(y)e^{W(y)} = y$.

**Proposition 2.** For $1 + \frac{\alpha_R}{\alpha_D} < m \leq m^*$, the optimal strategies for the candidates are:

\[
D_C^* = \bar{R}_C
\]

\[
R_L^* = \bar{R}_C - \exp\left[\frac{\ln[\frac{\alpha_D}{\alpha_R}]}{m-1}\right]
\]

**Proposition 3.** For $0 \leq m \leq 1 + \frac{\alpha_R}{\alpha_D}$, the optimal strategies for the candidates are:

\[
D_C^* = \bar{R}_C
\]

\[
R_L^* = \bar{D}_L
\]

For $\alpha_D = 2, \alpha_R = 1$, Figure 3 graphically shows Propositions 1 through 3 over a range of mass effect parameters.

For $m \leq m^*$, the above results show one or both candidates select full ambiguity as an optimal strategy. This appears in contrast to the current American political environment, in which presidential candidates tend to cater to voters ranging from their base to slightly beyond the median voter (Democrats try to appeal to Independents leaning Republican and vice versa). As such, I focus the remainder of the paper on scenarios in which $m > m^*$. For this range, Proposition 1 directly implies:

\[11\text{For ease of exposition, the case where } \alpha_D = \alpha_R \text{ is ignored. The closed-form solutions are identical, with } m^* = 2. \text{ It should be clear that the optimal solutions are symmetric, and no region will exist in which only one candidate will be fully ambiguous.}
\]
\[12\text{Numerically, } m^* \text{ is bounded below by 2.}
\]
\[13\text{\(W(y)\) is a multi-valued function over the complex reals. However, the real-valued branch of \(W(y)\), when restricted to } y \in \mathbb{R}_+, \text{ is a positive, strictly increasing, concave function. Please refer to Corless et al. (1996) for further details.} \]
Proposition 4. For \( m > m^* \), the size of the conflict zone, \( D_C^* - R_L^* \), is independent of the candidate affect potentials. Specifically,

\[
D_C^* - R_L^* = \frac{\bar{R}_C - \bar{D}_L}{m - 1}
\]

This proposition is the key finding of the paper. Along with Proposition 1, it tells us that for a given \( m > m^* \), while the optimal level of ambiguity might vary from election year to election year, the number of contested voters will not. Graphically, it indicates that different values of \( \alpha_D > \alpha_R \) will vertically shift Figure 3, but the interior shape will remain the same. This result will prove central in explaining recent trends in American presidential politics. Note that Proposition 1 also implies the candidate more likely to win will be more ambiguous than the losing candidate, a finding validated by Campbell (1983).

To relate Proposition 4 to recent trends in American presidential politics, it is first necessary to offer definitions of political affiliation. It should be noted that party labels in America are meaningless outside the context of the prevailing political parties. Even in the last twenty years, we have seen the term “Republican” take on a significantly more morally conservative connotation. I therefore offer the following definitions:

**Independent American Voter:** A voter whose ideal ideological stance is not consistently espoused by one and only one political party.

**Partisan American Voter:** A voter whose ideal ideological stance is consistently espoused by
one and only one political party.

**Political Party:** A party comprised of partisan voters.

In other words, an Independent American voter is one who cannot count on a specific party to be the sole party offering their ideal viewpoint. These definitions coincide with the trends found in Figure 4, which shows a strong relationship between partisanship and the voter’s perception that one party is superior to the other on their most important issue.\(^\text{14}\)

![Figure 4: Independent Voters](image)

With these definitions in mind, we return to the model. I assume that American presidential candidates have a bounded affect potential advantage over each other. This assumption is based on the inherent “political savvy” required to even be considered a presidential candidate. I model this assumption by assuming \(\alpha_D, \alpha_R \in \{A, B\}\), where \(\{A, B\} \in \mathbb{R}^2 \mid A < B\). I also assume that neither party consistently offers more personable candidates. Defining \(D_{\max}^*\) as the most conservative position a Democratic candidate will adopt, \(R_{\min}^*\) the most liberal position a Republican candidate will adopt, \([\bar{D}_L, D_{\max}^*]\) the voters targeted by Democratic candidates, \([R_{\min}^*, \bar{R}_C]\) the voters targeted by Republican candidates, and \(I\) the percentage of Independent voters, I find:

**Proposition 5.** For \(m > m^*\), changes due to specific candidate personalities will not affect either party’s targeted base, the number of self-proclaimed Independent voters, or the level of party ambiguity. Analytically,

\(^{14}\)As specified by ANES Cumulative Data variable VCF9012.
\[
\begin{align*}
\frac{\partial D^*_{\max}}{\partial \alpha_D} &= 0, & \frac{\partial D^*_{\max}}{\partial \alpha_R} &= 0, & \frac{\partial R^*_{\min}}{\partial \alpha_D} &= 0, & \frac{\partial I}{\partial \alpha_D} &= 0, \\
\frac{\partial R^*_{\min}}{\partial \alpha_R} &= 0, & \frac{\partial I}{\partial \alpha_R} &= 0.
\end{align*}
\]

**Proposition 6.** For \( m > m^* \), a decrease in \( m \) (corresponding to a decrease in the cost of being more ambiguous than the other candidate) will result in:

- The Democratic party will target a larger base.
- The Republican party will target a larger base.
- Candidates will become more ambiguous.
- The number of self-proclaimed Independent voters will increase

Analytically,

\[
\frac{\partial D^*_{\max}}{\partial m} < 0, \quad \frac{\partial R^*_{\min}}{\partial m} > 0, \quad \frac{\partial I}{\partial m} < 0
\]

Intuitively, as ambiguity becomes less harmful to candidates, they will choose to be more vague in their campaign platforms. This will have the effect of causing fewer voters to be convinced that a single party can implement their ideal point. For instance, consider an environment in which ambiguity is harmful. Assume that each candidate caters to their own base. Democrats would offer policy platforms that include the ideal points of voters between Strongly Democratic and Independent (1-4 on the ANES 1-7 scale). Republicans would offer the opposite policy platforms (4-7). Only true Independents (a 4 on the ANES scale) would identify themselves as an Independent, since every other voter type recognizes a single party who identifies with their ideal point. As ambiguity becomes less harmful, the Democratic candidate would offer a more conservative policy stand, including the ideal points of Independent leaning Republican voters (a 5 on the ANES scale). Likewise, Republicans would offer a more liberal policy stand, including the ideal points of Independent leaning Democratic voters (a 3 on the ANES scale). With decreased ambiguity cost, all voters between 3 and 5 would now identify themselves as Independent, since both parties offer their ideal point.

Note Proposition 6 also directly implies:

**Proposition 7.** For \( m > m^* \), a decrease in \( m \) (corresponding to a decrease in the cost of being more ambiguous than the other candidate) will result in:
• **The Democratic party, on average, will consist of more liberal voters, resulting in increased partisanship.**

• **The Republican party, on average, will consist of more conservative voters, resulting in increased partisanship.**

Analytically,

$$\frac{\partial \left( \frac{R_{\min} + D_L}{2} \right)}{\partial m} < 0$$

$$\frac{\partial \left( \frac{D_{\max} + R_C}{2} \right)}{\partial m} > 0$$

Propositions 5 through 7 are demonstrated in Figure 5. These results imply that a decrease in the cost of ambiguity will result in an increased number of independent voters, party partisanship, and candidate ambiguity. Arguably all three trends have been seen in recent American presidential elections, with timing coinciding with specific political events.

