Supplementary Appendix to

Prices and Heterogeneous Search Costs

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Introduction

In this supplementary appendix we present two extensions of our model. In Part A we study the same model as in the published article but for the case of duopoly. In Part B we examine a model with homogeneous products and an equilibrium in mixed strategies. Both extensions serve the purpose of illustrating that the main insights of our article carry over to other settings.

A Duopoly

In this section we study a duopoly version of the model in the article. Except that there are only two firms in the market, the rest of the model is exactly the same as in the article.[1]

We now present the derivations that are necessary to compute the SNE price p^*. For this we derive the (expected) payoff to a firm i that deviates by charging a price $p_i < p^*$. In order to compute firm i's demand, consider a consumer with search cost c who visits firm i in her first search. This happens with probability 1/2. Let $\varepsilon_i - p_i$ denote the utility the consumer derives from the product of firm i. Notice that search behavior is exactly the same as in the main text. The consumer expects the other firm to charge the equilibrium price p^*. The expected gains from searching one more time are equal to $\int_{\varepsilon_i - p_i + p^*}^{1} [\varepsilon_j - (\varepsilon_i - p_i + p^*)] f(\varepsilon_j) d\varepsilon_j$. It follows that the probability that the buyer visits firm i first and stops searching at firm i is equal to

$$\frac{1}{2} \Pr[\varepsilon_i - p_i > \max\{\hat{x}(c) - p^*, 0\}] = \frac{1}{2} [1 - F(\hat{x}(c) + p_i - p^*)] .$$

[1]Extending this analysis to the case of N firms is straightforward.

where $\hat{x}(c)$, as mentioned above, continues to be the solution to

$$h(x) \equiv \int_x^{\overline{\varepsilon}} (\varepsilon - x) f(\varepsilon) d\varepsilon = c.$$

Consumer c may find the product of firm i not good enough at first and may therefore continue searching. However, upon visiting the rival firm j, it may happen that consumer c returns to firm i because such a firm offers her the best deal after all. This occurs with probability

$$\frac{1}{2} \Pr[\max\{\varepsilon_j - p^*, 0\} < \varepsilon_i - p_i < \hat{x}(c) - p^*] = \frac{1}{2} \int_{p_i}^{\hat{x}(c)+p_i-p^*} F(\varepsilon - p_i + p^*) f(\varepsilon) d\varepsilon.$$

With probability 1/2 consumer c first visits the other firm, firm j. In that case, she will walk away from product j when searching again is more promising than buying j right away. Upon visiting firm i, she will buy product i when she finds product i better than j. This occurs with probability $(1/2) \Pr[\max\{\varepsilon_j - p^*, 0\} < \min\{\hat{x}(c) - p^*, \varepsilon_i - p_i\}]$, which is equal to

$$\frac{1}{2} \left[F(\hat{x}(c)) \left[1 - F(\hat{x}(c) + p_i - p^*)\right] + \int_{p_i}^{\hat{x}(c)+p_i-p^*} F(\varepsilon - p_i + p^*) f(\varepsilon) d\varepsilon \right].$$

To obtain the payoff of firm i we need to integrate over the consumers who decide to participate in the market. It can be shown that with two firms the surplus of a consumer with search cost c is given by the expression

$$CS(c) = \frac{1 - F(\hat{x}(c))^2}{1 - F(\hat{x}(c))} \left[\int_{\hat{x}(c)}^{\overline{\varepsilon}} (\varepsilon - p^*) f(\varepsilon) d\varepsilon - c \right] + 2 \int_{p^*}^{\hat{x}(c)} (\varepsilon - p^*) F(\varepsilon) f(\varepsilon) d\varepsilon. \tag{A1}$$

Setting this surplus equal to zero, we obtain the critical search cost value $\tilde{c}(p^*)$ above which consumers will refrain from participating in the market. Inspection of equation (A1) reveals that the last consumer who chooses to search has a search cost c such that $\hat{x}(c) = p^*$. Using again the notation $c_0(p^*) \equiv \min\{\overline{c}, \tilde{c}(p^*)\}$, the expected demand of firm i is:

$$d_i(p_i; p^*) = \frac{1}{2} \int_0^{c_0(p^*)} \left[(1 + F(\hat{x}))\left(1 - F(\hat{x} + p_i - p^*)\right) + 2 \int_{p_i}^{\hat{x}+p_i-p^*} F(\varepsilon - p_i + p^*) f(\varepsilon) d\varepsilon \right] g(c) dc,$$

where we have written $\hat{x}$ instead of $\hat{x}(c)$ to simplify the notation, but the reader should keep in mind the dependency of $\hat{x}$ on c.

