Persuasion by Cheap Talk

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Abstract

We consider the credibility, persuasiveness, and informativeness of multi-dimensional cheap talk by an expert to a decision maker. We find that an expert with state-independent preferences can always make credible comparative statements that trade off the expert's incentive to exaggerate on each dimension. Such communication benefits the expert – cheap talk is "persuasive" – if her preferences are quasiconvex. Communication benefits a decision maker by allowing for a more informed decision, but strategic interactions between multiple decision makers can reverse this gain. We apply these results to topics including media bias, advertising, product recommendations, voting, and auction disclosure. JEL: D82, L15, C72, D72.

1 Introduction

Experts often have strong biases. Lobbyists are paid to promote projects, stock analysts benefit from pushing stocks, and salespeople are paid to sell products. When an expert has a strong bias, can a decision maker still obtain useful information from the expert? If communication is via costless, unverifiable "cheap talk" then communication in a single dimension is possible if and only if the expert's incentive to exaggerate is not too large (Vincent Crawford and Joel Sobel, 1982). Consequently, most of the literature on communication by very biased experts assumes that credibility arises from the costliness of messages as in signaling games (e.g., Michael Spence, 1973) or from the verifiability of messages as in persuasion games (e.g., Paul Milgrom, 1981).

We reexamine the potential for cheap talk communication by an expert who is so biased as to always prefer the same action regardless of the state of the world. For instance, a biased salesperson wants to sell a product regardless of its true quality. Such state-independent preferences are common in the signaling and persuasion game literatures, but not in the cheap talk literature because they do not permit communication when the expert only has information in a single dimension. However,

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an expert often has information on multiple aspects of a decision-problem, e.g., a salesperson has information about multiple products or about multiple different attributes of a single product, so we consider environments with multidimensional information.

We find that the combination of multidimensional information and state-independent preferences is sufficient to ensure that the expert can credibly communicate through simple cheap talk and influence decision-making. The multidimensional nature of information allows the expert to make comparative statements that balance out her incentives to exaggerate, and the decision maker can use his knowledge of the expert's motives to infer the "dimensions" along which the expert has no incentive to deceive the decision-maker. As long as the expert's preferences are independent of her private information, the decision maker can adjust for the expert's biases and filter out the credible content of the expert's messages even when the expert favors some issues arbitrarily more than others.

Cheap talk is "persuasive" when it induces the decision maker to act to the benefit of the expert. Since influential communication makes the decision maker's updated estimates of the true state of the world more extreme relative to ex-ante expectations, the expert benefits from such communication when she prefers extreme estimates on some dimensions to average ones on every dimension. Therefore the expert benefits overall when her preferences over the decision maker's estimates are quasiconvex, i.e., her indifference curves are "bowed outwards." Indeed, cheap talk by an expert with quasiconvex preferences is persuasive not just in expectation but in all states of the world. Such preferences arise naturally in many environments where the literature has not recognized a role for cheap talk and we examine several in detail.

First, in a model of product recommendations, we show that a seller can credibly talk up a product by implicitly talking down another product. Such cheap talk helps the buyer make a better decision and, under the assumptions of standard discrete choice models, increases the probability of a sale. Second, in a voting model applied to jury decisions, we show how a defense attorney benefits from acknowledging the defendant's bad behavior in one dimension in the hope of persuading some jurors that he does not warrant conviction based on his better behavior in another dimension. Such cheap talk uniformly lowers the probability of conviction under the unanimity rule. Third, in an auction model, we show that a seller can credibly indicate to buyers the relative strengths of a product, thereby inducing a better match of the product with the buyer who values it most, but also weakening competition among buyers. Seller revenues rise as a result of the gain in allocational efficiency if and only if there are enough buyers to ensure sufficient competition. Fourth, in a monopoly model we show that a firm benefits from cheap talk advertising that emphasizes either the high quality of its product in a vertical dimension or the broadness of its appeal in a horizontal dimension.

In all of these situations the expert does not need to commit to a policy of revealing information. Instead the expert benefits from simple cheap talk that is credible and influential because of the state-independence of preferences and the multidimensional nature of communication, and that is persuasive because of the quasiconvexity of preferences. The information revealed is favorable on some dimensions and unfavorable on others but overall the expert faces no temptation to deviate from the communication strategy. This distinguishes our cheap talk approach from other models of communication that emphasize one-dimensional uncertainty and assume that the expert reveals even unfavorable information.[1]

Communication benefits a decision maker by allowing for more informed decision-making. However, if multiple decision makers play a game after hearing the expert's message, strategic interactions between them can sometimes offset the gains from better information.[2] In the auction and advertising models we find that cheap talk always benefits the buyers by allowing for more informed purchasing decisions, but in the voting model we find that cheap talk which benefits the speaker can hurt the voters. Each voter individually benefits from the ability to make a more informed decision, but this gain can be negated by the decisions of other voters with different preferences who also react to the information.

How much information can be revealed by cheap talk? Even if the expert has a strong incentive to exaggerate within each of N dimensions, there is always an $N-1$ dimensional subspace over which the expert has no incentive to deceive the decision maker. Therefore, full revelation on these $N-1$ "dimensions of agreement" may be possible.[3] With linear preferences we show that this limit is always attainable even with arbitrary distributions and arbitrary biases across dimensions through a series of increasingly detailed statements. For preferences that are strictly quasiconvex, detailed cheap talk using mixed strategies is also possible, with the expert's payoff strictly higher the more detailed is her speech.

These results rely on the assumption of state-independence, but some state-dependence will often be present in practice. In an online appendix containing extensions of the model, we show how communication is robust to small deviations from state-independence in three ways. First, if there are only a finite number of different types of expert preferences, then an influential equilibrium exists as long as the expert has information on a larger number of dimensions of interest to the decision maker(s). Second, if the probability that the expert's preferences are different from the expected type is sufficiently low, then under mild regularity conditions an equilibrium exists in which the decision maker still obtains information from the expected type while low probability types always recommend the same action. Finally, if preferences that are state-dependent converge uniformly to state-independent preferences, as is the case with Euclidean preferences converging to linear preferences, then any incentive to deviate from the cheap talk equilibria of the limiting preferences goes to zero.

We analyze how multiple dimensions of information can make cheap talk credible in environments where the literature has typically relied on the costliness or verifiability of messages to explain the credibility of communication. We do not consider other factors that can also generate credibility, such as reputational concerns (Joel Sobel, 1985; Roland Benabou and Guy Laroque, 1992; Stephen Morris, 2001; Matthew Gentzkow and Jesse Shapiro, 2006; Marco Ottaviani and Peter Sørenson, 2006), or competition between multiple experts (Thomas Gilligan and Keith Krehbiel, 1989; David Austen-Smith, 1993; Vijay Krishna and John Morgan, 2001; Marco Battaglini, 2002; Sendhil Mullainathan

[1] For instance, commitment to revealing unfavorable information is the assumption in most of the literature on seller communication in auctions following Paul Milgrom and Robert Weber (1982), and in most of the literature on information sharing between firms as surveyed by Xavier Vives (2001). Such revelation can also occur due to "unravelling" in a persuasion game with verifiable information (e.g., Milgrom, 2001).

[2] Following the analysis of Joseph Farrell and Robert Gibbons (1989), the literature on cheap talk with multiple audiences has focused on one-dimensional information.

[3] Marco Battaglini (2002) develops the idea of a dimension of agreement in the context of state-dependent preferences. See also David Spector's (2000) model with divergent priors in which debate reduces conflict to a single dimension.

and Vijay Shleifer, 2005; Attila Ambrus and Satoru Takahashi, 2008; Wolfgang Gick, 2006; Bauke Visser and Otto Swank, 2007).

In Section 2 we set up the model, demonstrate the existence of influential equilibria, identify conditions under which communication benefits the expert, and show when cheap talk can reveal fine information. Section 3 provides examples that illustrate the results, and Section 4 discusses the relationship of the results to the literature and concludes. All proofs are in the attached Appendix and an online Appendix contains robustness extensions of the model.

2 The Model

An expert privately knows the realization of a multidimensional state $\theta \in \mathbf{\Theta}$ where $\mathbf{\Theta}$ is a compact convex subset of $\mathbb{R}^N$ with a non-empty interior and $N \geq 2$. She sends advice in the form of a costless[4] unverifiable message m from an arbitrary set $\mathbf{M}$ to an uninformed decision maker (or decision makers) whose prior beliefs about θ are summarized by a joint distribution F with density f that has full support on $\mathbf{\Theta}$. A communication strategy for the expert specifies a probability distribution over messages in $\mathbf{M}$ as a function of the state θ. The decision maker estimates the expected value of θ given his priors, the expert's strategy, and the expert's message m. We represent these updated estimates or "actions" of the decision maker by $a = E[\theta|m]$. This standard behavioral assumption can be generated in different ways from the underlying preferences of the decision maker(s) as seen in Section 3.

The expert's preferences over the estimates a are described by a continuous utility function $U(a)$ that does not depend on the state θ and is common knowledge, i.e., are state-independent.[5] Given the specification of decision maker behavior, a (perfect Bayesian) equilibrium of this cheap talk game is fully specified by the expert's communication strategy.[6] A communication strategy is a cheap talk equilibrium if the expert has no incentive to deviate by "misreporting" information about θ. We say that an equilibrium is influential if different messages induce different estimates a with strictly positive probability.

As an initial example, consider a news network which has private information on the seriousness of two political scandals as represented by the random variables θ_1 and θ_2. Given a message m from the network, the network audience estimates the seriousness of the scandals as $a_1 = E[\theta_1|m]$ and $a_2 = E[\theta_2|m]$. The network's utility function is $U = \rho_1 a_1 + \rho_2 a_2$ where the network has a ratings bias to exaggerate the seriousness of either scandal, $\rho_i > 0$, and a possible partisan bias to exaggerate one scandal more, $\rho_1 \neq \rho_2$. Figure 1(a) shows the network's indifference curves for $\rho_1 = 4$, $\rho_2 = 1$ and state space $\mathbf{\Theta} = [0, 1]^2$.

Suppose this space is partitioned into regions $\mathbf{R}^+$ and $\mathbf{R}^-$ by a line h and the network's message m^+ or m^- indicates which region θ falls in. As h is spun around any interior point, $c = (1/2, 1/2)$ in this example, the estimates $a^+ = (E[\theta_1|m^+], E[\theta_2|m^+])$ and $a^- = (E[\theta_1|m^-], E[\theta_2|m^-])$ change continuously and eventually reverse themselves as seen by their "circular" path.[7] Therefore for some

[4] In practice the expert might have to pay to send a message (e.g., an advertising fee), or might receive a payment for sending a message (e.g., a subscription fee). This has no effect on equilibrium behavior as long as the amount paid or received does not vary with the message.

[5] As discussed in Section 4, this state-independence assumption is where we depart from the Crawford-Sobel model.

[6] We assume that all messages in $\mathbf{M}$ are used in equilibrium, and accordingly avoid specifying off-equilibrium-path beliefs. This is without loss of generality in a cheap talk game.