5 Trends in Modern American Presidential Politics

Since the late 1960’s, it has been well documented by Crotty and Jacobson (1984), Wattenberg (2000), Luttbeg and Gant (1995), and others that partisanship has had a declining effect on American voting decisions. Figure 6 demonstrates the rise of the self-proclaimed independent voter.\(^{15}\)

\(^{15}\) Independents are classified as those labeling themselves “Independent Democrat”, “Independent Republican”, or “Independent Independent” on the ANES survey. On a 1-7 scale, it is those individuals labeling themselves a 3, 4, or 5. See Figure 1(a) for the full ANES scale.
Despite the increase in independent voters, Groseclose et al. (1999), Jacobson (2000), Stonecash et al. (2003), and Brewer (2005) have shown that regardless of the measure used, the ideological distance between the parties has been growing, with the Democrats becoming more liberal and the Republicans becoming more conservative.

Coinciding with the increase in Independent voters is the common complaint that American presidential candidates are increasingly vague. Through the course of a presidential election, policy positions are rarely consistently unambiguous, causing Levine (1995) to claim “the major candidates rarely offer a clear choice of detailed, workable policy solutions on issues of importance to voters”.

On the issue of NAFTA, John McCain, Barack Obama, and Hillary Clinton’s websites offer the following proposals:16

- “The U.S. should engage in multilateral, regional and bilateral efforts to promote free trade, level the global playing field and build effective enforcement of global trading rules.”
- “Obama believes that NAFTA and its potential were oversold to the American people. Obama will work with the leaders of Canada and Mexico to fix NAFTA so that it works for American workers.”
- “[Hillary] will also ensure that trade policies work for average Americans. Trade policy must

---

16 [www.johnmccain.com](http://www.johnmccain.com), [www.barackobama.com](http://www.barackobama.com), and [www.hillaryclinton.com](http://www.hillaryclinton.com) as of 4/29/2008. Note Hillary Clinton and John McCain did not specifically mention NAFTA.
From these stances, it is extremely difficult to determine the exact measures each candidate will take on the issue. We seem far removed from the 1964 election when Barry Goldwater made his ideological position clear by proclaiming in his Republican National Convention acceptance speech, “Anyone who joins us in all sincerity, we welcome. Those, those who do not care for our cause, we don’t expect to enter our ranks, in any case. And let our Republicanism so focused and so dedicated not be made fuzzy and futile by unthinking and stupid labels. I would remind you that extremism in the defense of liberty is no vice! And let me remind you also that moderation in the pursuit of justice is no virtue!”.  

With the model proposed in this paper, the rise in independent voters, candidate ambiguity, and party partisanship can all be explained simultaneously with a decrease in the value of the mass effect parameter, \(m\). Or, put differently, over the past fifty years, ambiguity has become less detrimental for presidential candidates. Specifically, Figure 6 indicates ambiguity must have become significantly less detrimental in the late 1960’s and early 1970’s, and gradually less detrimental from 1980 - 2004. I argue there is reason to believe this is the case, especially due to two major political phenomena.

### 5.1 Death of the Party Bosses 1968-1976

In 1968, the Democratic party nominated Hubert Humphrey for their presidential candidate. This is despite the fact that he had not won (or entered) a single primary. Politics of the day were dictated by party bosses, “who from the sanctity of the so-called smoke-filled rooms at nominating conventions handpicked ‘their’ candidate to be the party nominee....Party bosses, through a system that combined the disposition of jobs with political favors, support, and even protection, controlled the votes, the party, and thus the selection of all candidates” (Trent and Friedenberg, 1995).

Humphrey’s nomination caused such disenfranchisement with Antiwar Democrats that the 1968 convention caused a riot in Chicago. Richard Nixon, the Republican nominee, ended up narrowly winning the general election. To avoid a recurrence, Democrats enacted party rule changes referred to as the McGovern-Fraser reforms. These reforms effectively took the convention out of the “back-rooms” and emphasized general primaries versus closed-room caucuses. Between 1968 and 1976, the percent of delegates selected via a Democratic primary (versus a caucus) increased from 38% to 73%. In addition, in 1974, the Federal Election Campaign Act (FECA) was passed into law, limiting the amount of private money available to candidates receiving public support. This had the effect of limiting the influence of a small number of wealthy donors.

Both changes had substantial effects on the American political landscape. Significant financial contributors could no longer require unambiguous promises from candidates. Party-bosses could not demand the candidate run on a specific party platform. There was no longer a small group of

---

17On specific policy issues, Goldwater was equally uncompromising. In a campaign brochure found at [http://www.4president.org/brochures/goldwater1964brochure.htm](http://www.4president.org/brochures/goldwater1964brochure.htm), Goldwater clearly states he is for an increase in State’s rights, against the Civil Rights Act of 1964’s public accommodations provision, for a decrease in Union power, and against expansion of government Welfare.

18In an effort to maintain the appearance of openness in the face of the Democratic changes, Republicans also enacted changes. Between 1968 and 1976, the percent of delegates selected via a Republican primary increased from 34% to 68%.
individuals to whom detailed promises must be made. As the role of closed-door caucuses became less pronounced in the nomination of presidential candidates, ambiguity became less harmful to individual nominees. Ambiguous messages could be delivered through the course of a campaign that could not occur in a single national convention during which the elected candidate was chosen internally.

In terms of the model, this had the effect of lowering the mass effect parameter, $m$, extremely quickly. In turn, this caused candidates to espouse more ambiguous campaign stances, increasing the number of self-proclaimed Independent voters. This downward trend in the caucus system lasted until the late 1970’s, at which point the rise in the number of political primaries leveled off. This coincides with the period during which the number of Independent voters stabilized. As shown in Figure 6, since the effect of the campaign reform measures stabilized in 1976, the number of Independents has remained reasonably steady.

5.2 Rise of the Partisan Media 1980-2008

While the number of Independents in the last three decades has not shown the drastic rise of the late 1960’s, Figure 6 does show a seemingly consistent increase. I argue this effect is in part due to cable and internet news, and the partisan bias consistent therein.

To explain this effect, note CNN, the first 24-hour cable news program, was founded in 1980. Since that time, the number of cable news outlets and internet news sources has grown exponentially, as well as the partisan slant of the media as a whole.\(^{19}\) I argue this ability to receive biased news has increased the value of ambiguity.

In particular, biased news outlets allow voters to self-select their information cues. In 2004, for instance, 52% of regular Fox viewers described themselves as politically conservative, while only 36% of CNN viewers and 33% of nightly network news viewers did the same.\(^{20}\) This corresponds to the findings of Groseclose and Milyo (2005), who show Fox News is more conservative than other news outlets. Using a biased news outlet that filters conflicting information allows voters to avoid the psychological cost of contradictory statements.\(^{21}\) This has the effect of allowing candidates to be more ambiguous in their campaign platforms, in that there is a reduced risk of voters hearing contradictory statements.

As an example, when asked in 2003, “Is it your impression that the US has or has not found clear evidence in Iraq that Saddam Hussein was working closely with the al Qaeda terrorist organization?” a study by the Program on International Policy Attitudes\(^{22}\) found that 67% of Fox News viewers believed the U.S. had found evidence linking the two. This is the highest of any other listed news organization, and is in considerable contrast to the 16% of NPR viewers who answered similarly.

\(^{19}\) Please see Goldberg (2003) and Alterman (2003) for details.