The expected payoff to firm i is

$$\pi_i(p_i;p^*) = (p_i - r)d_i(p_i;p^*).$$

Taking the FOC gives

$$\begin{aligned} 0 &= \int_0^{c_0(p^*)} \left[(1+F(\hat{x}))\left[1-F(\hat{x}+p_i-p^*)\right] + 2\int_{p_i}^{\hat{x}+p_i-p^*} F(\varepsilon - p_i + p^*) f(\varepsilon) d\varepsilon \right] g(c)dc \\ &\quad -(p_i - r)\int_0^{c_0(p^*)} (1+F(\hat{x}))\, f(\hat{x}+p_i-p^*) g(c)dc \\ &\quad -2(p_i - r)\int_0^{c_0(p^*)} \left[\int_{p_i}^{\hat{x}+p_i-p^*} f(\varepsilon - p_i + p^*) f(\varepsilon) d\varepsilon + F(\hat{x}) f(\hat{x}+p_i-p^*) - F(p^*) f(p_i) \right] g(c)dc. \end{aligned}$$

Applying symmetry, i.e. $p_i = p^*$, gives

$$\begin{aligned} 0 &= \int_0^{c_0(p^*)} \left[(1+F(\hat{x}))\,(1-F(\hat{x})) + 2\int_{p^*}^{\hat{x}} F(\varepsilon) f(\varepsilon) d\varepsilon \right] g(c)dc \\ &\quad -2(p^* - r)\int_0^{c_0(p^*)} \left[(1+F(\hat{x}))\, f(\hat{x}) + \int_{p^*}^{\hat{x}} f(\varepsilon)^2 d\varepsilon - F(\hat{x}) f(\hat{x}) + F(p^*) f(p^*) \right] g(c)dc. \end{aligned} \tag{A2}$$

If a symmetric equilibrium exists, then the equilibrium price has to satisfy the FOC in equation (A2). We will not pursue here the conditions under which the equilibrium exists (cf. footnote 6 in the article). Instead, we will check how the candidate equilibrium price changes when search costs go up. For this, we proceed by solving the FOC in equation (A2) numerically. We use the uniform distribution for the match values. For search costs we use the Kumaraswamy distribution (see Kumaraswamy, 1980):

Definition: *The Kumaraswamy distribution has a cumulative distribution function $G(\cdot)$ and a probability distribution function $g(\cdot)$ given by*

$$\begin{aligned} G(c) &= 1 - \left[1 - \left(\frac{c}{\beta}\right)^a\right]^b, \; c \in [0,\beta], \quad a, b > 0; \\ g(c) &= \frac{ab}{\beta}\left(\frac{c}{\beta}\right)^{a-1}\left[1 - \left(\frac{c}{\beta}\right)^a\right]^{b-1}. \end{aligned}$$

The Kumaraswamy distribution is often used as a substitute for the beta-distribution (see, e.g., Ding and Wolfstetter, 2011). This distribution turns out to be quite useful in our setting because its likelihood ratio is increasing (for $b > 1$), decreasing (for $0 < b < 1$), or constant (for $b = 1$) with respect to the shifter parameter β.[2] Note that the β parameter multiplies the search cost c and scales the support of the distribution. An increase in β therefore shifts the search cost distribution rightward, which signifies that search costs are higher for all consumers.

The focus is on the case in which the upper bound of the search cost distribution β is sufficiently high (cf. Theorem 1B in the article). For the case of the uniform distribution of match values we have that

$$CS(c) = \hat{x}(c) - p^* - \frac{\hat{x}(c)^3 - p^{*3}}{3},$$

whereas the critical search cost value above which consumers refrain from searching the market is equal to

$$\tilde{c}(p^*) = \frac{(1-p^*)^2}{2}$$

In Table A1, we set $r = 0$ and let search costs be distributed according to the Kumaraswamy distribution with parameter $a = 1$ and various levels of the parameters b and β. The table shows that prices decrease when search costs increase when $b = 1/2$, in which case the search cost density has the DLRP property. For the $b = 1$ case (uniform distribution), prices are independent of the search cost upper bound. Finally, when $b = 3/2$ and the search cost density has the ILRP property, we get the standard result that prices increase with higher search costs.