[7] For the pictured case of θ_i i.i.d. uniform, as h is rotated from the 45 degree line, the estimates shift from

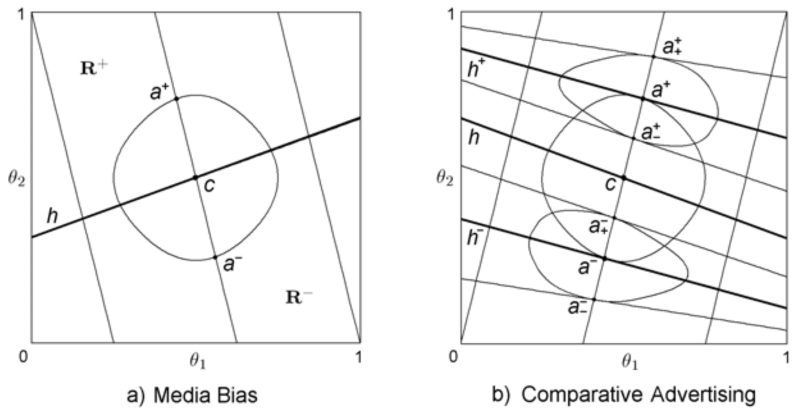

Figure 1: Equilibrium Construction

h the estimates a^+ and a^- must fall on the same indifference curve, implying that the incentive to lie about which region θ falls in is eliminated. In equilibrium if the network "plays up" the first scandal (message m^-) the estimate of its seriousness goes up only slightly, while if the network plays up the second scandal (message m^+) the estimate of its seriousness rises substantially.

The following Theorem shows that this argument does not depend on the specific choice of preferences and priors nor on the number of dimensions $N \geq 2$. Fixing any point c in the interior of $\mathbf{\Theta}$, a hyperplane h through c is identified by its orientation s, a point on the unit sphere $\mathbb{S}^{N-1} \subset \mathbb{R}^N$, that provides the gradient of h. Since the difference $U(a^+) - U(a^-)$ is a continuous odd map as a function of s, the difference must be zero for some orientation s^* (and associated hyperplane).[8] Therefore the expert gains nothing from misrepresenting which region the state θ is in, i.e., the communication strategy represented by the partition is an equilibrium. Since the regions $\mathbf{R}^+$ and $\mathbf{R}^-$ are convex the estimates are contained in each region and therefore distinct, $a^+ \neq a^-$, and since c is in the interior and f has full support, each estimate is induced with strictly positive probability. Therefore the equilibrium is influential.

Theorem 1 *An influential cheap talk equilibrium exists for all U and F.*

Cheap talk is credible because of an endogenous tradeoff whereby the expert gains in some dimensions but loses in others. This tradeoff allows for credible communication even when the expert wants a higher estimate in one dimension and a lower estimate in another dimension. Figure 1(b) shows such a case where $U = 4a_1 - a_2$, e.g., a political campaign wants to raise the perceived quality of its candidate and to lower that of the competing candidate. In equilibrium, the credibility of a positive message about its own candidate is sustained by an endogenous cost in the form of an improved public perception of the competing candidate. Similarly if the campaign says something negative about the competing candidate perceptions of both candidates must fall.[9] The figure shows

$a^+ = (E[\theta_1|\theta_1 < \theta_2], E[\theta_2|\theta_1 < \theta_2]) = (1/3, 2/3)$ and $a^- = (E[\theta_1|\theta_1 \geq \theta_2], E[\theta_2|\theta_1 \geq \theta_2]) = (2/3, 1/3)$, in which case $U(a^-) > U(a^+)$, to $a^+ = (2/3, 1/3)$ and $a^- = (1/3, 2/3)$, in which case $U(a^-) < U(a^+)$, and back again.

[8] This is a direct application of the Borsuk-Ulam theorem: every continuous odd map g from $\mathbb{S}^{N-1}$ to $\mathbb{R}^{N-1}$ must have the origin in its image, i.e., $g(s^*) = 0$ for some $s^* \in \mathbb{S}^{N-1}$ (see, e.g., Jiri Matousek, 2003). A map g is odd if $g(-s) = -g(s)$ for all s.

[9] This pattern is observed in political advertising (Richard Lau et al., 1999) and product advertising (Shailendra

this tradeoff for hyperplane h with estimates a^+ and a^- on the same rising indifference curve. The additional hyperplanes and estimates illustrate the multi-message equilibria in Theorems 3 and 4.

Theorem 1 establishes existence for any interior c so its position can be chosen to increase the value of cheap talk to the decision maker or the expert. For the decision maker, the value of cheap talk is limited if the space is partitioned such that almost all the probability mass corresponds to one of the messages, but this is avoided if c is the centerpoint of $\mathbf{\Theta}$, in which case by the centerpoint theorem for any h through c the probability mass of each half-space is at least $1/(N+1)$.[10] For the expert, Section 3 provides examples in which the choice of c ensures that the expert benefits strictly from cheap talk.

Any influential communication strategy induces a mean-preserving spread in the decision maker's updated estimates, implying that the expert benefits relative to remaining silent or "babbling" if her preferences are convex.[11] In any cheap talk *equilibrium* the weaker condition of quasiconvexity is sufficient. Since the lower contour sets of a quasiconvex U are convex, and since the prior estimate $E[\theta]$ is a convex combination of the posterior estimates $E[\theta|m]$ that lie on the same indifference curve in equilibrium, the prior estimate must lie on a lower indifference curve. In such cases we say that cheap talk is "persuasive."[12]

Theorem 2 *Relative to no communication, any influential cheap talk equilibrium (strictly) benefits the expert if U is (strictly) quasiconvex and (strictly) hurts the expert if U is (strictly) quasiconcave.*

Quasiconvexity implies that all the equilibrium estimates $E[\theta|m]$ offer higher utility than $E[\theta]$. So the expert benefits from communication not just in expected terms before learning her information, but benefits equally for every realization of her private information. As shown in Section 3, quasiconvexity emerges in many standard economic environments from the structure of the decision problem or from interactions between multiple decision makers, creating a role for persuasive pre-play cheap talk.

Regarding the payoffs to the decision maker(s), the partitional equilibria we consider always benefit a decision maker by allowing for a more informed decision (Blackwell, 1953), but if there are multiple decision makers that play a game after hearing the expert their interactions can offset this gain. The examples in Section 3 provide two cases where each of multiple decision makers benefits, and one case where the decision makers are all worse off from more information.

To investigate how an expert can reveal more information than in the equilibria of Theorem 1, we now consider "k-message equilibria" which induce the decision maker to make k distinct estimates. Consider again Figure 1(b) and suppose we follow the same procedure for the upper region to spin a new hyperplane h^+ around a^+, and also follow the same procedure for the lower region to spin a new hyperplane h^- around a^-. The resulting estimates for the top region are then a^+_+ and a^+_-, and

Jain and Steven Posovac, 2004). Mattias Polborn and David Yi (2006) model the phenomenon in a disclosure game.

[10] See, e.g., Matousek (2003). The centerpoint is a multidimensional generalization of the median and need not be unique. Note that the choice of h can also affect informativeness. For $N > 2$ the Borsuk-Ulam theorem implies that for *any* c in the interior of $\mathbf{\Theta}$ there is at least one h where both the differences $U(a^+)-U(a^-)$ and $\Pr[\theta \in \mathbf{R}^+]-\Pr[\theta \in \mathbf{R}^-]$ are equal to zero, implying each equilibrium estimate is induced with ex ante probability 1/2.

[11] A babbling equilibrium in which messages convey no information exists in any standard cheap talk game.

[12] The proof of Theorem 2 uses the even weaker condition that the lower contour set for the indifference curve through $E[\theta]$ is convex, rather than all lower contour sets being convex as implied by quasiconvexity. Also, by relying on the law of iterated expectations, the proof uses the fact that the a's are expectations, in contrast to Theorem 1 which only uses the continuity of the estimates a in the orientation s. Nothing substantive changes if the a_i's are instead expectations of monotone transforms of the θ_i's.

for the lower region are a_+^- and a_-^-. Since the estimates are all on the same linear indifference curve through $E[\theta]$, the expert has no incentive to lie and the partition is a four-message equilibrium. To extend this procedure to more messages and more dimensions, consider linear preferences of the form

$$U(a) = \rho \cdot a \tag{1}$$

where the $\rho = (\rho_1, ..., \rho_N)$ is a vector of real numbers that captures the expert's bias or "slant" across dimensions. For the following result we show that it is possible to repeatedly apply Theorem 1 as described above to obtain an arbitrarily fine partition in $N - 1$ dimensions.

Theorem 3 *An influential cheap talk equilibrium revealing almost all information on $N-1$ dimensions exists if U is linear.*

To see the result for two dimensions, consider a 2^k-message equilibrium where each region is repeatedly subdivided by a hyperplane through the centerpoint of the region as in Figure 1(b). By the centerpoint theorem each element has probability mass at most $(1 - 1/(N+1))^k$ which goes to zero as k increases. The full support assumption implies that there must lie an equilibrium estimate within any $\varepsilon > 0$ of any point on the equilibrium indifference line for k large enough. In this sense, the expert reveals almost all information in one dimension of the two-dimensional space as k becomes large. To extend this to three or more dimensions, we utilize the extra degrees of freedom given by the fact that $N > 2$ and "slice and dice" the successive partition elements in order to obtain equilibrium estimates that for large k form an arbitrarily fine grid of the expert's $N-1$ dimensional equilibrium indifference hyperplane.

Theorem 3 returns to an idea due to Marco Battaglini (2002) that in two dimensions it can be possible to reveal full information in a one-dimensional subspace (the dimension of agreement) on which there is no conflict of interest. As Battaglini noted, for state-dependent preferences such revelation can only occur in special cases with special distributions.[13] We find that such revelation is possible generally for linear preferences. Moreover, as shown in the online Appendix, linear preferences are of particular interest since they are the limiting case of standard Euclidean preferences as biases become large.

Now consider the potential for multiple messages when preferences are not linear. When we divide the initial two regions further as in Figure 1(b), we run into the problem that application of Theorem 1 to each region ensures that $U(a_+^+) = U(a_-^+)$ and $U(a_+^-) = U(a_-^-)$, but without linearity there is no assurance that all four estimates are on the same indifference curve. The following Theorem shows that, when preferences are quasiconcave or quasiconvex, mixed messages can be used to equalize the payoffs and ensure that a cheap talk equilibrium with an arbitrarily large number of influential messages exists.

Theorem 4 *A 2^k-message influential cheap talk equilibrium exists for every $k \geq 1$ if U is strictly quasiconvex, with the expert's payoff strictly increasing in k.*

The proof uses an inductive argument that creates a 2^k-message equilibrium from a 2^{k-1}-message equilibrium. We illustrate the logic here by creating a four-message equilibrium from a two-message one. Starting from a two-message equilibrium with induced estimates a^+, a^- we find a hyperplane

[13] Battaglini discusses the difficulty of such revelation as a step toward understanding why multiple experts are needed to obtain full revelation in all dimensions.

for each of its halfspaces and four corresponding estimates satisfying $U(a_+^+) = U(a_-^+)$ and $U(a_+^-) = U(a_-^-)$. If, coincidentally, $U(a_+^+) = U(a_-^+) = U(a_+^-) = U(a_-^-)$, we have a four-message equilibrium. Suppose instead that $U(a_+^+) = U(a_-^+) > U(a_+^-) = U(a_-^-)$. Due to the strict quasiconvexity of U, we must also have $U(a_+^-) = U(a_-^-) > U(a^+) = U(a^-)$. If the expert mixes over the messages for a_+^+ and a_-^+ when $\theta \in \mathbf{R}^+$, the informativeness of each message falls and the expectations for the messages become closer, eventually both equalling a^+. By continuity, there must be some mix such that the expert's utility from mixing is the same as $U(a_+^-) = U(a_-^-)$, implying a four-message equilibrium exists in which two of the messages are mixed over.

This procedure can be extended to generate equilibria with 2^k messages for any k and all $N \geq 2$. By strict quasiconvexity, the expert's payoff is strictly increasing in k. Since all the induced estimates lie on the same indifference curve, this mixed message equilibrium is also a sequential cheap talk equilibrium in that the expert can make successive statements corresponding to the successive stages of the procedure. In this sense the "longer" that cheap talk continues, the more information that is revealed.[14]

3 Applications

We now analyze several applications where the richness of the expert's information ensures the credibility of cheap talk, and the quasiconvexity of the expert's preferences implies that the expert benefits from such communication. The first application considers the simplest case of a single decision maker and the subsequent applications consider multiple decision makers who may play a game after hearing the expert's message.