\(^{21}\) Burke (2007) offers an alternative rationale for using biased news outlets that does not rely on the concept of psychological cost.

6 Efficacy of the Heuristic

One of the requirements for a heuristic to be ecologically rational is that it performs “almost as well” as a more complicated decision process. In terms of our model, this implies for voters to be using the “hear what you want to hear” heuristic, it must perform reasonably well compared to a fully informed process. To claim a heuristic exists that systematically resulted in the “wrong” decision would be fundamentally suspect.

To judge the efficacy of the heuristic, it is impossible to use isolated voters. In particular, the assumption of any heuristic implies that voters will never become fully informed, and as such, never determine what the “right” decision should have been. A voter (at the time their vote is cast) always views their decision as correct. As such, to determine if the heuristic is valid, I will use an aggregate performance measure.

Specifically, I will assume that winning candidates enact policy stands at the mean of their issue interval. Voters view their initial vote as “right” if the enacted policy stand is “closer” (in a fully informed spatial sense) than their estimate of the losing candidate’s policy interval, which we also assume is at the mean of their issue interval. In addition, I assume that the share of voters voting for the Republican candidate are those on the more conservative side of the conflict zone (and vice versa). Call this metric the efficacy metric.

Proposition 8. Using the efficacy metric, for scenarios in which $\frac{\alpha_D}{\alpha_R} < 20$, the percentage of voters voting “correctly” equals

$$\%\text{ Voting \ “Correctly” (PVC)} = 1 + \frac{1}{m-1} \min \left( \frac{2 - m}{4} + \frac{m}{2 \left(1 + \exp \left[ \frac{\ln \left( \frac{\alpha_D}{m-1} \right)}{\alpha_D + \alpha_R \exp \left[ \frac{m \ln \left( \frac{\alpha_D}{m-1} \right)}{m-1} \right] } \right] } \right),$$

$$- \frac{1}{m-1} \max \left( \frac{2 - m}{4} + \frac{m}{2 \left(1 + \exp \left[ \frac{\ln \left( \frac{\alpha_R}{m-1} \right)}{\alpha_D + \alpha_R \exp \left[ \frac{m \ln \left( \frac{\alpha_D}{m-1} \right)}{m-1} \right] } \right] } \right),$$

Figure 7 shows this percentage of voters for which the heuristic is effective. Affect potentials are shown such that a candidate facing another candidate with identical ambiguity will receive anywhere from 25% to 75% of the vote. Mass effect parameters are shown such that equilibrium conflict zones will range from 1/4 to 1/2 of the total possible political ideology. Assuming that American presidential politics exists within these ranges, approximately 95%+ of voters will view their choice as correct, ex-post.

As the justification for using a fast and frugal heuristic is its tradeoff between speed and accuracy, it useful to examine how the party heuristic, in which voters vote for the candidate promoted by

\footnote{As long as the assumed policy skew is identical for each party, the mean assumption is not critical. It is worth noting that a candidate who enacts policies considerably more extreme than expected will cause a significantly higher number of voters to be disappointed with their initial vote.}

\footnote{Given the requirements necessary to win a party primary, it is reasonable to assume no two presidential candidates are significantly different in affect potential. Also, modern campaigns show a propensity for “appealing to the independent voter without alienating your base”, implying a moderate level of conflict.}
their preferred party, compares to the “hear what you want to hear” heuristic. In particular, voters should not use the more complex “hear what you want to hear” heuristic unless it offers an accuracy advantage over the party heuristic. Figure 8 shows the efficacy of voters using the “hear what you want to hear” heuristic versus the party heuristic. Numerically, as \( m \) decreases, the greater the accuracy benefit to using the “hear what you want to hear” heuristic. Thus, as the cost of ambiguity decreases, we should see a movement away from strict party voters.\(^{25}\)

7 Multidimensional Issue Space

It is worth noting the effect of a multidimensional issue space on the model. If we assume that ambiguity in one issue does not affect the share of the contested vote in another issue,\(^{26}\) then we can view each issue space as a separate game. A candidate’s affect potential might vary from issue to issue, allowing one candidate to be specific on some issues and ambiguous on others. However,\(^{25}\)

\(^{25}\)Two notes are worth mentioning. First, by assuming the share of contested voters who vote for Republicans are those whose positions are more conservative, I am assuring the highest efficacy level possible for the “hear what you want to hear” heuristic. If I assume the shares of each type of voter are distributed randomly throughout the conflict zone, then the results are not as clear cut. In particular, a large affect differential need exist in order for the “hear what you want to hear” heuristic to clearly surpass the party heuristic. Second, the efficacy of the party heuristic is based on the best responses of candidates assuming voters are using the “hear what you want to hear” heuristic. Thus, this analysis should not be viewed as a causal justification for the “hear what you want to hear” heuristic, but instead be viewed as an argument for its ecological rationality in its current environment.

\(^{26}\)For instance, a devote pro-life candidate’s share of the pro-life vote will not be affected by an ambiguous stance on welfare reform.
20 Conclusion

(a) \( m = 5 \)

(b) \( m = 3 \)

Figure 8: “Hear What You Want To Hear” vs. Party Heuristic

the best responses in each game will follow the form of Proposition 1.

Note the only significant change to the unidimensional model is the higher likelihood that voters will choose not to vote. If, for instance, a candidate offers the voter’s ideal point in Issue 1, but does not in Issue 2, the heuristic will mandate the voter does not vote. This again coincides with the findings that individuals avoid making trade-offs between cues pointing in opposite directions. This result also offers an explanation for the highly correlated party stances on highly disparate issues (pro-social welfare, pro-union, pro-choice, etc.). Because a voter claiming to be pro-choice is likely to be pro-welfare, the optimal strategy for a Democrat is to offer both issue stances.

Lastly, note that the multidimensional model allows the concept of a voter satisficing strategy. In particular, because each game is separate, voters choosing to focus on only a few key issues do not affect any of the results.

8 Conclusion

This paper has applied a voter heuristic to American presidential elections and applied the economic theory of conflict to help explain recent trends in American presidential politics. The model’s key results are that individual candidate personality does not influence the trends of Independent voters, party partisanship, or long-term ambiguity. Instead, only the cost of being ambiguous is significant, with lower ambiguity costs related to increases in these trends. I have also made the case that the primary system, campaign finance reform, and changing media climate have all resulted in lower ambiguity costs. Lastly, I have shown that the proposed heuristic is ecologically rational.

While the model offers a starting point to understanding candidate and voter behavior, it has not addressed the rich environment inherent to campaign advertising strategy. I assumed the level of
affect potential is fixed for each candidate, but this is an obvious simplification, given the effectiveness of negative campaigning, image consultants, etc. Further, I have not explored how the primary campaign game affects the general election. Exploring this subgame would likely offer further insight into optimal presidential campaign strategies. In particular, examining the tradeoff between more extreme and more personable candidates would likely prove valuable.

9 Appendix†

I will solve for the Nash equilibrium using the necessary first order conditions for an interior solution. I will then verify the solution is, in fact, a maximum using the second order conditions. Lastly, for \( m > m^* \), I will confirm the constraints \( D_C \in [\bar{D}_L, \bar{R}_C] \) and \( R_L \in [\bar{D}_L, \bar{R}_C] \) are not violated.