Table A1: Duopoly model (uniform-Kumaraswamy with $a = 1$)

	$b = 3/2$			$b = 1$			$b = 1/2$		
	$\beta = 1$	$\beta = 2$	$\beta = 3$	$\beta = 1$	$\beta = 2$	$\beta = 3$	$\beta = 1$	$\beta = 2$	$\beta = 3$
p^*	0.4713	0.4715	0.4716	0.4717	0.4717	0.4717	0.4722	0.4720	0.4719
π	0.0370	0.0188	0.0126	0.0255	0.0127	0.0085	0.0132	0.0065	0.0043
CS	0.0225	0.0113	0.0076	0.0153	0.0076	0.0051	0.0078	0.0038	0.0025
CS^*	0.1116	0.1107	0.1105	0.1100	0.1100	0.1100	0.1084	0.1092	0.1095
Welfare	0.0966	0.0491	0.0329	0.0665	0.0332	0.0221	0.0343	0.0168	0.0112

Notes: CS^* is consumer surplus conditional on participating, i.e., $CS = CS/\int_0^{\tilde{c}(p^*)} gdc$.

Table A1 also reports profits, consumer surplus and welfare. In all cases, profits, consumer surplus and welfare decrease as search costs go up; this is mainly due to the fall in consumer participation. Interestingly, because the price decreases when search costs increase for $b = 1/2$, consumer surplus conditional on participating (4th row in the table) does increase in search costs.

[2] A proof can be obtained from the authors upon request. Observe that the uniform density case is obtained by setting $a = b = 1$ in the Kumaraswamy distribution above.

B Homogeneous products and non-sequential search

In this part of the supplementary appendix we study the effects of lowering search costs in an environment where firms sell homogeneous products and consumers search non-sequentially for lower prices. Our purpose is to illustrate that our results in the main body of the article also carry over to this quite different setting.

The model we study is similar to that in Burdett and Judd (1983). The main difference is that we examine the case of duopoly and that we allow for heterogeneous search costs. The N-firm case has been studied in Moraga-González, Sándor, and Wildenbeest (forthcoming). We use some of their results on existence and uniqueness of symmetric equilibrium in mixed strategies; however, the comparative statics effects of lower search costs presented here are new. We note that the comparative statics effects are very hard to generalize to the N-firm case. The reason is that the equilibrium in mixed strategies is given implicitly by the solution to an N-dimensional nonlinear system of equations. The study of how such a solution changes when the search cost distribution varies is a problem open for future research.[3]

B.1 Model and preliminary results

Two firms produce a homogeneous good at constant unit cost r. There is a unit mass of buyers. Each buyer inelastically demands one unit of the good and is willing to pay a maximum of $v > r \geq 0$. Let $\theta \equiv v - r$. Consumers search for prices non-sequentially and buy from the cheapest store they know. Search costs are distributed on $(0, \overline{c})$, with distribution G and density g. Searching n times costs the consumer nc, $n = 0, 1, 2$.

Firms and buyers play a simultaneous moves game. An individual firm chooses its price taking rivals' prices as well as consumers' search behavior as given. A firm i's strategy is denoted by a distribution of prices $F_i(p)$. Let $F_{-i}(p)$ denote the vector of prices charged by firms other than i. The (expected) profit to firm i from charging price p_i given rivals' strategies is denoted by $\Pi(p_i, F_{-i}(p))$. Likewise, an individual buyer takes as given firm pricing and decides on his/her optimal search strategy to maximize his/her expected utility. The strategy of a consumer with

[3] The focus on the case in which consumers search non-sequentially is not accidental. The case of sequential search has been studied by Stahl (1996). Stahl shows that in the absence of an atom of shoppers (i.e. a strictly positive fraction of consumers with zero search costs), the search model suffers from a problem of multiplicity of symmetric equilibria. The monopoly price is always an equilibrium (Corollary to Proposition 4.1) and therefore does not depend on the distribution of search costs. Further, there may be a continuum of pure-strategy equilibria (Proposition 4.3). Furthermore, with log-concavity of the search cost density, pure-strategy equilibria are the only possible equilibria. With an atom of shoppers, the model has symmetric equilibria in mixed strategies but the characterization of the equilibrium price distribution is analytically non-tractable.