3.1 Persuasive Recommendations

Consider a decision maker who faces a choice between multiple alternatives and consults an expert who has a strong bias against one choice. For instance, a salesperson advises a customer who might buy one of multiple products, but might also walk away and leave the salesperson without a sale. Or an industry lobbyist advises a senator who might support one of several proposals to help the industry, but might also decline to support any. Can cheap talk be credible and benefit the expert by lowering the probability of the undesired choice?

To analyze this question, suppose a buyer might purchase one of N different products or nothing at all from a seller who seeks to maximize the probability of a sale.[15] The utility of purchasing product i is v_i, which is a strictly increasing linear function of a quality variable θ_i known only by the seller.[16] The utility from not buying is ε, a random variable with prior G that is independent of θ and known only by the buyer. Given estimated quality $a_i = E[\theta_i|m]$ the probability that the

[14] Vijay Krishna and John Morgan (2004) and Robert Aumann and Sergio Hart (2003) consider mixed strategies in multi-stage cheap talk games. Whereas the multi-stage aspect of communication is central to their results, in our game the communication may also be one-stage.

[15] In this application we focus on the asymmetry between the choices that the expert prefers and the one that she does not. Using the multinomial logit model introduced below, the analysis can be extended to allow the expert to favor some of the preferred choices more than others. Quasiconvexity holds as long as the payoffs from the preferred choices are sufficiently similar relative to that from the unpreferred choice.

[16] Utility v_i is net of a fixed price p_i. In Application C below the price is endogenously determined by bidders in an auction while in Application D it is set in competition between sellers.

buyer buys a product is just $\Pr[\max_i\{v_i(a_i)\} \geq \varepsilon]$, so the seller's utility is

$$U = G(\max_i\{v_i(a)\}). \tag{2}$$

Since U is continuous, influential cheap talk is possible by Theorem 1, and since U is a monotonic function of the convex $\max\{\cdot\}$ function and therefore quasiconvex, the seller benefits from such cheap talk by Theorem 2.[17] The buyer also benefits from the extra information since he can make a more informed choice. The next result follows immediately.

Proposition 1 *Seller recommendations are credible, benefit the seller by increasing the probability of a sale, and benefit the buyer by providing more information.*

Figure 2(a) shows the case of $N = 2$ where $v_i = a_i$ and the θ_i and ε are i.i.d. uniform on $[0, 1]$. Without communication the expected quality of either good is $E[\theta_i] = 1/2$ so the probability of a sale is $G(1/2) = 1/2$, while with cheap talk that indicates which product is better the expected quality of the better good is $E[\max_i\{\theta_i\}] = 2/3$ so the probability of a sale is $G(2/3) = 2/3$. Therefore both the buyer and seller are better off. As seen in the figure from the indifference curves representing iso-probability contours of U, the lower contour sets are convex, so U is quasiconvex and a^+ and a^- are on a higher indifference curve than $c = E[\theta]$.

Following a standard discrete choice model, the analysis can be extended to accommodate more uncertainty over the buyer's choices. In particular, suppose that the buyer's utility from each product is $v_i + \varepsilon_i$ where ε_i is known only by the buyer and independent of θ. Under the standard multinomial logit model in which ε_i and ε follow independent Type 1 extreme value distributions, Daniel McFadden (1973) shows that the probability of good i being purchased is $e^{v_i(a_i)}/(1+\Sigma_i e^{v_i(a_i)})$, implying that $U(a) = 1-1/(1+\Sigma_i e^{v_i(a_i)})$. Note that U is a monotonic function of $\Sigma_i e^{v_i(a_i)}$, the sum of strictly convex monotone transformations of linear functions. Therefore, although U is not convex, it is strictly quasiconvex, implying by Theorem 2 that the seller always benefits from influential cheap talk.[18]

3.2 Influencing Voters

Now consider a discrete choice problem involving multiple decision makers voting on a collective decision after listening to an expert. In particular, consider a jury trial with heterogeneous jurors influenced by different aspects of a case, e.g., whether or not the defendant is technically guilty, and whether or not the defendant deserves punishment. Let θ_i capture the strength of the case on aspect i which is the defense attorney's private information. Jurors in group $i = 1, 2$ have payoff $\theta_i - \tau_i$ from conviction and payoff 0 from acquittal. The threshold τ_i privately known by the jurors is distributed on $[0, 1]$ independently of θ and τ_{-i} according to G_i with continuous density g_i. The probability that jurors in group i vote for conviction given estimate a_i is then $G_i(a_i)$ so, if conviction requires a unanimous vote, the probability of conviction is the intersection $G_1(a_1)G_2(a_2)$ and the probability of acquittal is

$$U(a) = 1 - G_1(a_1)G_2(a_2). \tag{3}$$

[17] Even though U is not strictly quasiconvex, the seller can reach a strictly higher indifference curve via cheap talk if $\max_i\{v_i(\theta_i)\}$ varies in θ and c is chosen to be at the kink of the indifference curve through $E[\theta]$. The choice of c in ensuring a strict gain to the expert is discussed further in Application C.

[18] More generally, if G is the joint distribution of $\varepsilon_1 - \varepsilon, ..., \varepsilon_N - \varepsilon$, then the probability of no sale is $G(-v(a))$ so $U(a) = 1 - G(-v(a))$ is quasiconvex if $G(-v(a))$ is quasiconcave.

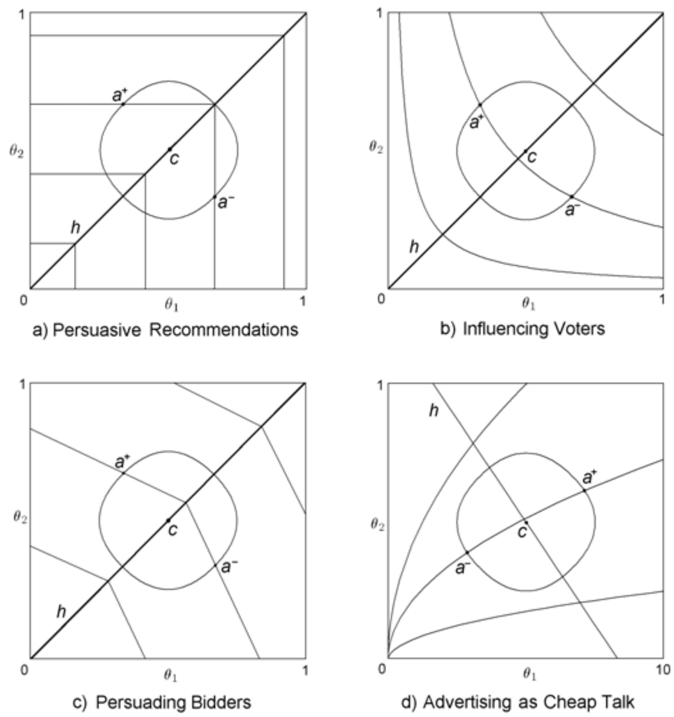

Figure 2: Applications

Since U is continuous, Theorem 1 implies that a defense attorney maximizing U can engage in credible and influential cheap talk, e.g., emphasize that the defendant is of strong moral character (even if he might be guilty), or emphasize that the client is innocent (even if he might be a scoundrel). As shown in the proof of the following Proposition, if G_i is log-concave then U is a strictly quasiconvex function in the interior of $[0,1]^2$, so by Theorem 2 the defense benefits from communication regardless of the merits of the case θ.[19] More information benefits each individual juror in isolation, but under the unanimity rule the jurors favoring conviction are hurt if communication induces other jurors to switch to favoring acquittal. As a result jurors can lose overall in expected terms and always do so if, for instance, G_i is uniform.

Proposition 2 *Under the unanimity rule cheap talk by the defense strictly lowers the probability of conviction if each G_i is log-concave. Jurors can each be worse off in expectation from such cheap talk.*

The unanimity rule encourages the defense to provide information in the hope that at least one type of voter will be persuaded to vote for acquittal. By the same token if the prosecution knew θ it would be worse off from any communication via cheap talk. Figure 2(b) shows a two-message equilibrium for i.i.d. uniform θ_i and i.i.d. uniform τ_i with the defense's quasiconvex

[19] Since the probability of conviction is strictly quasiconcave for any N for logconcave G_i, the result extends to $N > 2$ different voters interested in different aspects of the case. Interdependence between the τ_i, which can capture different weights on different issues and differently sized groups of voters with similar preferences, can be allowed for by considering the quasiconcavity of the joint distribution G evaluated at a.

preferences $U = 1 - a_1 a_2$ where utility is decreasing away from the origin as the probability of conviction increases. Since τ_i is uniform, given estimates a, each juror's expected payoff conditional on conviction, $a_i - E[\tau_i | \tau_i < a_i]$, is linear in a_i. Therefore, on average across messages, each juror earns the same expected payoff conditional on conviction in any equilibrium. More information then hurts each juror by lowering the probability with which conviction occurs. By Theorem 4 this two-message equilibrium can be extended to an arbitrarily "long" informative speech involving additional mixed messages where the probability of ultimate acquittal, and the loss to the jurors, are both strictly increasing in the length of the speech.

In the context of elections, this result suggests that a candidate who needs support from only one of two groups of voters can always benefit from being informative about intended policies, while a candidate with a smaller base who needs bipartisan support from both groups is better off babbling. Generally, persuasive cheap talk must have a divisive effect in equilibrium by attracting one group of voters at the expense of alienating the other, so voting rules affect communication via the incentive they generate to be divisive or not. This result adds to the long literature following Condorcet on how voting rules affect the aggregation of information among voters (e.g., Timothy Feddersen and Wolfgang Pesendorfer, 1998; Cesar Martinelli, 2002). Peter Coughlan (2000) and David Austen-Smith and Timothy Feddersen (2006) extend this literature to consider cheap talk between voters before voting, and show that the unanimity rule for jury convictions discourages information flows among informed jurors with different preferences. Our results show that an outside expert can benefit from cheap talk to voters, and such communication can make all voters worse off in expectation.

3.3 Disclosure in Auctions

Now consider when a seller with information about multiple attributes of a single product can benefit from cheap talk that favors one attribute over another. We analyze this question in the context of a private value auction in which buyers care differently about different product attributes. Information about the product's true strengths increases the allocational efficiency of the auction by making it more likely that buyers who value those strengths win the auction. But such information also softens competition by increasing the spread in buyer valuations between those buyers and the buyers who value other attributes.

To see whether there is a net benefit to the seller from cheap talk, suppose a seller with information on $N = 2$ attributes of an object engages in cheap talk with $n \geq 2$ potential buyers prior to holding a second-price auction. Buyers have correlated values, $v_j = \lambda_j \theta_1 + (1 - \lambda_j)\theta_2$, where the seller privately knows $\theta \in [0,1]^2$, each bidder j privately knows $\lambda_j \in [0,1]$, where λ_j is independent of θ and of λ_i, $i \neq j$.[20] For instance, we may think of θ_1 (θ_2) as the short-run (long-run) value of a product with λ_j capturing the time preference of buyer $j = 1, ..., n$. Each buyer's dominant strategy is to bid his expected value $E[v_j|m] = \lambda_j a_1 + (1 - \lambda_j) a_2$ given the seller's message m and associated estimates $a = E[\theta|m]$. Therefore the seller's expected revenue is the expected second-highest bid,

$$U(a) = E[2^{nd} \max_{\lambda_j} \{\lambda_j a_1 + (1 - \lambda_j) a_2\}]. \tag{4}$$

[20] This formulation of buyer valuations ensures that the identity of the winning bidder varies with (buyer estimates) of the seller's information θ. As long as this is guaranteed, the results remain qualitatively unchanged. See Simon Board (2009) for similar conditions for the case where the seller can commit to a disclosure policy.