9.0.1 First Order Conditions

Rewriting the objective functions 4 and 5,

\[
V_D(D_C) = \nu \left( \frac{1}{R_C - D_L} \left( R_L - \bar{D}_L + s_D(D_C - R_L) \right) \right), \\
V_R(R_L) = 1 - V_D = 1 - \nu \left( \frac{1}{R_C - D_L} \left( R_L - \bar{D}_L + s_D(D_C - R_L) \right) \right). 
\]

With Candidate R selecting his most liberal campaign position, \( R_L^* \), the necessary condition for an interior optimum for candidate D is

\[
\frac{\partial V_D}{\partial D_C} = \nu (\cdot)' \left( \frac{1}{R_C - D_L} \left[ \alpha_D \alpha R \left( \frac{D_C^* - D_L}{R_C - R_L} \right)^m \right] \\
- \frac{(D_C^* - R_L^*) \alpha D \alpha R m (D_C^* - D_L)^{m-1} (R_C - R_L^*)^{-m}}{\left( \alpha_D + \alpha R \left( \frac{D_C^* - D_L}{R_C - R_L} \right)^m \right)^2} \right) = 0, \tag{6} 
\]

or, simplified,

\[
\alpha_D^2 (\bar{R}_C - R_L^*)^m + \alpha_D \alpha R (D_C^* - \bar{D}_L)^m - D_C^* \alpha D \alpha R m (D_C^* - \bar{D}_L)^{m-1} \\
+ R_L^* \alpha D \alpha R m (D_C^* - \bar{D}_L)^{m-1} = 0. 
\]

Setting \( \mu \equiv \frac{2 \alpha}{\alpha_D} \) and rearranging,

\[
\left( \frac{(\bar{R}_C - R_L^*)}{(D_C^* - \bar{D}_L)} \right)^m + \mu \frac{D_C^*}{(D_C^* - \bar{D}_L)} + \mu \frac{R_L^*}{(D_C^* - \bar{D}_L)} = 0. \tag{7} 
\]

†I use considerable algebraic simplification throughout the Appendix. To view additional intermediate algebraic steps, please refer to www.unc.edu/~petranka/PetrankaCTV_Extended.pdf
With Candidate D selecting her most conservative campaign position, $D^*_C$, the necessary condition for an interior optimum for candidate R is

$$
\frac{\partial V_R}{\partial R_L} = \nu \left( -1 + \frac{\alpha_D}{\alpha_D + \alpha_R \left( \frac{D^*_C - D^*_L}{R_C - R^*_L} \right)^m} + \frac{(D^*_C - R^*_L) \alpha_D \alpha_R (D^*_C - \tilde{D}_L) \left( \frac{(D^*_C - R^*_L)^{(m-1)} - 1}{(\alpha_D + \alpha_R \left( \frac{D^*_C - D^*_L}{R_C - R^*_L} \right)^m} \right)}{\left( \frac{D^*_C - D^*_L}{R_C - R^*_L} \right)^m} \right) = 0,
$$

or, simplified,

$$
-\alpha_D \alpha_R \frac{(D^*_C - \tilde{D}_L)^m}{(R_C - R^*_L)^m} - \alpha_R \left( \frac{D^*_C - \tilde{D}_L}{R_C - R^*_L} \right)^{2m} \alpha_R \frac{(D^*_C - \tilde{D}_L)^m}{(R_C - R^*_L)^{m+1}} - R^*_C \alpha_D \alpha_R m \frac{(D^*_C - \tilde{D}_L)^m}{(R_C - R^*_L)^{m+1}} = 0.
$$

Setting $\phi \equiv \frac{\alpha_D}{\alpha_R}$ and simplifying,

$$
-\phi (D^*_C - \tilde{D}_L)^m (R_C - R^*_L)^m - (D^*_C - \tilde{D}_L)^{2m} + D^*_C \phi m (D^*_C - \tilde{D}_L)^m (R_C - R^*_L)^{(m-1)} - \phi (D^*_C - \tilde{D}_L)^m (R_C - R^*_L)^{(m-1)} = 0.
$$

Setting $\eta \equiv (R_C - R^*_L)$ and $\psi \equiv (D^*_C - \tilde{D}_L)$, Eqn. (9) can be written

$$
-\phi \psi^m \eta^m - \psi^{2m} + (\psi + \tilde{D}_L) \phi m \psi^m \eta^{(m-1)} - (R_C - \eta) \phi m \psi^m \eta^{(m-1)} = 0,
$$

or, further simplified,

$$
\left( \frac{\eta}{\psi} \right)^{(m-1)} = \frac{\psi}{\phi(-\eta + \psi m + \tilde{D}_L m - R_C m + \eta m)}.
$$

Also note Eqn. (7) implies

$$
\left( \frac{\eta}{\psi} \right)^m = \mu \frac{-\psi + \psi m + \tilde{D}_L m - R_C m + \eta m}{\psi},
$$

which further implies

$$
\left( \frac{\eta}{\psi} \right)^m \frac{\psi}{\eta} = \mu \frac{-\psi + \psi m + \tilde{D}_L m - R_C m + \eta m}{\eta}.
$$

Combined, Eqns. (10) and (12) tell us
\[ \mu \frac{-\psi + \psi m + \bar{D}_L m - \bar{R}_C m + \eta m}{\eta} = \frac{\psi}{\phi(-\eta + \psi m + \bar{D}_L m - \bar{R}_C m + \eta m)}, \]

which, after recognizing \( \mu \phi = 1 \), implies

\[ \eta \psi = (-\psi + \psi m + \bar{D}_L m - \bar{R}_C m + \eta m)(-\eta + \psi m + \bar{D}_L m - \bar{R}_C m + \eta m), \]

or, after further simplification,

\[ \psi = -\eta + \frac{m(\bar{R}_C - \bar{D}_L)}{m - 1}. \]  

(13)

Also note from Eqn. (10),

\[ \left( \frac{\psi}{\eta} \right)^m = \frac{\phi(-\eta + \psi m + \bar{D}_L m - \bar{R}_C m + \eta m)}{\eta}. \]

Substituting for \( \psi \) from Eqn. (13) and simplifying,

\[ \left( \eta + \frac{m(\bar{R}_C - \bar{D}_L)}{m - 1} \right)^m = \frac{\phi(-\eta - \eta m + \frac{m^2(\bar{R}_C - \bar{D}_L)}{m - 1} + \bar{D}_L m - \bar{R}_C m + \eta m)}{\eta} = \frac{\phi(-\eta(m - 1) + \bar{R}_C m - \bar{D}_L m)}{(m - 1)\eta}. \]

Multiplying both sides by \( \eta \),

\[ \eta \left( \eta + \frac{m(\bar{R}_C - \bar{D}_L)}{m - 1} \right)^m = \phi \left( \eta + \frac{m(\bar{R}_C - \bar{D}_L)}{m - 1} \right), \]

which, after taking the natural log of both sides, implies,

\[ \ln \left( \eta + \frac{m(\bar{R}_C - \bar{D}_L)}{m - 1} \right) = \frac{\ln(\phi)}{m - 1}, \]

or, simplified,

\[ \eta = \frac{m(\bar{R}_C - \bar{D}_L)}{(m - 1)[1 + \exp\left(\ln(\phi)\right)_{m - 1}]} \cdot \]

Changing \( \eta \) back to \( (\bar{R}_C - \bar{R}_L') \) and \( \psi \) back to \( (\bar{D}_C - \bar{D}_L) \),
Using the first order condition of Eqn. (6) and simplifying Eqn. (16), an interior optimum for Candidate D is

$$R_L^* = R_C - \frac{m(R_C - D_L)}{(m-1)(1 + \exp \left[\frac{\ln(\frac{\nu_d}{m-1})}{m-1}\right])}.$$  \hspace{1cm} (14)