search cost c is then choosing a number k of prices to sample, $k = 0, 1, 2$. Let the fraction of consumers sampling k firms be denoted by μ_k. We shall concentrate on symmetric Nash equilibria (SNE). An SNE is a distribution of prices $F(p)$ and a collection $\{\mu_0, \mu_1, \mu_2\}$ such that (a) $\Pi_i(p, F(p))$ is equal to a constant $\overline{\Pi}$ for all p in the support of $F(p)$, $i = 1, 2$; (b) $\Pi_i(p, F(p)) \leq \overline{\Pi}$ for all p, $i = 1, 2$; (c) a consumer with search cost c chooses to sample $k(c)$ firms such that $k(c) = \arg\min_{k\in\{0,1,2\}} \left[kc + \int_{\underline{p}}^{v} pk(1 - F(p))^{k-1} f(p)dp\right]$; and (d) $\sum_{k=0}^{2} \mu_k = 1$. Let us denote the equilibrium density of prices by $f(p)$, with maximum price $\overline{p}$ and minimum price $\underline{p}$.

The following two lemmas follow directly from Burdett and Judd (1983). The first indicates that, for an equilibrium to exist, there must be some consumers who search just once and others who search twice. The second shows that prices must be dispersed in equilibrium.

Lemma 1 *If an SNE exists, then* $1 > \mu_k > 0$, $k = 1, 2$, *and* $\mu_0 \geq 0$.

The intuition behind this result is simple. Suppose all searching consumers did search twice ($\mu_0 + \mu_2 = 1$); then pricing would be competitive. This, however, is contradictory because then consumers would not be willing to search that much in the first place. Suppose now that no consumer did compare prices ($\mu_0 + \mu_1 = 1$); then firms would charge the monopoly price. This is also contradictory because in that case consumers would not be willing to search at all.[4]

Lemma 2 *If an SNE exists,* $F(p)$ *must be atomless with upper bound equal to* v.

This is easily understood. If a particular price is chosen with strictly positive probability then a deviant firm can gain by undercutting such a price and attracting all price-comparing consumers. This competition for the price-comparing consumers cannot drive the price down to zero because in that case a deviant firm would prefer to raise its price and sell to the consumers who do not compare prices.

We now turn to consumers' search behavior. Expenditure minimization requires a consumer with search cost c to continue to draw prices from the price distribution $F(p)$ untill the expected gains of drawing one more price fall below her search cost. The expected net gains from searching once rather than not searching at all are given by $v - E[p] - c$, whereas the expected net gains from searching twice rather than once are given by $E[p] - E[\min\{p_1, p_2\}] - c$, where E denotes the expectation operator. Because the search cost distribution has support on $[0, \bar{c}]$, we can define the

[4] In the original model of Burdett and Judd (1983), it is assumed that the search cost is lower than the surplus consumers get at the monopoly price. As a result, all consumers buy no matter the equilibrium price distribution and therefore there always exists an equilibrium where all firms charge the monopoly price (cf. Diamond, 1971). As we have arbitrary search cost heterogeneity, this assumption is relaxed. A by-product is that a Diamond-type of result can no longer be an equilibrium.

search costs c_0 and c_1 as follows:

$$c_0 = \min\left\{\bar{c}, v - E[p]\right\}, \tag{A3}$$

$$c_1 = E[\min\{p_1, p_2\}] - E[p]. \tag{A4}$$

From Lemma 1, it must be the case that $c_1 > 0$ and $c_0 > c_1$. The critical search cost c_0 is the minimum of the search cost of the consumer who is indifferent between searching and not searching at all and of the upper bound of the search cost distribution. When the upper bound of the search cost distribution $\bar{c}$ is sufficiently high $c_0 = v - E[p]$ and all consumers with search cost above c_0 will not search at all. When $\bar{c}$ is small enough, all consumers will search at least once. The critical search cost c_1 is the search cost of the consumer who is indifferent between searching once and twice. In particular, consumers for whom $c_1 < c \leq c_0$ will indeed search once and consumers for whom $c \leq c_1$ will search twice.

Lemma 3 *Given any atomless price distribution $F(p)$, optimal consumer search behavior is uniquely characterized as follows: the fractions of consumers searching once and twice are given by*

$$\mu_1 = \int_{c_1}^{c_0} dG(c) > 0; \;\; \mu_2 = \int_{0}^{c_1} dG(c) > 0 \tag{A5}$$

whereas the fraction of consumers not searching at all is

$$\mu_0 = \int_{c_0}^{\bar{c}} dG(c) \geq 0, \tag{A6}$$

where c_0 and c_1 are given by equations (A3)–(A4).