Since U is continuous in a, by Theorem 1 the seller can credibly disclose information through cheap talk for all n and any priors on θ and λ.

The following Proposition shows that (4) reduces to a concave $\min\{\cdot\}$ function for $n = 2$ since the second highest of two bids is the minimum bid. It is linear for $n = 3$ and reduces to a convex $\max\{\cdot\}$ function for $n > 3$. Since concavity (convexity) implies quasiconcavity (quasiconvexity), the seller benefits from communication if and only if there are four or more bidders, where a strict benefit is ensured by selection of the point c. The proposition also confirms that buyers are always at least weakly better off from the gain in allocational efficiency, and that they are strictly better off if there are four or more bidders.

Proposition 3 *Seller cheap talk strictly increases seller revenues if and only if $n \geq 4$. Expected buyer rents and total surplus are weakly higher regardless of n and strictly so if $n \geq 4$.*

As an example consider the symmetric case where both the θ_i and the λ_j are i.i.d. uniformly distributed on $[0, 1]$. If there is no communication then $a_1 = a_2 = 1/2$ so every bidder has the same expected value and seller revenues are $1/2$ for any n. If the seller discloses whether or not $\theta_1 \geq \theta_2$, then $a_1 = 2/3$ and $a_2 = 1/3$ or vice versa. In the former case the expected price is the bid of the buyer with the second highest value of λ_j, and in the latter case it is the bid of the buyer with the second lowest value of λ_j. Following standard order statistic calculations, $E[\lambda_{j:n}] = \frac{j}{n+1}$, so the expected second highest bid in either case is $\frac{2}{n+1}\frac{1}{3} + \frac{n-1}{n+1}\frac{2}{3} = \frac{2}{3}\frac{n}{n+1}$ which is increasing in n, approaches $2/3$ in the limit of perfect competition between buyers, and exceeds $1/2$ for $n \geq 4$. Figure 2(c) shows the seller's quasiconvex preferences when $n = 5$.

If instead of a second price auction we had a first price auction, then by the revenue equivalence theorem, for each value of the estimates a the expected revenue of the seller would still be given by the expression for $U(a)$ derived above. It follows that the set of informative cheap talk equilibria and their revenue implications are identical across all auction formats for which standard revenue equivalence results obtain.[21]

This result adds to the existing literature on auction disclosure which assumes commitment to revealing truthful, verifiable information. This literature shows that with a large enough number of bidders, the seller gains from disclosure *on average* (Juan-José Ganuza, 2004; Simon Board, 2009),[22] but disclosure may increase or decrease revenues depending on the seller's exact information. With multidimensional information the seller can credibly communicate through unverifiable cheap talk and, with sufficiently many bidders, the seller benefits for any realization of her information so there is no need for commitment.

3.4 Advertising as Cheap Talk

Due to the conflict of interest between an advertiser and buyers, the advertising literature has not examined whether the content of an advertisement can be credible even when it cannot be verified.

[21] We assume there are no reserve prices which is optimal when the seller's reservation value is lower than the lowest possible virtual valuation of the buyers (e.g., Roger Myerson, 1981).

[22] The ex ante gains in allocation efficiency due to seller communication are also addressed by Juan-José Ganuza and José Penalva (2006) and Dirk Bergemann and Martin Pesendorfer (2007). Archishman Chakraborty, Nandini Gupta, and Rick Harbaugh (2006) consider the gains from credible seller communication about multiple goods based on the linkage principle (Paul Milgrom and Robert Weber, 1982). Colin Campbell (1998) and Antonio Miralles (2008) consider the gains to buyers from credible communication among them about which goods to bid on.

As an example of how information about multiple attributes of a product allows for cheap talk to have a role in advertising, we consider a model where consumers are unsure of the nature of a good supplied by a monopolist both along the vertical and the horizontal dimension.

Suppose that a monopolist firm, located at 0 faces a unit mass of consumers distributed uniformly on $[0, 1]$. A consumer located at x values the product of the firm at $v - tx$ less the price p where v captures a vertical aspect that is independent of consumer characteristics and t captures a horizontal aspect that measures whether the product is a niche product or has more mass appeal. For instance, v could represent the quality of a camera and t could represent its difficulty of use, or v could represent the expected return on an asset and t could represent its riskiness. We suppose that both v and t are the firm's private information, marginal costs are zero, and as detailed in the proof of Proposition 4, the parameters are such that some but not all consumers buy the product.

Prior to setting its price the firm can engage in cheap talk advertising about v and t. Consistent with the cheap talk assumption, any costs of advertising do not vary with the firm's private information or the content m. Let $a_1 = E[t|m]$ and $a_2 = E[v|m]$ be the estimates commonly held by consumers given a message m from the entrant. The firm's equilibrium profits (excluding any fixed advertising costs) are, as shown later in the proof of the proposition,

$$U(a) = \frac{a_2^2}{4a_1}. \tag{5}$$

Since $U(a)$ is continuous, by Theorem 1 there exists an influential cheap talk equilibrium. Furthermore, $U(a)$ is also strictly quasiconvex so that by Theorem 2 cheap talk strictly raises the firm's profits with probability one. The following Proposition shows generally that the payoffs of the firm and each type of consumer are convex so everyone is better off in ex-ante expected terms from a better understanding of the monopolist's product.[23]

Proposition 4 *Cheap talk advertising by a monopolist is Pareto improving.*

As seen in the two-message equilibrium of Figure 2(d) for the case where t $(= \theta_1)$ and v $(= \theta_2)$ are independently and uniformly distributed, the firm can credibly emphasize either quality or breadth of appeal.[24] Emphasizing the product's quality allows for a higher price to a smaller set of buyers, while emphasizing its mass appeal allows for more sales but at a lower price. In the first case the demand curve becomes higher and steeper, while in the second case it becomes lower and flatter, and in either case the firm benefits relative to not communicating at all. Justin Johnson and David Myatt (2006) show general conditions under which such rotations in the demand curve increase a firm's profits - this example shows that such rotations can be induced by advertising that is pure cheap talk. Unlike in a signaling model of advertising (e.g., Paul Milgrom and John Roberts, 1986), the costs and benefits of advertising are not state-dependent so consumers do not learn about the product from the expense of advertising, but rather learn from the content of advertising. And unlike in a disclosure/persuasion game model of advertising (e.g., Simon Anderson and Régis Renault, 2006) there is no restriction that the content must be verifiable.

[23] Advertising costs are given so the comparison is between advertising with and without content on product attributes.

[24] The restriction that not all consumers is consumed is satisfied for $a_1 \geq a_2/2$, so the extreme upper left region does not satisfy this restriction.

4 Discussion and Conclusion

In this section we compare our approach to multidimensional cheap talk with the literature, and indicate some of the remaining issues to be resolved. In a model with two competing experts, Battaglini (2002) shows how multidimensional information permits communication in settings where credible communication would not always be possible in one dimension. Using a multidimensional extension of the Crawford and Sobel (1982) model of communication, he finds that in equilibrium each expert can reveal information orthogonal to her own interests, and that the decision maker can thereby infer the exact state of the world from both sources of information. However, it is only the presence of the other expert that allows each expert to credibly reveal information, so this approach does not work for a single expert.

Archishman Chakraborty and Rick Harbaugh (2007) analyze communication in which a single expert discloses complete or partial rankings of multiple variables, e.g., a stock analyst provides a ranking of different stocks or a categorization of stocks into "buy", "hold" and "sell" groupings. They find that a complementarity condition on preferences is sufficient for such "comparative cheap talk" to be influential when preferences and priors are sufficiently symmetric across dimensions.[25]CHANGED As shown by Gilat Levy and Ronny Razin (2007), the assumption of sufficient symmetry can be necessary in that communication can sometimes break down when preferences or priors are sufficiently asymmetric.

The present paper combines insights from these approaches to show how, even with arbitrary asymmetries, a single expert can reveal detailed comparative information along all but one dimension. The key assumption that differs from the literature is that the expert's preferences over the decision maker's estimates are state-independent and common knowledge. In contrast, in the Crawford and Sobel model the expert prefers to induce an estimate that differs from the true state of the world by the expert's bias. Therefore, even when this bias is common knowledge, the expert's preferences over the decision maker's estimates are state-dependent.

State-dependence is necessary in a one-dimensional model to introduce non-trivial commonality of interest between the expert and decision maker, e.g., the expert has a bias toward the decision maker having a higher estimate but this bias is limited so the expert does not want to exaggerate too much.[26] With multidimensional information, even if the expert has a strict preference ordering over the decision maker's estimates in each dimension, e.g., always prefers a higher estimate, the expert will not have a strict ordering over all estimate vectors if preferences are continuous. Therefore the absence of conflict along some dimensions – and the potential for cheap talk along those dimensions when they are common knowledge – is inherent to multidimensional environments. By assuming state-independence, we are able to focus purely on this property of multiple dimensions of information.

Our model applies best to situations where the expert has a strong incentive to exaggerate,

[25]The complementarity condition is satisfied weakly in this paper due to the state-independence assumption. In two dimensions the equilibria in Theorem 1 of this paper are a generalization of the comparative cheap talk equilibria they examine. Campbell (1998) provides an early application of comparative cheap talk in auctions.

[26]If $\boldsymbol{\Theta}$ were one-dimensional in an environment with state-independent preferences, the hyperplane h corresponding to the construction in Theorem 1 would just be a point dividing the interval into regions corresponding to a "good" or "bad" message. An influential equilibrium exists in this case if and only if the expert switches which of the two estimates associated with each region is preferred as h varies over the interval. Such a switching condition cannot be satisfied, for example, by monotonic preferences.

implying both that the potential for communication in a single dimension is limited and that the intricacies of state-dependence can be abstracted away from. In fact, the assumption of an arbitrarily large bias toward a higher estimate is standard in the literatures on signaling (Spence, 1973), screening (Stiglitz, 1975), and persuasion/disclosure (Milgrom, 1981; Masahiro Okuno-Fujiwara, Andrew Postlewaite, and Kotaro Suzumura, 1990; Jacob Glazer and Ariel Rubinstein, 2004). Therefore our focus on such preferences shows that cheap talk can be applied to the wide range of environments which previously have been analyzed using only these other models of communication.

This idea that a strong incentive to exaggerate in each dimension can be captured by a simple model of state-independent preferences is formalized in Proposition 7 in the online Appendix. We show that standard state-dependent Euclidean preferences converge uniformly to transparent linear preferences as the biases in each dimension increase in a fixed ratio. Moreover, the expert's biases across dimensions (the expert's "slant") converges to that of the ratios of the biases within dimensions. This provides a close link between the idea of an expert being biased towards a higher action as developed in the Crawford and Sobel model, and the idea of an expert being biased across dimensions as emphasized in this paper.

Recent research has considered how uncertainty over the expert's bias in a single dimension in the Crawford and Sobel model affects communication.[27] In our context the corresponding question is how communication is affected by uncertainty over the expert's bias across dimensions. Our assumption of state-independence implies that any such biases are transparent to the decision maker, but in the online Appendix we analyze two situations where our approach can be used to show that communication remains possible despite limited uncertainty. Proposition 5 shows that communication is still possible when there are more dimensions of information than types of experts, e.g., a salesperson might be biased or not toward the products of one of two companies and there are at least three products the salesperson is informed about. And Proposition 6 shows that communication is still possible when the expert is likely to be biased in one direction but there is a small chance that she is biased in another direction. The general question of how uncertainty about the expert's biases affects communication in multidimensional environments remains open.