Eqns. (13) and (14) imply

$$D_C^* - D_L = \frac{m(D_L - R_C)}{(m-1)(1 + \exp \left[\frac{\ln(\frac{\nu_d}{m-1})}{m-1}\right])} + \frac{m(R_C - D_L)}{m-1},$$

which, when simplified, implies

$$D_C^* = \frac{\exp \left[\frac{\ln(\frac{\nu_d}{m-1})}{m-1}\right](mR_C - D_L)}{(m-1)(1 + \exp \left[\frac{\ln(\frac{\nu_d}{m-1})}{m-1}\right])}.$$  \hspace{1cm} (15)

### 9.0.2 Second Order Conditions

With Candidate R again selecting his most liberal campaign position, $R_L^*$, a sufficient condition for an interior optimum for Candidate D is $\frac{\partial^2 V_D}{\partial D_C^2}(D_C^*) < 0$. To verify this condition holds,

$$\frac{\partial^2 V_D}{\partial D_C^2}(D_C^*) = \nu(\cdot)^{\prime} \left( -2\alpha_D\alpha_Rm(R_C - R_L^*)^{-m}(D_C^* - D_L)(m-1) \right) \left[ \frac{\alpha_D + \alpha_R(D_C^* - D_L)}{[\alpha_D + \alpha_R(D_C^* - D_L)]^2} \right]$$

$$+ \nu(\cdot)^{\prime} \left( \frac{1}{R_C - D_L} \left[ \frac{\alpha_D}{\alpha_D + \alpha_R(D_C^* - D_L)} \right] \left( \frac{D_C^* - R_L^*}{(D_C^* - R_L^*)^{-m}} \right) \left( \frac{\alpha_D + \alpha_R(D_C^* - D_L)}{[\alpha_D + \alpha_R(D_C^* - D_L)]^2} \right) \right).$$  \hspace{1cm} (16)

Using the first order condition of Eqn. (6) and simplifying Eqn. (16),

$$\frac{\partial^2 V_D}{\partial D_C^2}(D_C^*) = \nu(\cdot)^{\prime} \left( 2\alpha_D\alpha_Rm(R_C - R_L^*)^{-m}(D_C^* - D_L)(m-1) \left[ \frac{\alpha_D + \alpha_R(D_C^* - D_L)}{[\alpha_D + \alpha_R(D_C^* - D_L)]^2} \right] \right)$$

$$- 2\alpha_Rm(D_C^* - R_L^*)(D_C^* - D_L)(m-1)(R_C - R_L^*)^{-m}\left[ \frac{\alpha_D + \alpha_R(D_C^* - D_L)}{[\alpha_D + \alpha_R(D_C^* - D_L)]^2} \right] - (m-1)(D_C^* - D_L)(D_C^* - R_L^*)$$

$$\hspace{1cm} \left( \frac{(R_C - R_L^*)m-1[D_C^* - D_L]}{(D_C^* - D_L)^m} \right).$$  \hspace{1cm} (17)
Appendix

To further evaluate $\frac{\partial^2 V_D}{\partial D_C^2}$ at the optimum, $D_C^*$, note Eqn. (6) implies

$$\frac{(D_C^* - R_L^*)\alpha_R m (D_C^* - D_L)^{(m-1)(\bar{R}_C - R_L)^{-m}}}{\alpha_D + \alpha_R \left(\frac{D_C^* - D_L}{R_C - R_L}\right)^m} = 1.$$ 

In addition, Proposition 2 implies

$$-(m-1)(D_C^* - \bar{D}_L)^{-1}(D_C^* - R_L^*) = -\frac{R_C - \bar{D}_L}{D_C^* - D_L}.$$ 

Substituting these two equalities into Eqn. (17),

$$\frac{\partial^2 V_D}{\partial D_C^2} = \frac{\nu(\cdot)\alpha_D\alpha_R m (\bar{R}_C - R_L)^{-m}(D_C^* - D_L)^{(m-1)}}{[\alpha_D + \alpha_R \left(\frac{D_C^* - D_L}{R_C - R_L}\right)^m]^2} \left( -\frac{\nu(\cdot)(\bar{R}_C - \bar{D}_L)}{D_C^* - D_L} \right).$$

The first term in the above equation is greater than zero, while the second term is less than zero, implying

$$\frac{\partial^2 V_D}{\partial D_C^2}(D_C^*) < 0.$$ 

Symmetrically,

$$\frac{\partial^2 V_R}{\partial R_L^2}(R_L^*) < 0,$$

verifying we have solved for the Nash Equilibrium if the constraints are not violated.

9.0.3 Requirements to Avoid Constraint Violation

W.L.O.G. assume $\alpha_R > \alpha_D$, making $R_L^* > \bar{D}_L$ the relevant constraint. We will assume $R_L^* > \bar{D}_L$, and determine the required conditions on $m$, $\alpha_D$, and $\alpha_R$. All variables should be assumed to be at the optimum.

$R_L^* > \bar{D}_L$ and Eqn. (14) imply

$$\bar{D}_L < \bar{R}_C - \frac{m(\bar{R}_C - \bar{D}_L)}{(m-1)(1 + \exp \left[\frac{\ln[\frac{\alpha_D}{\alpha_R}]}{m-1}\right])},$$

which, further simplified, requires

$$\exp \left[\frac{\ln[\frac{\alpha_D}{\alpha_R}]}{m-1}\right] > \frac{1}{m-1},$$
which, after taking the natural log of both sides and simplifying, tells us

\[
\ln \left[ \frac{\alpha_R}{\alpha_D} \right] < (m - 1) \ln[m - 1].
\] (18)

Setting \( y \equiv \ln[\frac{\alpha_R}{\alpha_D}] \) and \( x \equiv (m - 1) \), Eqn. (18) can be rewritten

\[
y < x \ln[x],
\]

which implies

\[
\frac{\exp[y/x]}{x} < 1.
\]

Multiplying both sides by \( y \),

\[
\frac{y}{x} \exp[y/x] < y,
\]

which implies

\[
\frac{y}{x} < W(y)
\] (19)

where \( W(y) \) is the Lambert W-function (also called the omega function), which is defined as the inverse function of \( f(W) = We^W \).

Changing \( y \) back to \( \ln[\frac{\alpha_R}{\alpha_D}] \) and \( x \) back to \( (m - 1) \) in Eqn. (19),

\[
R^*_L > \bar{D}_L \iff m > \frac{\ln[\frac{\alpha_R}{\alpha_D}]}{W\left(\ln[\frac{\alpha_R}{\alpha_D}]\right)} + 1 \text{ when } \alpha_R > \alpha_D.
\]

Symmetrically,

\[
D^*_C < \bar{R}_C \iff m > \frac{\ln[\frac{\alpha_D}{\alpha_R}]}{W\left(\ln[\frac{\alpha_D}{\alpha_R}]\right)} + 1 \text{ when } \alpha_D > \alpha_R.
\]

W.L.O.G., assume \( \alpha_D > \alpha_R \). Defining \( \phi \equiv \frac{\alpha_D}{\alpha_R} \), for \( 1 + \frac{1}{\gamma} < m \leq m^* \), the above analysis shows a fully interior solution does not exist. To solve for the Nash equilibrium, I will instead assume Candidate D is fully ambiguous, show Candidate R will select an interior solution, then verify Candidate D’s best response to Candidate R is, in fact, full ambiguity.