We now examine firm pricing behavior taking consumer search strategies as given. Following Burdett and Judd (1983), a firm i charging a price p_i sells to a consumer who searches one time provided the consumer samples firm i, which happens with probability $\frac{1}{2}$, and sells to a consumer who searches twice provided the rival firm charges a price higher than p_i, which happens with probability $1 - F(p_i)$. Therefore the expected profit to firm i from charging price p_i when its rivals draw a price from the cumulative distribution function (CDF) $F(p)$ is

$$\Pi_i(p_i; F(p)) = (p_i - r)\left\{\tfrac{1}{2}\mu_1 + \mu_2\left[1 - F(p_i)\right]\right\}.$$

In equilibrium, a firm must be indifferent between charging any price in the support of $F(p)$ and charging the upper bound $\overline{p}$. Thus, any price in the support of $F(p)$ must satisfy $\Pi_i(p_i; F(p)) =$

$\Pi_i(\overline{p}; F(p))$. Because $\Pi_i(\overline{p}; F(p))$ is monotonically increasing in $\overline{p}$, it must be the case that $\overline{p} = v$. As a result, equilibrium pricing requires

$$(p_i - r)\left\{\mu_1 + 2\mu_2\left[1 - F(p_i)\right]\right\} = \mu_1(v - r).$$

Solving this equation for $F(p_i)$ leads to the following result, which is also from Burdett and Judd (1983):

Lemma 4 *Given μ_1 and μ_2, there exists a unique symmetric equilibrium price distribution $F(p)$. In equilibrium firms charge prices randomly chosen from the set $\left[\frac{(v-r)\mu_1}{\mu_1+2\mu_2} + r, v\right]$ according to the price distribution*

$$F(p) = 1 - \frac{\mu_1}{2\mu_2}\frac{v-p}{p-r}. \tag{A7}$$

Notice that $F(p)$ depends on the search cost distribution via its effect on μ_1 and μ_2; moreover, notice that $F(p)$ is increasing in μ_2 and decreasing in μ_1. Hence, if an increase in search costs results in a higher (lower) ratio of "price-comparing to non-price-comparing" consumers, then the price distribution will shift up (down) and prices will correspondingly decrease (increase).

For the price distribution (A7) to be an equilibrium of the game, the conjectured groupings of consumers have to be the outcome of optimal consumer search. This requires that

$$c_0 = \min\left\{\overline{c}, \int_0^v F(p)dp\right\} \text{ and } c_1 = \int_0^v F(p)(1 - F(p))dp.$$

As the price distribution $F(p)$ in equation (A7) is strictly increasing in p, we can find its inverse:

$$p(z) = \frac{v - r}{1 + 2\frac{\mu_2}{\mu_1}(1 - z)} + r.$$

Using this inverse function, integration by parts and the change of variables $z = F(p)$, we can state that:

Proposition 1 *If an SNE exists in the non-sequential search duopoly model with homogeneous products then consumers search according to Lemma 3, firms set prices according to Lemma 4, and*

c_0 and c_1 are given by the solution to the following system of equations:

$$c_0 = \min\left\{\bar{c}, (v-r)\left[1 - \int_0^1 \frac{G(c_0) - G(c_1)}{G(c_0) - G(c_1)(1-2u)} du\right]\right\}, \tag{A8}$$

$$c_1 = (v-r)\int_0^1 \frac{[G(c_0) - G(c_1)](1-2u)}{G(c_0) - G(c_1)(1-2u)} du. \tag{A9}$$

The proof involves relatively standard manipulations and is therefore omitted. Because of the heterogeneous search costs, this result is not in Burdett and Judd (1983). It is useful because of two reasons. First, it provides a straightforward way to compute the market equilibrium. For fixed v, r, $\bar{c}$, and $G(c)$, the system of equations (A8)–(A9) can be solved numerically. If a solution exists, then the consumer equilibrium is given by equations (A5)–(A6) and the price distribution follows readily from equation (A7). Secondly, this result enables us to address the issues of existence and uniqueness of SNE by studying whether the system (A8)–(A9) has a solution and whether such a solution is unique.

B.2 Equilibrium and comparative statics for sufficiently high search costs

We start with the most interesting case, which arises when $c_0 > \bar{c}$ and therefore not all consumers search in market equilibrium.