A Appendix[28]

Proof of Theorem 1: We look for an influential cheap talk equilibrium involving a single hyperplane $h_{s,c}$ of orientation $s \in \mathbb{S}^{N-1}$ passing through $c \in int(\mathbf{\Theta})$ that partitions $\mathbf{\Theta}$ into two nonempty sets $\mathbf{R}^+(h_{s,c})$ and its complement $\mathbf{R}^-(h_{s,c})$, with corresponding receiver actions $a^+(h_{s,c})$ and $a^-(h_{s,c})$. Let $\mathbf{R}^+(h_{s,c})$ be in the halfspace that contains the point $s + c$. Notice first that under the assumed conditions on priors, $a^+(h_{s,c}) \in int(\mathbf{R}^+(h_{s,c}))$ and $a^- \in int(\mathbf{R}^-(h_{s,c}))$, implying in particular that $a^+(h_{s,c}) \neq a^-(h_{s,c})$ so that any such equilibrium, if it exists, is influential. Furthermore, $a^+(h_{s,c})$ and $a^-(h_{s,c})$ are continuous functions of s (with the subspace topology for $\mathbb{S}^{N-1}$) for any

[27] John Morgan and Phillip Stocken (2003) consider the Crawford and Sobel model when the expert might be biased or unbiased and their results imply that such uncertainty can improve communication compared to the case where the expert has a known but intermediate bias. Vassilios Dimitrakas and Yianis Sarafidis (2005) show that revelation of the expert's bias can hurt communication when the size of the possible bias is uncertain, and Ming Li and Kristof Madarasz (2008) show that revelation always hurts communication when the direction of the bias is uncertain. An uncertain expert bias is also analyzed by Joel Sobel (1985), Roland Benabou and Guy Laroque (1992), Sidartha Gordon (2006), Stephen Morris (2001), Roman Inderst and Marco Ottaviani (2007), and Giuseppe Moscarini (2007).

[28] Additional Appendix available online.

fixed $c \in int(\mathbf{\Theta})$. Notice next that for any two antipodal orientations $-s, s \in \mathbb{S}^{N-1}$, we must have $\mathbf{R}^{+}(h_{s,c}) = \mathbf{R}^{-}(h_{-s,c})$ and so $\mathbf{R}^{-}(h_{s,c}) = \mathbf{R}^{+}(h_{-s,c})$. It follows that $a^{+}(h_{s,c}) = a^{-}(h_{-s,c})$ implying in particular that the map $\Delta(\cdot;c) : \mathbb{S}^{N-1} \to \mathbb{R}$ defined by $\Delta(s,c) = U(a^{-}(h_{s,c})) - U(a^{+}(h_{s,c}))$ is a continuous odd function of s. By the Borsuk-Ulam theorem, there exists $s^{*} \in \mathbb{S}^{N-1}$ such that $\Delta(s^{*}, c) = 0$. The hyperplane through c with orientation s^{*} generates a two-message convex partitional equilibrium.

Proof of Theorem 2: Recall that U is a (strictly) quasiconvex function iff every lower contour set $W(x) = \{a|U(a) \leq U(x)\}$ is (strictly) convex, i.e., for all $x', x'' \in W(x)$, $x' \neq x''$ and $\lambda \in (0,1)$, $U(\lambda x' + (1-\lambda)x'') < U(x)$. Similarly, U is (strictly) quasiconcave iff every upper contour set $B(x) = \{a|U(a) \geq U(x)\}$ is (strictly) convex. Given prior F, we prove the slightly stronger result that communication (strictly) benefits the expert if $W(E[\theta])$ is (strictly) convex and (strictly) hurts if $B(E[\theta])$ is (strictly) convex. Consider any k message equilibrium with induced actions $a^{1}, ..., a^{k}$ such that the actions satisfy $a^{j} = E[\theta|m^{k}]$ where m^{k} is a message that induces action a^{k}. Since all the induced actions belong to the same level set of U in equilibrium, they must all belong either to $W(E[\theta])$ or to $B(E[\theta])$. Suppose $W(E[\theta])$ is strictly convex, but contrary to the claim, the induced actions $a^{1}, ..., a^{k}$ belong to $W(E[\theta])$. By strict convexity of $W(E[\theta])$, every convex combination of the induced actions must then yield utility strictly less than $E[\theta]$. But this is impossible since, by the law of iterated expectations, $E[\theta]$ is itself a convex combination of the induced actions, $E[\theta] = \sum_{j=1}^{k} p^{j} a^{j}$ where $p^{j} > 0$ is the probability with which a^{j} is induced. This proves the strict version of the first part of the claim. The argument also shows that if $W(E[\theta])$ is convex but not strictly so, communication may only weakly raise the expert's payoffs. Symmetric remarks apply to the case where $B(E[\theta])$ is (strictly) convex.

Proof of Theorem 3. The discussion in the text establishes the existence of 2^{k} message equilibrium from a 2^{k-1} message equilibrium, $k \geq 2$, by choosing an interior point c for each partition element, applying Theorem 1 to find an equilibrium hyperplane, and then exploiting the linearity of U and applying the law of iterated expectations. Note now that for any such 2^{k}-message equilibrium, all induced actions belong to the set $\overline{\mathbf{A}} = \{a|\rho \cdot a = \rho \cdot E[\theta]\}$. We wish to demonstrate that for every $\varepsilon > 0$, we can find a 2^{k}-message equilibrium such that there exists an induced action $a^{*} \in \overline{\mathbf{A}}$ that is is within ε distance of a for every $a \in \overline{\mathbf{A}}$. In this sense, the set $\overline{\mathbf{A}}$ is asymptotically (in k) dense in the induced actions and we say that the expert reveals all information in the $N-1$ dimensions corresponding to the $N-1$ dimensional compact set $\overline{\mathbf{A}}$.

First consider the case where $N = 2$ and fix $\varepsilon > 0$. Consider the ball $B_{\varepsilon}(a)$, open in $\mathbb{R}^{2}$, of radius ε and centered around $a \in \overline{\mathbf{A}}$. Notice next that for k large enough, there exists an element $\mathbf{P}$ of the equilibrium partition such that $\mathbf{P} \cap \overline{\mathbf{A}} \subset B_{\varepsilon}(a)$. This follows from the centerpoint theorem that each element has probability mass at most $(1 - 1/(N+1))^{k} < \min_{a \in \overline{\mathbf{A}}} \Pr\{\theta \in B_{\varepsilon}(a)\}$, for k large enough, provided that in each stage of creating successively finer partitions in the equilibrium construction above we choose the interior point c through which the corresponding hyperplane h passes as the centerpoint of the corresponding convex compact partition element. But then the equilibrium action $a^{*} \in \mathbf{P} \cap \overline{\mathbf{A}}$ corresponding to the element $\mathbf{P}$ must lie within ε of a.

Next consider the case where $N > 2$. We construct a 2^{k}-message equilibrium with the desired property as follows. Introduce $N-2$ fictitious linear preferences $U^{j_1}(a) = \rho^{j_1} \cdot a$ for $j_1 = 1, ..., N-2$. We use the index $j_1 = 0$ to denote the actual expert preferences, i.e., $\rho^{0} = \rho$, and assume that all ρ^{j_1} are linearly independent. Using the Borsuk-Ulam theorem and the law of iterated expectations, we can construct a 2^k-element partition of $\mathbf{\Theta}$ with the property that the resulting 2^k action profiles lie on a unique line $\overline{\mathbf{A}}_1 = \{a|\rho^{j_1} \cdot a = \rho^{j_1} \cdot E[\theta], j_1 = 0, ..., N-2\} \subset \overline{\mathbf{A}}$. We can then choose a second distinct set of $N-2$ linearly independent fictitious types and construct a 2^k-element partition, for each element $i = 1, ..., 2^k$ of the 2^k-element partition obtained in the previous step, treating that element as the entire state-space, following the same procedure as above. The resulting 2^k action profiles lie on a unique line $\overline{\mathbf{A}}_{2,i} = \{a|\rho^{j_{2,i}} \cdot a = \rho^{j_{2,i}} \cdot E[\theta], j_{2,i} = 0, ..., N-2\} \subset \overline{\mathbf{A}}$ that can be chosen to be orthogonal to $\overline{\mathbf{A}}_1$ for each $i = 1, ..., 2^k$, for a total of 2^{2k} actions. Repeat this procedure $N-1$ times, at each step using a new set of $N-2$ distinct fictitious preferences to obtain successively the lines $\overline{\mathbf{A}}_1, \{\overline{\mathbf{A}}_{2,i}\}_{i=1}^{2^k}, ..., \{\overline{\mathbf{A}}_{N-1,i}\}_{i=1}^{2^{(N-2)k}}$. Notice that the resultant actions and associated partition of $\mathbf{\Theta}$ is a $2^{(N-1)k}$-message equilibrium with each induced action on some line $\overline{\mathbf{A}}_{N-1,i}$, $i = 1, ..., 2^{(N-2)k}$. For suitable choices of c, by the centerpoint theorem it follows, as for the case $N = 2$, that for any $\varepsilon > 0$ and k large enough, it follows that for any $a \in \overline{\mathbf{A}}$, there must be an equilibrium action a^* within ε of a.

Proof of Theorem 4: We prove the result by induction on $k \geq 1$. Suppose, as part of the inductive hypothesis, that we have a 2^k-message equilibrium associated with a 2^k-element partition of $\mathbf{\Theta}$ created by 2^{k-1} hyperplanes, $k \geq 1$. Identify the j-th partition element μ^j (a compact convex subset of θ with non-empty interior) by the message m^j and the corresponding induced action by a^j. We suppose that message m^j is sent by all $\theta \in int\{\mu^j\}$ with probability $p^j \in (0,1]$ that does not depend on θ. Message m^j may also be sent by other types $\theta \notin \mu^j$ with positive probability. Let $z^j_{in} = E[\theta|m^j, \theta \in \mu^j]$ and $z^j_{out} = E[\theta|m^j, \theta \notin \mu^j]$, whenever defined. By the law of iterated expectations a^j is a probability weighted average of z^j_{in} and z^j_{out}, so that the line joining the latter two points must pass through a^j. Since we have an equilibrium, $U(a^1) = ... = U(a^{2^k})$. We proceed by induction on k by first creating a 2^{k+1}-element partition of $\mathbf{\Theta}$ from the given 2^k-element partition. Next, we adjust the induced actions via mixed strategies in order to obtain an equilibrium with 2^{k+1} messages.