**Step 1:** Assume \( D^*_C = \bar{R}_C \) and \( \alpha_D > \alpha_R \). Find candidate R’s optimal strategy, \( R^*_L \), assuming an interior solution. 

Eqn. (9), with the assumption that $D_C^* = \bar{R}_C$, implies at an interior optimum for candidate $\hat{R}$,

$$- \phi(\bar{R}_C - \bar{D}_L)^m (\bar{R}_C - \bar{R}_L^*)^m - (\bar{R}_C - \bar{D}_L)^{2m} + \bar{R}_C \phi m (\bar{R}_C - \bar{D}_L)^m (\bar{R}_C - \bar{R}_L^*)^{m-1}$$

$$- R_L^* \phi m (\bar{R}_C - \bar{D}_L)^m (\bar{R}_C - \bar{R}_L^*)^{m-1} = 0,$$

or, simplified,

$$(\bar{R}_C - \bar{D}_L) \left( -1 - \frac{(\bar{R}_C - \bar{D}_L)^m}{(\bar{R}_C - \bar{R}_L^*)^m \phi} + m \right) = 0.$$  

Further simplification and taking the natural log of both sides tells us

$$m \ln [\bar{R}_C - \bar{R}_L^*] = \ln \left[ \frac{(\bar{R}_C - \bar{D}_L)^m}{(m - 1) \phi} \right],$$

or, simplified,

$$\bar{R}_L^* = \bar{R}_C - \exp \left[ \frac{\ln \left[ \frac{(\bar{R}_C - \bar{D}_L)^m}{(m - 1) \phi} \right]}{m} \right]. \tag{20}$$

**Step 2:** When $R_L = R_L^*$, show $\frac{\partial V_D}{\partial D_C} > 0$, implying $D_C = \bar{R}_C$ is an optimum for candidate $\hat{D}$.

Equations (6) and (20) imply $\frac{\partial V_D}{\partial D_C}$ evaluated at $D_C = \bar{R}_C$ equals

$$\frac{\partial V_D}{\partial D_C}(\bar{R}_C) = \nu(\cdot)^m \left( \frac{\alpha D (\alpha_D + \alpha_R \frac{\bar{R}_C - \bar{D}_L}{\Omega})^m}{R_C - D_L} \right)^{-1}$$

$$- m \alpha D \alpha_R (\bar{R}_C - \bar{R}_L^*) (\Omega)^{-m} (\bar{R}_C - \bar{D}_L)^{m-1} \left( \frac{\alpha D + \alpha_R \frac{\bar{R}_C - \bar{D}_L}{\Omega}}{(m - 1) \phi} \right)^{-2}$$

$$= \frac{\alpha D}{R_C - D_L} \exp \left[ \ln \left[ \frac{(R_C - D_L)^m}{(m - 1) \phi} \right] \right] m \alpha D \left( \frac{m - 1 \phi}{(R_C - D_L)^m} \right) (\bar{R}_C - \bar{D}_L)^{m-1} \left( \frac{\alpha D + \alpha_R (m - 1) \phi}{(R_C - D_L)^m} \right)^{-2}.$$

Recognizing $\phi = \frac{\alpha_D}{\alpha_R}$, this further reduces to

$$\frac{\partial V_D}{\partial D_C}(\bar{R}_C) = \nu(\cdot)^m \left( \frac{\alpha D}{R_C - D_L} \left( \frac{1}{m \alpha D} - \exp \left[ \ln \left[ \frac{(R_C - D_L)^m}{(m - 1) \phi} \right] \right] m \alpha D \left( \frac{m - 1}{R_C - D_L (m \alpha D)^2} \right) \right) \right).$$
\[
\nu(\cdot) f \left( \frac{\alpha_D}{(\bar{R}_C - \bar{D}_L) m \alpha_D} \left( 1 - \frac{(m - 1)}{(m - 1) \phi} \right) \right).
\] (21)

Using the same techniques as in Section (9.0.3), note when \(\alpha_D > \alpha_R\), \(m < m^*\) if and only if

\[
\ln[\phi] > (m - 1) \ln[m - 1],
\]

which implies

\[
\phi > (m - 1)^{m-1}.
\]

Multiplying both sides by \((m - 1)\), raising each side to the \(1/m\) power, and simplifying,

\[
1 - \frac{(m - 1)}{(m - 1) \phi} > 0.
\] (22)

Combining Eqns. (22) and (21), we find \(\frac{\partial V_D}{\partial D_C}(\bar{R}_C) > 0\), implying \(D_C = \bar{R}_C\) is, in fact, candidate D’s best response when candidate R is playing \(R^*_L\).

Step 3: Determine the requirement on \(m\) that ensures \(R_L = R^*_L\) does not violate the constraints.

Eqn. (20) implies

\[
R^*_L = \bar{R}_C - \left( (\bar{R}_C - \bar{D}_L)^m (m - 1)^{-1} \phi^{-1} \right)^{\frac{1}{m}}.
\]

To ensure the constraints are not violated, we need

\[
\bar{D}_L \leq R^*_L = \bar{R}_C - \left( (\bar{R}_C - \bar{D}_L)^m (m - 1)^{-1} \phi^{-1} \right)^{\frac{1}{m}},
\]

or, simplified,

\[
1 \geq \left( (m - 1)^{-1} \phi^{-1} \right)^{\frac{1}{m}}.
\]

Raising both sides to the \(-m\) power (which reverses the inequality) and rearranging, we need

\[
m \geq 1 + \frac{1}{\phi}.
\] (23)
9.1 Proof of Proposition 3

W.L.O.G., assume $\alpha_D > \alpha_R$. Again defining $\phi \equiv \frac{\alpha_D}{\alpha_R}$, for $m < 1 + \frac{1}{\phi}$ the above analysis shows neither candidate will have an interior solution. To show both Candidates will be fully ambiguous, I will instead assume Candidate D is fully ambiguous, show Candidate R’s best response to Candidate D is full ambiguity, then verify Candidate D’s best response to Candidate R is, in fact, full ambiguity.

**Step 1:** When $D_C = \bar{R}_C$, show $\frac{\partial V}{\partial R_L} < 0$, implying $R_L = \bar{D}_L$ is an optimum for candidate R.

Equation (8) implies when $D_C = \bar{R}_C$,

$$\frac{\partial V}{\partial R_L} = \nu(\cdot)\left(-1 + \alpha_D(\alpha_D + \alpha_R \frac{\bar{R}_C - \bar{D}_L}{R_C - R_L})^m\right)^{-1}$$

$$+ m\alpha_D \alpha_R \left(\alpha_D + \alpha_R \frac{\bar{R}_C - \bar{D}_L}{R_C - R_L}\right)^{-2} \left(\bar{R}_C - \bar{D}_L\right)^m (R_C - R_L)^{-m-1},$$

which, when simplified, implies $\frac{\partial V}{\partial R_L} \leq \nu(\cdot)$ equals

$$\nu(\cdot)\left(\alpha_D + \alpha_R \frac{\bar{R}_C - \bar{D}_L}{R_C - R_L}\right)^{-2} \left(\frac{\bar{R}_C - \bar{D}_L}{R_C - R_L}\right)^m \left(- 2\alpha_D - \alpha_R \frac{\bar{R}_C - \bar{D}_L}{R_C - R_L}\right)$$

$$+ \alpha_R \left(\frac{\alpha_D + \alpha_R \frac{\bar{R}_C - \bar{D}_L}{R_C - R_L}}{\bar{R}_C - \bar{D}_L}\right)^{-2} \left(\frac{\bar{R}_C - \bar{D}_L}{R_C - R_L}\right)^m \left(\alpha_D + m\alpha_D\right)\right).$$

Note that when $0 < m < 1 + \frac{1}{\phi}$

$$\left(\frac{\bar{R}_C - \bar{D}_L}{R_C - R_L}\right)^m \geq 1 > (m - 1)\phi,$$

which implies

$$- \left(\frac{\bar{R}_C - \bar{D}_L}{R_C - R_L}\right)^m + (m - 1)\phi < 0. \quad (25)$$

Combining Eqns. (24) and (25) we find $\frac{\partial V}{\partial R_L} < 0$, implying $R_L = \bar{D}_L$ is Candidate R’s best response when Candidate D is playing $\bar{R}_C$ and $0 < m < 1 + \frac{1}{\phi}$.

**Step 2:** When $R_L = \bar{D}_L$, show $\frac{\partial V}{\partial D_C} > 0$, implying $D_C = \bar{R}_C$ is an optimum for Candidate D.

Equation (6) implies when $R_L = \bar{D}_L$,
\[
\frac{\partial V_D}{\partial D_C} = \nu(\cdot)^{\prime} \left( \alpha_D \left( \alpha_D + \alpha_R \left( \frac{D_L - \tilde{D}_L}{R_C - D_L} \right)^m \right)^{-1}
\right.
\]
\[
- \left( m \alpha_D \alpha_R \left( \alpha_D + \alpha_R \left( \frac{D_L - \tilde{D}_L}{R_C - D_L} \right)^m \right)^{-2} \left( \tilde{R}_C - \tilde{D}_L \right)^{-m} \left( D_C - \tilde{D}_L \right)^m \right)
\]
which, when simplified, implies
\[
\frac{\partial V_D}{\partial D_C} = \nu(\cdot)^{\prime} \left( \alpha_D \left( \alpha_D + \alpha_R \left( \frac{D_L - \tilde{D}_L}{R_C - D_L} \right)^m \right)^{-2} \left( \tilde{R}_C - \tilde{D}_L \right)^{-m} \left( D_C - \tilde{D}_L \right)^m \right). \tag{26}
\]
Note that since \(\frac{D_C - \tilde{D}_L}{R_C - D_L} \leq 1\), when \(0 < m < 1 + \frac{1}{\phi}\)
\[
\phi - (m - 1) \left( \frac{D_C - \tilde{D}_L}{R_C - D_L} \right)^m > \phi - \frac{1}{\phi} \left( \frac{D_C - \tilde{D}_L}{R_C - D_L} \right)^m \geq \phi - \frac{1}{\phi}. \tag{27}
\]
Since we have assumed that \(\alpha_D > \alpha_R\),
\[
\phi - \frac{1}{\phi} = \frac{\alpha_D}{\alpha_R} - \frac{\alpha_R}{\alpha_D} > 0,
\]
which, along with Eqns. (27) and (26), tell us \(\frac{\partial V_D}{\partial D_C} > 0\), implying \(D_C = \tilde{R}_C\), is, in fact, candidate D’s best response when candidate R is playing \(\tilde{D}_L\) and \(0 < m < 1 + \frac{1}{\phi}\).

9.2 Proof of Proposition 5
Within the affective potential bounds of \([A, B]\), note that Eqns. (14) and (15) imply the Democratic and Republican parties will, at most, cater to voters in the ranges:

**Targeted by Democratic Candidates:**
\[
\tilde{D}_L, D_C^{\text{max}} = \frac{\exp\left[\frac{\ln\left(\frac{A}{B}\right)}{m-1}\right] \left( m \tilde{R}_C - \tilde{D}_L \right) + \tilde{D}_L \left( m - 1 \right) \right]}{\left( m - 1 \right) \left( 1 + \exp\left[\frac{\ln\left(\frac{A}{B}\right)}{m-1}\right] \right)}
\]

**Targeted by Republican Candidates:**
\[
R_L^{\text{min}} \equiv \tilde{R}_C - \frac{m \left( \tilde{R}_C - \tilde{D}_L \right)}{\left( m - 1 \right) \left( 1 + \exp\left[\frac{\ln\left(\frac{A}{B}\right)}{m-1}\right] \right)} \tilde{R}_C.
\]

The percentage of Independent voters equals
\[
I = \frac{D_C^{\text{max}} - R_L^{\text{min}}}{\tilde{R}_C - \tilde{D}_L}.
\]

It is straight-forward that \(\frac{\partial D_C^{\text{max}}}{\partial \alpha_D} = 0, \frac{\partial D_C^{\text{max}}}{\partial \alpha_R} = 0, \frac{\partial R_L^{\text{min}}}{\partial \alpha_D} = 0, \frac{\partial R_L^{\text{min}}}{\partial \alpha_R} = 0, \frac{\partial I}{\partial \alpha_D} = 0, \frac{\partial I}{\partial \alpha_R} = 0\). Intuitively, individual candidates do not affect a voter’s stated ideology. Instead, only the recognized party bounds are critical. A single Democratic candidate running on a highly conservative platform would not systematically make Republicans declare themselves Democratic.
Appendix

9.3 Proof of Proposition 6

Using the definitions above,

\[
\frac{\partial R^*_L}{\partial m} = -\frac{(\bar{R}_C - \bar{D}_L)}{(m-1)(1+\exp[\frac{\ln A}{(m-1)}])} + \frac{m(\bar{R}_C - \bar{D}_L)}{(m-1)^2(1+\exp[\frac{\ln A}{(m-1)}])}
\]

\[
-\frac{m(\bar{R}_C - \bar{D}_L)\ln[\frac{A}{B}](\exp[\frac{\ln A}{(m-1)}])}{(m-1)^2(1+\exp[\frac{\ln A}{(m-1)}])^2}
\]

\[
= -\frac{(\bar{R}_C - \bar{D}_L)}{(m-1)^2(1+\exp[\frac{\ln A}{(m-1)}])^2}(\exp[\frac{\ln A}{(m-1)}](\frac{m}{m-1} \ln[\frac{A}{B}] - 1)).
\]

The first term in the above equation is negative. The second term is also negative, since \(A < B\) and \(m > 1\) by the assumption of an interior optimum and Proposition 1. Therefore,

\[
\frac{\partial R^*_L}{\partial m} > 0.
\]  

(28)

As \(m\) decreases, candidate R becomes more ambiguous. Symmetrically,

\[
\frac{\partial D^*_C}{\partial m} < 0.
\]  

(29)

To show \(\frac{\partial I}{\partial m} < 0\),

\[
\frac{\partial I}{\partial m} = \frac{\partial D^*_C}{\partial m} - \frac{\partial R^*_L}{\partial m}
\]

\[
= \frac{\partial D^*_C}{\partial m} - \frac{\partial R^*_L}{\partial m}
\]

\[
= \frac{(\bar{R}_C - \bar{D}_L)}{(R_C - D_L)}
\]

which, using Eqns. (28) and (29), is less than 0.

9.4 Derivation of Figure 6

W.L.O.G., assume \(\alpha_D > \alpha_R\). In this scenario, there are two classes of problem as demonstrated in Figure 9:

9.4.1 Case 1: Indifferent Voter (IV) < \(R^*_L\)

All variables should be assumed to be at the optimal values, as determined in Proposition 1.