For each m^j, consider a hyperplane through $z^j_{in} \in int(\mu^j)$ of orientation $s \in \mathbb{S}^{N-1}$ that splits μ^j into two regions $\mu^{j+}(s)$ and $\mu^{j-}(s)$ and expected values $z^{j+}_{in}(s)$ and $z^{j-}_{in}(s)$. Relative to the original 2^k-element partition, we think of this new 2^{k+1}-element partition where each corresponding message m^j is split into two messages m^{j+} and m^{j-} such that (i) each $\theta \in \mu^j$ sends message m^{j+} (resp., m^{j-}) with the same probability $p^j > 0$ as the original message m^j if $\theta \in \mu^{j+}$ (resp., μ^{j-}) and does not send the other message m^{j-} (resp., m^{j+}); and, (ii) each $\theta \notin \mu^j$ who sent m^j with positive probability, now splits that probability equally between the messages m^{j+} and m^{j-}. Accordingly, the corresponding actions can be written as $a^{j+}(s) = E[\theta|m^{j+}] = \pi^{j+}(s) z^{j+}_{in}(s) + (1 - \pi^{j+}(s)) z^j_{out}$ and $a^{j-}(s) = E[\theta|m^{j-}] = \pi^{j-}(s) z^{j-}_{in}(s) + (1 - \pi^{j-}(s)) z^j_{out}$ where the conditional probabilities are

$$\pi^{j+}(s) \quad = \quad \Pr[\theta \in \mu^{j+}(s)|m^{j+}] = \frac{p^j \Pr[\theta \in \mu^{j+}(s)]}{p^j \Pr[\theta \in \mu^{j+}(s)] + \frac{1}{2}\Pr[m^j, \theta \notin \mu^j]} \tag{6}$$

$$\pi^{j-}(s) \quad = \quad \Pr[\theta \in \mu^{j-}(s)|m^{j-}] = \frac{p^j \Pr[\theta \in \mu^{j-}(s)]}{p^j \Pr[\theta \in \mu^{j-}(s)] + \frac{1}{2}\Pr[m^j, \theta \notin \mu^j]}. \tag{7}$$

Since $\mu^{j+}(-s) = \mu^{j-}(s)$ for all s, we have $\pi^{j+}(-s) = \pi^{j-}(s)$. Since in addition $z^{j+}_{in}(-s) = z^{j-}_{in}(s)$, it follows that $a^{j+}(-s) = a^{j-}(s)$ for all s, and symmetrically $a^{j-}(-s) = a^{j+}(s)$. But then the difference $U(a^{j+}(s)) - U(a^{j-}(s))$ is a continuous odd function of s, so that by the Borsuk-Ulam theorem there exists $s^{j*} \in \mathbb{S}^{N-1}$ such that $U(a^{j+}(s^{j*})) - U(a^{j-}(s^{j*})) = 0$, for each $j = 1, ..., 2^k$. Furthermore, by the law of iterated expectations, there exists $\delta^j \in (0,1)$ such that $a^j = \delta^j a^{j+}(s^{j*}) +$

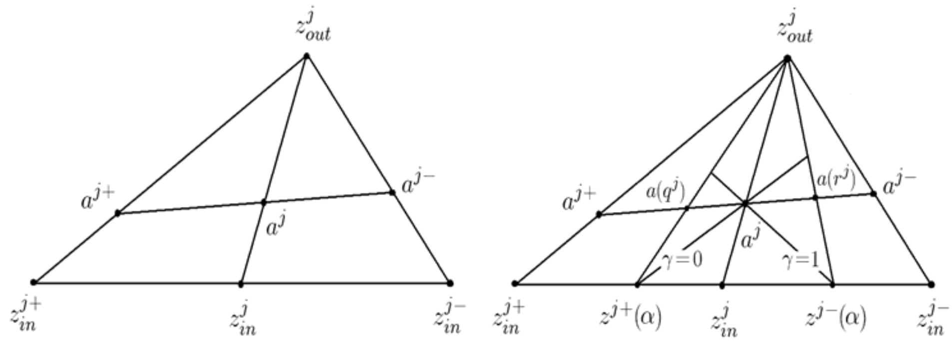

Figure 3: Mixed message construction

$(1-\delta^j)a^{j-}(s^{j^*})$ for each j. Since the orientation s^{j*} will be fixed for all j for the remainder of the proof we suppress it in what follows. The left panel of Figure 3 depicts the typical situation with respect to the new actions and expectations obtained for the j-th element of the original 2^k-element partition and it will be useful for the reader to consult it for the rest of the proof.[29]

If $U(a^{j+}) = U(a^{j-})$ does not vary with j, we have created a 2^{k+1}-message equilibrium. If not, suppose without loss of generality that $U(a^{j+}) = U(a^{j-})$ is the lowest for $j = 1$. Since for the original 2^k-message equilibrium $U(a^j)$ did not depend on j, exploiting the strict quasiconvexity of U we have for all $j > 1$,

$$U(a^{j+}) = U(a^{j-}) \geq U(a^{1+}) = U(a^{1-}) > U(a^1) = U(a^j). \tag{8}$$

For each j for which the first inequality in (8) holds with equality we do not alter the probabilities with which the messages m^{j+} and m^{j-} are sent. In contrast, for j for which the first inequality in (8) is strict, we adjust the induced actions after messages m^{j+} and m^{j-} respectively by suitably altering the probabilities by which these messages are sent, as follows.

First, since preferences are continuous, from (8) there must exist $q^j, r^j \in (0,1)$ and actions $a(q^j) = q^j a^{j+} + (1-q^j)a^j$ and $a(r^j) = r^j a^{j+} + (1-r^j)a^{j-}$ such that $U(a(q^j)) = U(a(r^j)) = U(a^{1+}) = U(a^{1-})$. Indeed, we must have $1 > q^j > \delta^j > r^j > 0$, i.e., $a(q^j)$ and $a(r^j)$ both lie on the line joining a^{j+} and a^{j-} that passes through a^j, on either side of a^j. This is depicted in the right panel of Figure 3. We wish to adjust the induced actions to $a(q^j)$ and $a(r^j)$, after messages m^{j+} and m^{j-} respectively, by suitably altering the probabilities by which these messages are sent.

Let $\alpha^{j+}p^j$ (resp., $\alpha^{j-}p^j$) be the probability with which any $\theta \in \mu^{j+}$ (resp., μ^{j-}) sends message m^{j+} (resp., m^{j-}), with the remaining probability $(1-\alpha^{j+})p^j$ (resp., $(1-\alpha^{j^-})p_j$) on the other message m^{j-} (resp., m^{j+}), $\alpha^{j+}, \alpha^{j-} \in (0,1)$. Similarly, let $\theta \notin \mu^j$, divide the probability with which they sent the original message m^j into the messages m^{j+} and m^{j-} in the ratio $\gamma/(1-\gamma)$, $\gamma \in (0,1)$. We wish to find α^{j+}, α^{j-} and γ such that $a(q^j) = E[\theta|m^{j+}]$ and $a(r^j) = E[\theta|m^{j-}]$. To this end, produce the line joining z^j_{out} with $a(q^j)$ till it meets the line joining z^{j+}_{in} and z^{j-}_{in} (that must pass through z^j_{in}) to obtain the point $z^{j+}(\alpha)$ as depicted in the right panel of Figure 3; similarly, obtain

[29] The figure depicts the typical situation where the expectations z^{j+}_{in} and z^{j-}_{in} are not co-linear with the original expecations z^j_{in}, z^j_{out} and a^j. The arguments go through in the co-linear case, except for non-generic situations where, in addition, $a^{j+} = a^{j-} = a^j$. For $N > 2$, this can be ruled out generally by choosing s^{j*} to guarantee also that $\pi^{j+}(s^{j*}) = \pi^{j-}(s^{j*})$.

$z^{j-}(\alpha)$. Using Bayes' Rule, it is not difficult to verify that there exist $\alpha^{j+}, \alpha^{j-} \in (0,1)$ such that $E[\theta|m^{j+}, \theta \in \mu^j] = z^{j+}(\alpha)$ and $E[\theta|m^{j-}, \theta \in \mu^j] = z^{j-}(\alpha)$, implying that there exists $\gamma \in (0,1)$ such that

$$\begin{aligned} a(q^j) &= E[\theta|m^{j+}] = \Pr[\theta \in \mu^j|m^{j+}]z^{j+}(\alpha) + \Pr[\theta \notin \mu^j|m^{j+}]z^j_{out} && (9) \\ a(r^j) &= E[\theta|m^{j-}] = \Pr[\theta \in \mu^j|m^{j-}]z^{j-}(\alpha) + \Pr[\theta \notin \mu^j|m^{j-}]z^j_{out} && (10) \end{aligned}$$

and we suppress the details.

This completes the construction of the communication strategies and induced actions that constitute a 2^{k+1}-message equilibrium from a 2^k message equilibrium. Since, by Theorem 1 such an equilibrium exists for the case $k = 1$, this completes the induction. By the strict quasiconvexity of U, payoffs are strictly increasing in k.

Proof of Proposition 2: U is strictly quasiconvex if $G_1(a_1)G_2(a_2)$ is strictly quasiconcave. Since a sufficient condition for strict quasiconcavity is strict logconcavity, U is strictly quasiconvex if $\ln G_1(a_1) + \ln G_2(a_2)$ is strictly concave. But if each $G_i(a_i)$ is strictly logconcave, the last expression is the sum of strictly concave functions and so strictly concave.

Regarding payoffs to the jurors, in any equilibrium and given a message m, the expected payoff to juror $i = 1, 2$ unconditional on τ_i is equal to the probability of conviction times the expected payoff given conviction and m,

$$G_1(E[\theta_1|m])G_2(E[\theta_2|m])\left(E[\theta_i|m] - E[\tau_i|\tau_i < E[\theta_i|m]]\right). \tag{11}$$

Notice first that the equilibrium probability of conviction $G_1(E[\theta_1|m])G_2(E[\theta_2|m])$ does not depend on m. Furthermore, this probability is strictly lower in an influential equilibrium compared to the babbling equilibrium. Letting $h(x) \equiv x - E[\tau_i|\tau_i < x]$, notice also from (11) that the expected payoff to a juror given conviction and m is equal to $h(E[\theta_i|m])$ in an influential equilibrium and equal to $h(E[\theta_i])$ in the babbling equilibrium. To show that jurors may be strictly worse off in expectation (over m) in an influential equilibrium, it suffices then to show that the function h is weakly concave in x and apply Jensen's inequality. This is equivalent to $E[\tau_i|\tau_i < x]$ being weakly convex in x which holds for many logconcave distributions, including the uniform.

Proof of Proposition 3: From (4) we can write

$$U(a) = \begin{cases} E[\lambda_{2:n}]a_1 + (1 - E[\lambda_{2:n}])a_2 & \text{if } a_1 \leq a_2 \\ E[\lambda_{n-1:n}]a_1 + (1 - E[\lambda_{n-1:n}])a_2 & \text{if } a_1 > a_2 \end{cases} \tag{12}$$

where $\lambda_{j:n}$ is the jth lowest buyer signal, $j = 1, ..., n$. Notice from (12) that the (expected) price equals either the (expected) bid of the buyer with the second-lowest private signal $\lambda_{2:n}$ or that of the buyer with the second-highest one $\lambda_{n-1:n}$. For $n = 2$, $\lambda_{2:n} > \lambda_{n-1:n}$ almost surely so (4) is $U = \min\{E[\lambda_{2:n}]a_1 + (1 - E[\lambda_{2:n}])a_2, E[\lambda_{n-1:n}]a_1 + (1 - E[\lambda_{n-1:n}])a_2\}$, a concave and hence quasiconcave function of a, implying from Theorem 2 influential cheap talk cannot raise expected revenue when $n = 2$. Similarly, when $n = 3$, $\lambda_{2:n} = \lambda_{n-1:n}$ and U is linear in a, so from Theorem 2 all cheap talk equilibria must yield the same expected revenue. However, for $n \geq 4$, $\lambda_{2:n} < \lambda_{n-1:n}$ almost surely and so (4) reduces to $U = \max\{E[\lambda_{2:n}]a_1 + (1 - E[\lambda_{2:n}])a_2, E[\lambda_{n-1:n}]a_1 + (1 - E[\lambda_{n-1:n}])a_2\}$, a convex and hence quasiconvex function of a, implying from Theorem 2 influential cheap talk cannot lower expected revenue when $n \geq 4$. To ensure a strict gain, we choose the reference point c to be at the kink of the seller's indifference curve through $E[\theta]$ so that the induced actions lie on different linear segments of the same higher indifference curve.