\(IV < R^*_L\) implies

\[
\frac{\bar{D}_L + \bar{D}^*_C}{2} + \frac{R^*_L + \bar{R}_C}{2} = \bar{D}_L + D^*_C + R^*_L + \bar{R}_C < R^*_L.
\]  

(30)
Simplified, this implies

\[ 2R_L^* > \bar{D}_L + (D_C^* - R_L^*) + \bar{R}_C, \]

which, along with Proposition 2, requires

\[ 2R_L^* > \bar{D}_L + \bar{R}_C + \frac{\bar{R}_C - \bar{D}_L}{m - 1}. \]

Combined with Eqn. (14), this tells us

\[ 2\bar{R}_C - \frac{2m(\bar{R}_C - \bar{D}_L)}{(m - 1)(1 + \exp\left[\frac{\ln\left[\frac{\alpha_D}{\alpha_R}\right]}{m - 1}\right])} > \bar{D}_L + \bar{R}_C + \frac{\bar{R}_C - \bar{D}_L}{m - 1}, \]

or, simplified,

\[ \exp\left[\frac{\ln\left[\frac{\alpha_D}{\alpha_R}\right]}{m - 1}\right] > \frac{m + 2}{m - 2}. \]

(31)

Proposition 1 implies \( m > 2 \). With Eqn. (31), this implies

\[ \frac{\alpha_D}{\alpha_R} > \exp\left[(m - 1)\ln\left[\frac{m + 2}{m - 2}\right]\right]. \]

Figure 10 represents this inequality for values of \( m \) for which the equilibrium solutions would range from full ambiguity to 5% issue overlap, a seemingly reasonable range for American presidential politics. As shown, in order for Case 1 to occur, the affect potential ratio must exceed 20, implying that for an identical level of ambiguity, one candidate would win 95%+ of the contested votes.
9.4.2 Case 2: Indifferent Voter (IV) \( \geq R'_L \)

Having established Case 1 does not occur if \( \alpha_D < 20 \), I focus on Case 2. Note the assumption that \( \alpha_D > \alpha_R \) ensures \( IV < D'_C \), allowing us to focus on the scenario shown in Figure 9(b). Again, all variables should be assumed to be at the optimal interior values, as determined in Proposition 1.

\[
\% \text{ Voting "Correctly" (PVC)} \equiv 1 + \frac{(D'_C - R'_L) \min\left(\frac{D_L + R_C + D'_C + R'_L - R'_L}{D'_C - R'_L}, \Upsilon\right)}{R'_C - D_L} - \frac{(D'_C - R'_L) \max\left(\frac{D_L + R_C + D'_C + R'_L - R'_L}{D'_C - R'_L}, \Upsilon\right)}{R'_C - D_L}
\]

where \( \Upsilon \equiv \frac{\alpha_D + \alpha_R (D'_C - R'_L)}{\alpha_D - \alpha_R (D'_C - R'_L)} \).

Note Eqn. (15) implies

\[
D'_C - D_L = \frac{m \exp\left[\ln\left(\frac{\alpha_D}{\alpha_R}\right)\right] (R'_C - D'_L)}{(m - 1) \left(1 + \exp\left[\ln\left(\frac{\alpha_D}{\alpha_R}\right)\right]\right)}
\]

and Eqn. (14) implies

\[
R'_C - R'_L = \frac{m (R'_C - D'_L)}{(m - 1) \left(1 + \exp\left[\ln\left(\frac{\alpha_D}{\alpha_R}\right)\right]\right)}
\]
Together, Eqns. (33) and (34) tell us
\[
\frac{D_C^* - \bar{D}_L}{R_C - R^*_L} = \exp \left[ \ln \left( \frac{\alpha_D}{\alpha_R} \right) \right].
\] (35)

Plugging Eqn. (35) into Eqn. (32) and simplifying,
\[
PVC = 1 + \frac{D_C^* - R^*_L}{R_C - D_L} \min \left( \frac{\bar{D}_L + \bar{R}_C + D_C^* - R^*_L - 2R^*_L}{4(D_C^* - R^*_L)}, \Xi \right)
- \frac{D_C^* - R^*_L}{R_C - D_L} \max \left( \frac{\bar{D}_L + \bar{R}_C + D_C^* - R^*_L - 2R^*_L}{4(D_C^* - R^*_L)}, \Xi \right),
\] (36)

where \( \Xi \equiv \frac{\alpha_D + \alpha_R \exp \left[ \frac{m \ln \left( \frac{\alpha_D}{\alpha_R} \right)}{m - 1} \right]}{\alpha_D + \alpha_R} \). Plugging Proposition 4 into Eqn. 36 and simplifying,
\[
PVC = 1 + \frac{1}{m - 1} \min \left( \frac{\bar{D}_L + \bar{R}_C + \frac{\bar{R}_C - \bar{D}_L}{m - 1} - 2R^*_L}{4 \frac{R_C - \bar{D}_L}{m - 1}}, \Xi \right)
- \frac{1}{m - 1} \max \left( \frac{\bar{D}_L + \bar{R}_C + \frac{\bar{R}_C - \bar{D}_L}{m - 1} - 2R^*_L}{4 \frac{R_C - \bar{D}_L}{m - 1}}, \Xi \right),
\] (37)

Plugging Eqn. (14) into Eqn. (37) and simplifying,
\[
PVC = 1 + \frac{1}{m - 1} \min \left( \frac{\bar{D}_L + \bar{R}_C + \frac{\bar{R}_C - \bar{D}_L}{m - 1} - 2 \left( \frac{\bar{R}_C - \bar{D}_L}{m - 1} \right) \left( 1 + \exp \left[ \frac{m \ln \left( \frac{\alpha_D}{\alpha_R} \right)}{m - 1} \right] \right)}{4 \frac{R_C - \bar{D}_L}{m - 1}}, \Xi \right)
- \frac{1}{m - 1} \max \left( \frac{\bar{D}_L + \bar{R}_C + \frac{\bar{R}_C - \bar{D}_L}{m - 1} - 2 \left( \frac{\bar{R}_C - \bar{D}_L}{m - 1} \right) \left( 1 + \exp \left[ \frac{m \ln \left( \frac{\alpha_D}{\alpha_R} \right)}{m - 1} \right] \right)}{4 \frac{R_C - \bar{D}_L}{m - 1}}, \Xi \right),
\]

which, after simplification, implies
\[
PVC = 1 + \frac{1}{m - 1} \min \left( \frac{2 - m}{4} + \frac{m}{2 \left( 1 + \exp \left[ \frac{m \ln \left( \frac{\alpha_D}{\alpha_R} \right)}{m - 1} \right] \right)}{\alpha_D + \alpha_R \exp \left[ \frac{m \ln \left( \frac{\alpha_D}{\alpha_R} \right)}{m - 1} \right]}, \Xi \right)
- \frac{1}{m - 1} \max \left( \frac{2 - m}{4} + \frac{m}{2 \left( 1 + \exp \left[ \frac{m \ln \left( \frac{\alpha_D}{\alpha_R} \right)}{m - 1} \right] \right)}{\alpha_D + \alpha_R \exp \left[ \frac{m \ln \left( \frac{\alpha_D}{\alpha_R} \right)}{m - 1} \right]}, \Xi \right),
\]

which can be graphed as a function of mass effect parameter, \( m \), and the affect potential ratio \( \frac{\alpha_D}{\alpha_R} \).
References