Notice next that the expected total surplus (sum of buyer rents and seller revenue) equals the expected value of the winning bidder which in turn equals

$$\max\{E[\lambda_{1:n}]a_1 + (1 - E[\lambda_{1:n}])a_2, E[\lambda_{n:n}]a_1 + (1 - E[\lambda_{n:n}])a_2\} \tag{13}$$

for all $n \geq 2$. Since this is a convex function of a, total surplus must weakly rise from influential communication regardless of n. Since seller revenues are weakly lower from communication when $n < 4$, it follows that buyer rents must weakly rise for such n.

It remains to show that both buyer rents and total surplus must strictly rise when $n \geq 4$. Assume wlog that $E[\theta_2] \geq E[\theta_1]$ so that under babbling the sum of buyers' expected payoffs is $E[\lambda_{n:n} - \lambda_{n-1:n}]E[\theta_2 - \theta_1]$. Because U has piecewise linear indifference curves with a kink on the diagonal, it suffices to consider the two-message cheap talk equilibrium with c chosen as above to ensure different winning and second highest bidders across messages. Let P^+ and P^- are the equilibrium probabilities of messages m^+ and m^- respectively, and suppose wlog that m^+ boosts the second attribute and m^- the first attribute, implying $E[\theta_2^+ - \theta_1^+] > 0 > E[\theta_2^- - \theta_1^-]$. The sum of buyers' expected payoffs is then $P^+E[\lambda_{n:n} - \lambda_{n-1:n}]E[\theta_2^+ - \theta_1^+] + P^-E[\lambda_{1:n} - \lambda_{2:n}]E[\theta_2^- - \theta_1^-]$, implying buyer rents strictly rise if

$$\begin{aligned} & P^+E[\lambda_{n:n} - \lambda_{n-1:n}]E[\theta_2^+ - \theta_1^+] + P^-E[\lambda_{1:n} - \lambda_{2:n}]E[\theta_2^- - \theta_1^-] \\ > \; & E[\lambda_{n:n} - \lambda_{n-1:n}]\left(P^+E[\theta_2^+ - \theta_1^+] + P^-E[\theta_2^- - \theta_1^-]\right) \end{aligned} \tag{14}$$

where we have used $E[\theta_2 - \theta_1] = P^+E[\theta_2^+ - \theta_1^+] + P^-E[\theta_2^- - \theta_1^-]$ by the law of iterated expectations. This reduces to $E[\lambda_{n:n} - \lambda_{n-1:n}] > E[\lambda_{1:n} - \lambda_{2:n}]$ which always holds. Since seller revenues also rise, total surplus is also strictly higher.

Proof of Proposition 4: First suppose v and t are common knowledge. Then given a price p the cutoff consumer x (supposing one exists) who is indifferent between purchasing the product or not satisfies $v - tx - p = 0$, where the value of non-consumption is zero. This yields $D = x = (v-p)/t$ as the demand function faced by the monopolist. The profit maximizing price is $p^* = \frac{v}{2}$, yielding quantity $x^* = \frac{v}{2t}$, profit $\Pi^* = \frac{v^2}{4t}$, and equilibrium consumer surplus for consumer $x \in [0,1]$ of $S_x^* = \max[v - tx - p^*, 0]$. Assuming that $t \geq v/2$ ensures that not all consumers consume.

In the present setting, t and v are known only by the firm, but since consumer utility is linear in t and v, and since these variables only affect firm payoffs via consumer decisions, the above expressions apply to the estimates $a_1 = E[t|m]$ and $a_2 = E[v|m]$ of t and v given a message m. We may then write

$$U(a) = \Pi^* = \frac{a_2^2}{4a_1}. \tag{15}$$

Since this is continuous and strictly quasiconvex in a, it follows via Theorems 1 and 2 that the entrant firm's profits are strictly higher because of cheap talk. Since S_x^* is also convex in a, each consumer x is weakly better off, with those who switch from not buying the product to buying it (or vice versa) as a function of the entrant's message strictly so.

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B Online Appendix

Our results in the main text concern the case where the expert's preferences $U(a)$ over the decision maker's actions are common knowledge and so independent of the state θ. But often the decision maker might believe that he has a good sense of the expert's preferences, and yet still face some uncertainty over the exact preferences. In this online Appendix we extend our results to allow the decision maker to face some limited uncertainty about the expert's preferences, including the possibility that preferences are correlated with the state θ.

DROPPED Suppose that the expert's preferences are given by a function $U(a,t)$, continuous in a, where $t \in \mathbf{T}$ is the type of the expert. The expert knows her own type t (in addition to θ), but the decision maker only has a prior Φ on t. Let $F(\theta|t)$ summarize the conditional distribution of θ given $t \in \mathbf{T}$. This approach allows t to be independent of θ or to be correlated with θ. It also covers the Crawford-Sobel model where $t = \theta$. More importantly, it allows us to conceptually separate uncertainty about the expert's motives t from that about the decision maker's ideal course of action θ. Notice that the expert's type t is fully specified by the pair $U(.,t)$ and $F(.|t)$.

We analyze three extensions in this Appendix which show the robustness of our main results. The first extension allows for arbitrary Φ but supposes that T is small. This captures situations where the decision maker attaches positive probability to only a few possible expert types. We find that if there are only a finite number of different types of expert preferences, then an informative equilibrium exists as long as there are a larger number of dimensions of interest to the decision maker(s).

The second concerns the case where the prior Φ is concentrated on a particular type $t^* \in \mathbf{T} = \{1, ..., T\}$. This captures situations where the decision maker is almost certain that the expert has preferences $U(a, t^*)$, although there are potentially many other unlikely possibilities. We find that when the expert's preferences are sufficiently likely to be from one type, then under mild regularity conditions an equilibrium exists in which the decision maker listens to the expert and adjusts his use of the expert's information to reflect the fact that low probability types always recommend the same action.

The third extension relaxes the equilibrium notion from cheap talk to epsilon cheap talk and focuses on the case where the expert's type space is rich, i.e., $t = \theta$, but her type has a limited effect on (differences in) her utility. In particular we consider Euclidean preferences which are state-dependent but that converge uniformly to state-independent preferences as the bias in each dimension increases. We show that any incentive to deviate from the cheap talk equilibria of the limiting preferences goes to zero.

B.1 More Issues Than Motives

We first consider the case where there is a larger number of issues N relative to the number of types T. For instance, suppose a car magazine is biased toward one of several car manufacturers and the reader is unsure of which manufacturer is favored, so that there are several types of expert preferences T. If the magazine has information on multiple models for some manufacturers and/or on multiple attributes of some models, then the dimensionality of information N is larger than T and it would seem that comparative statements about different models or about the relative strengths of particular models should be credible. Application of the Borsuk-Ulam theorem confirms this intuition quite generally.

Proposition 5 *Suppose $N > T$. Then an influential cheap talk equilibrium exists for all* $\mathbf{T}$ *and* Φ.

Proof: The arguments are identical to that for Theorem 1. The only difference is that now we look for an s that simultaneously sets the $T \leq N-1$ maps $\Delta(s,c,t) = U(a^{-}(h_{s,c}),t) - U(a^{+}(h_{s,c}),t) = 0$, one for each $t \in \mathbf{T}$. ■

When $N > T$, it is possible to find a two-message partition of $\mathbf{\Theta}$ that is an equilibrium for every type $t \in \mathbf{T}$. In other words, it is possible to find an informative communication strategy that induces actions in $\mathbb{R}^N$ that the T possible types of the expert agree on. This is not surprising for linear $U(.,t)$ since satisfying all the T experts simultaneously still leaves $N-1-T$ degrees of freedom. Proposition 5 shows that it suffices simply to count equations and unknowns in general for all continuous preferences. Hence even in an environment where an expert's motives are unclear, an expert can often gain credibility via knowledge of a large number of decision-relevant issues. In the next section we consider the case where N is instead small relative to T but only one type is very likely.

B.2 Almost Certain Motives

Now suppose that the prior $\Phi = (\phi_1, ..., \phi_T)$ is close to the degenerate distribution Φ^* on type $t^* \in \mathbf{T}$ (i.e., $\phi^*_{t^*} = 1$). We use the implicit function theorem to look for equilibria in the neighborhood of the equilibria identified by Theorem 1 for the degenerate case, for general preferences and conditional distributions of θ. In such equilibria the decision maker anticipates that the low probability types will not be indifferent so they will always offer the same advice. For instance, the decision maker thinks that the expert is probably unbiased across dimensions, $U = a_1 + a_2$, but there is some chance that the expert has a relatively extreme slant, $U = 4a_1 + a_2$, in which case she will offer the same advice regardless of the state θ.

To apply the implicit function theorem we assume that $U(a,t)$ is continuously differentiable in a for each $t \in \mathbf{T}$ and, for simplicity, consider only the case $N = 2$. We also suppose that the types $t \in \mathbf{T}$ have different preferences from each other in the following sense: for any two action profiles a, a' with $a \neq a'$, if $U(a,t^*) = U(a',t^*)$, then $U(a,t) \neq U(a',t)$ for all $t \neq t^*$, $t \in \mathbf{T}$. We call this condition (S). Notice that it will hold if, for instance, the indifference curves of the different types satisfy a single-crossing property in $\mathbb{R}^2$.[30]

Proposition 6 *Suppose U satisfies (S) and $N = 2$. Generically in $U(.,t^*)$, $F(.|t^*)$, there exists $\varepsilon > 0$ such that for each Φ with $||\Phi - \Phi^*|| < \varepsilon$ an influential cheap talk equilibrium exists.*

Proof: For any fixed $c \in int(\mathbf{\Theta})$, let s^* be the orientation of an equilibrium hyperplane through c with corresponding actions $a^{+}(h_{s^*,c})$ and $a^{-}(h_{s^*,c})$ when priors are degenerate on t^*, i.e., given by Φ^*. Such s^* exists by Theorem 1 and we have $U(a^{+}(h_{s^*,c}),t^*) = U(a^{-}(h_{s^*,c}),t^*)$. By condition (S), for all $t \neq t^*$, $U(a^+,t) \neq U(a^-,t)$. Wlog, rename types so that $U(a^+,t) > U(a^-,t)$ for all $t > t^*$ and $U(a^+,t) < U(a^-,t)$ for all $t < t^*$ and consider the actual priors Φ. Pick a hyperplane of arbitrary

[30] Proposition 5 provides insight on how the following result can be extended to the case where $N > 2$. In brief, one applies the implicit function theorem to the equilibrium of Proposition 5 constructed with $N-1$ types (including t^*) disclosing a partition and the remaining types sending only one message. Condition (S) then has to be suitably amended.

orientation s through c (the same c as above) and, using usual notation, let the actions or expected values corresponding to each message be

$$\alpha^+(h_{s,c};\Phi) = \Pr[t^*|m^+]E[\theta|t^*,m^+] + \sum_{t>t^*}\Pr[t|m^+]E[\theta|t] \tag{16}$$

$$\alpha^-(h_{s,c};\Phi) = \Pr[t^*|m^-]E[\theta|t^*,m^-] + \sum_{t<t^*}\Pr[t|m^-]E[\theta|t] \tag{17}$$

That is, we assume type t^* discloses the partition of $\mathbf{\Theta}$ associated with $h_{s,c}$ truthfully, while all types $t > t^*$ (resp., $t < t^*$) send only message m^+ (resp., m^-). Thus,

$$\Pr[t^*|m^+] = \frac{\Pr[\theta \in m^+|t^*]\phi_{t^*}}{\Pr[\theta \in m^+|t^*]\phi_{t^*} + \sum_{t>t^*}\phi_t} \tag{18}$$

and

$$\Pr[t|m^+] = \frac{\phi_t}{\Pr[\theta \in m^+|t^*]\phi_{t^*} + \sum_{t<t^*}\phi_t} \tag{19}$$

if $t > t^*$ and is 0 otherwise, and similarly for the message m^-. Let

$$\Delta(s,\Phi,t) = U(\alpha^+(h_{s,c};\Phi),t) - U(\alpha^-(h_{s,c};\Phi),t) \tag{20}$$

which is a continuously differentiable function of s. We know that $\Delta(s^*,\Phi^*,t^*) = 0$. We wish to show via the implicit function theorem that there exists $\varepsilon > 0$, such that for $||\Phi - \Phi^*|| < \varepsilon$, there exists $s(\Phi)$ close to s^* for which $\Delta(s(\Phi);\Phi,t^*) = 0$. This is enough to show the result, since when $s(\phi)$ is close to s^* the corresponding actions $\alpha^+(s(\Phi);\phi)$ and $\alpha^-(s(\phi);\phi)$ are close to a^+ and a^- respectively, so that $\Delta(s^*,\Phi^*,t) < 0$ if $t < t^*$ and $\Delta(s^*,\Phi^*,t) > 0$ if $t > t^*$. Then type t^* has the right incentives to disclose the partition of $\mathbf{\Theta}$ associated with $h_{s(\Phi),c}$ truthfully, while all types $t > t^*$ (resp., $t < t^*$) send only message m^+ (resp., m^-).

We use the fact that the circle is locally like the line. That is, we set $s_1(z) = z \in [-1,1]$ and, when $s_2^* = \sqrt{1 - s_1^{*2}}$ set $s_2(z) = \sqrt{1-z^2}$ (and, similarly, when $s_2^* = -\sqrt{1-s_1^{*2}}$ set $s_2(z) = -\sqrt{1-z^2}$). We then consider the function $\Delta(s(z),\Phi,t^*)$ as a function of z in a neighborhood of s^*. To apply the implicit function theorem we have to show that $\partial\Delta(s(z),\Phi^*,t^*)/\partial z \neq 0$. It is easy to see that this derivative consists of terms involving the derivative of $U(.,t^*)$ with respect to the actions (that do not depend on $F(.|t^*)$) and terms involving the expected actions α^+ and α^- (that depend on $F(.|t^*)$ but not on $U(.,t^*)$). Since we can vary $F(.|t^*)$ and $U(.,t^*)$ independently, $\frac{\partial\Delta(s(z),\Phi^*,t^*)}{\partial z}$ can vanish only in non-generic cases, establishing the result. ■

In the influential equilibrium of Proposition 6, type t^* discloses a two-message partition of $\mathbf{\Theta}$ similar to the equilibria of Theorem 1. By condition (S), no other type can be indifferent between two induced actions and so will send one of the two messages with probability one. Since Φ is close to Φ^*, the induced actions (and the equilibrium partition) are close to the equilibrium of the case where the expert's likely type t^* is common knowledge. The decision maker essentially ignores the implications of messages from unlikely types $t \neq t^*$ in determining his action.[31]

[31] Since unlikely types always influence the decision maker in their preferred direction, this provides a multidimensional perspective on the finding by Stefano DellaVigna and Ethan Kaplan (2007) that, in an environment where few viewers expected a news network to have a conservative bias, Fox News had a large influence on voting behavior by viewers.

B.3 Epsilon Cheap Talk

For our third robustness test we consider preferences that are highly state-dependent, but that converge to state-independent preferences. As a notable example of such preferences, we consider Euclidean preferences where the expert's utility is based on the distance between the expert's ideal action and the decision maker's ideal action,

$$U(a;\theta) = -d(a, \theta + b) = -\left(\sum_{i=1}^{N}(a_i - (\theta_i + b_i))^2\right)^{1/2} \tag{21}$$

where $d(\cdot,\cdot)$ is the Euclidean distance function and $b = (b_1, ..., b_N) \in \mathbb{R}^N$ is the vector of known biases representing the distance between the decision maker's ideal action θ, and the expert's ideal action $\theta + b$. Distance preferences are used in the leading example from Crawford and Sobel and in a wide variety of applications.

As the expert's bias in each dimension increases, Euclidean preferences converge uniformly to state-independent linear preferences with known biases across dimensions equal to the ratios of these biases within dimensions. More precisely, if we write $b = \rho B$ for some vector $\rho \in \mathbb{R}^N$, $\rho \neq 0$, and real number $B \geq 0$, then for any θ, as B increases without bound the expert's ideal point $\theta + b$ becomes more and more distant from θ, and the circular indifference curves for Euclidean preferences become straighter, and converge to those of known linear preferences of the form $\rho \cdot a$ given by (1).[32]

To use this convergence, we modify the game so that the expert's payoff from any action a and message m given θ is $U(a, m; \theta) = -d(a, \theta + b)$ less an arbitrarily small cost $\varepsilon > 0$ of lying if the message m is not consistent with θ.[33] We study if influential equilibria exist in the modified game with distance preferences (21) and an arbitrarily small cost of lying. We say that a communication strategy is an *ε-cheap talk equilibrium for large biases* of the game with distance preferences if and only if for each $\varepsilon > 0$ there exists $\overline{B}$ such that for all $B > \overline{B}$ and any θ, the incentive to lie for an expert is at most ε. Our next result shows an equivalence between such equilibria and the cheap talk equilibria for linear preferences characterized by Theorem 3.[34, 35]

Proposition 7 *Suppose U is Euclidean. Then for all F and all k, a communication strategy is a k-message ε–cheap talk equilibrium for large biases if and only if it is a cheap talk equilibrium for the limiting linear U with slant ρ.*

[32] All arguments go through unchanged if each b_i is of the form $b_i = \kappa_i + \rho_i B$, for constants κ_i. In such cases, as B becomes large the expert becomes infinitely biased in dimensions where $\rho_i \neq 0$ but has finite (possibly, no) bias in dimensions where $\rho_i = 0$.

[33] Since the meaning of a message is derived from a (candidate) equilibrium communication strategy, the notion of what constitutes a "lie" is endogenous here. Therefore this equilibrium notion is distinct from that of an "almost cheap talk" equilibrium (Kartik, 2008) or a "costly talk" equilibrium (Kartik, Ottaviani, and Squintanni, 2007) in which the sender's reports have an exogenous meaning corresponding to the true value of the state and any deviation from this true value is costly. Our notion corresponds to that of an ε-equilibrium of a cheap talk game.

[34] In an extension of their analysis of lexicographic preferences, Levy and Razin (2007) show for Euclidean preferences that even slight asymmetries in distributions can preclude pure cheap talk for sufficiently large but finite B. Our result shows that for sufficiently large B any incentive to deceive the decision maker is arbitrarily close to 0.

[35] Epsilon equilibria are not invariant to monotonic transformations of the underlying preferences. For instance, with a quadratic variant of the Euclidean specification which drops the square root term in (21), the difference in the sender's utilities from actions a and a' is unbounded in B, implying that our equivalence result obtains only if the cost of lying also increases in the unit of payoffs B, e.g., if it is equal to εB for any $\varepsilon > 0$. Note however that indifference curves corresponding to such quadratic preferences are also linear *at* the limit of infinite biases.

Proof: We show that for all F and all k, a communication strategy is a k-message ε–cheap talk equilibrium for Euclidean U with large B if and only if it is a cheap talk equilibrium for the limiting linear preferences $\rho \cdot a$. To do this first pick an arbitrary hyperplane $h_{s,c}$ of orientation $s \in \mathbb{S}^{N-1}$ passing through $c \in int(\mathbf{\Theta})$ and let L be the line joining the corresponding actions $a^+ = a^+(h_{s,c})$ and $a^- = a^-(h_{s,c})$. Pick any $\theta \in \mathbf{\Theta}$ and let $p(\theta)$ be the point where the perpendicular from $\theta + B\rho$ on to L meets L. Then

$$p(\theta) = q(\theta)a^+ + (1 - q(\theta))a^- \tag{22}$$

where $q(\theta) \in \mathbb{R}$ is given by

$$q(\theta) = \frac{(\theta - a^-) \cdot (a^+ - a^-) + B\rho \cdot (a^+ - a^-)}{(a^+ - a^-) \cdot (a^+ - a^-)}. \tag{23}$$

Notice that this is well-defined since $a^+ \neq a^-$. Notice next that

$$\begin{aligned} d(a^+, \theta + b) - d(a^-, \theta + b) &= \frac{d^2(a^+, \theta + b) - d^2(a^-, \theta + b)}{d(a^+, \theta + b) + d(a^-, \theta + b)} \\ &= \frac{d^2(a^+, p(\theta)) - d^2(a^-, p(\theta))}{d(a^+, \theta + b) + d(a^-, \theta + b)}. \end{aligned} \tag{24}$$

For the if part consider first a two-message cheap talk equilibrium with induced actions a^+ and a^- when U is given by (1), so that $\rho \cdot (a^+ - a^-) = 0$. Then $q(\theta)$ and so $p(\theta)$ do not depend on B. Furthermore, using (24),

$$\begin{aligned} \left|U(a^+; \theta + b) - U(a^-; \theta + b)\right| &= \left|d(a^+, \theta + b) - d(a^-, \theta + b)\right| \\ &= \left|\frac{d^2(a^+, p(\theta)) - d^2(a^-, p(\theta))}{d(a^+, \theta + b) + d(a^-, \theta + b)}\right| \\ &\leq \max_{\theta \in \mathbf{\Theta}} \left|\frac{d^2(a^+, p(\theta)) - d^2(a^-, p(\theta))}{d(a^+, \theta + B\rho) + d(a^-, \theta + B\rho)}\right|. \end{aligned} \tag{25}$$

Let θ_B be the solution to the last maximization problem. As B rises, θ_B stays bounded in the compact set $\mathbf{\Theta}$, so that $p(\theta_B)$ stays bounded as well, implying that the numerator stays bounded. However the denominator becomes arbitrarily large. It follows that for any $\varepsilon > 0$, for B large enough, $|U(a^+, \theta + b) - U(a^-, \theta + b)| < \varepsilon$ for all θ. An analogous argument obtains for the k-message equilibria if we consider pairs of equilibrium actions that must all lie on the same line L and use the logic above.

For the only if part, suppose that two actions a^+ and a^- do not constitute a two-message cheap talk equilibrium when U is given by (1). Wlog, suppose that $\rho \cdot a^+ < \rho \cdot a^-$. Consider type $\theta = a^+$ and observe via (24) that

$$\begin{aligned} \lim_{B \to \infty} \left[U(a^+; a^+ + b) - U(a^-; a^+ + b)\right] &= \lim_{B \to \infty} \left[\frac{d^2(a^+, p(a^+)) - d^2(a^-, p(a^+))}{d(a^+, a^+ + B\rho) + d(a^-, a^+ + B\rho)}\right] \\ &= \frac{\rho \cdot (a^- - a^+)}{\sqrt{\rho \cdot \rho}} \end{aligned} \tag{26}$$

where we have used (22) and (23) in the last line. It follows that when $\varepsilon < \rho \cdot (a^- - a^+)/\sqrt{\rho \cdot \rho}$, and B is large enough, type $\theta = a^+$ would gain by more than ε from lying (i.e., by inducing the decision maker to choose the action a^- instead of a^+), implying in turn that h is not an ε-cheap talk equilibrium for large B when U is given by (21). An identical argument obtains for the k-message equilibria of Theorem 3, $k \geq 2$ and finite, if we consider some pair of actions for which $\rho \cdot a^+ \neq \rho \cdot a^-$. ■

This convergence result implies that a strong incentive to exaggerate in each dimension can be captured by our simple model of linear preferences. Moreover, since the expert's biases or slant across dimensions converges to that of the ratios of the biases within dimensions, the result formally links the idea of an expert being biased towards a higher action as developed in the Crawford and Sobel model, and the idea of an expert being biased across dimensions as emphasized in this paper.