Proposition 2 *In the non-sequential search duopoly model with homogeneous products, assume that $\bar{c}$ is sufficiently large so that c_0 defined in equation (A8) satisfies $c_0 < \bar{c}$. Then: (A) For any search cost distribution function $G(c)$ with support $(0, \bar{c})$ such that either $g(0) > 0$ or $g(0) = 0$ and $g'(0) > 0$, an SNE exists. (B) For the family of power distribution functions $G(c) = (c/\bar{c})^a$, $a > 0$, the SNE is unique.*

Proof. Define $\theta \equiv v - r$. Changing variables $x_0 = G(c_0)$ and $x_1 = G(c_1)$ in the system of equations (A8)–(A9) gives:

$$x_0 = G\left(\theta - \theta\int_0^1 \frac{x_0 - x_1}{x_0 - x_1 + 2x_1 u} du\right);$$

$$x_1 = G\left(\theta\int_0^1 \frac{(x_0 - x_1)(1-2u)}{x_0 - x_1 + 2x_1 u} du\right).$$

Let $y \equiv x_1/x_0 \in [0,1]$. An equilibrium of the model is given by a solution to

$$H(y) \equiv yG(c_0(y)) - G(c_1(y)) = 0, \tag{A10}$$

where

$$c_0(y) = \theta - \theta\,(1-y)\,I(y), \tag{A11}$$

$$c_1(y) = \theta\,(1-y)\,J(y), \tag{A12}$$

and

$$I(y) = \int_0^1 \frac{1}{1-y+2yu}du = \frac{\log(1+y)-\log(1-y)}{2y},$$
$$J(y) = \int_0^1 \frac{1-2u}{1-y+2yu}du = \frac{\log(1+y)-\log(1-y)-2y}{2y^2}.$$

We note that for $y=0$ and $y=1$ we have

$$H(0) = 0\cdot G\left(c_0\left(0\right)\right) - G\left(c_1\left(0\right)\right) = -G\left(0\right) = 0,$$
$$H(1) = G\left(c_0\left(1\right)\right) - G\left(c_1\left(1\right)\right) = G\left(1\right) - G\left(0\right) = G\left(1\right) > 0.$$

Consider now the value of $\partial H\left(y\right)/\partial y$ at $y=0$. Because $0 = c_1\left(0\right) = c_0\left(0\right)$ and the derivative $c_1'\left(0\right) > 0$ we have

$$\frac{\partial H(0)}{\partial y} = G\left(0\right) - g\left(0\right)c_1'\left(0\right) = -g\left(0\right)c_1'\left(0\right) < 0.$$

Given these three observations (i.e. $H\left(0\right) = 0, H\left(1\right) > 0$ and $\partial H\left(0\right)/\partial y < 0$), we conclude that there exists at least one equilibrium.

We now prove uniqueness of equilibrium. Let $G\left(c\right) = \left(c/\beta\right)^a$ for some $a > 0$ with support $[0,\beta]$. Because the case $y=0$ is not interesting and $G\left(c_0\left(y\right)\right) > 0$ for $y > 0$, it is sufficient to prove that the equation

$$y = \frac{G\left(c_1\left(y\right)\right)}{G\left(c_0\left(y\right)\right)} \tag{A13}$$

has a unique solution. As the LHS of equation (A13) is increasing in y, it suffices to show that the RHS decreases in y. Let $h\left(y\right)$ denote the RHS of equation (A13):

$$h\left(y\right) = \frac{\left(\frac{c_1(y)}{\beta}\right)^a}{\left(\frac{c_0(y)}{\beta}\right)^a} = \frac{c_1\left(y\right)^a}{c_0(y)^a}.$$

The derivative of $h(y)$ is

$$\begin{aligned}\frac{dh(y)}{dy} &= \frac{ac_1'(y)\,c_1^{a-1}(y)\,c_0^a(y) - ac_1^a(y)\,c_0'(y)\,c_0^{a-1}(y)}{c_0^{2a}(y)};\\ &= \frac{ac_1^{a-1}(y)\,c_0^{a-1}(y)}{c_0^{2a}(y)}\left(c_1'(y)\,c_0(y) - c_1(y)\,c_0'(y)\right).\end{aligned}$$

Because

$$\begin{aligned}c_1'(y) &= \frac{2y(2+y) - (1+y)(2-y)\ln\frac{1+y}{1-y}}{2y^3(1+y)},\\ c_0'(y) &= \frac{-2y + (1+y)\ln\frac{1+y}{1-y}}{2y^2(1+y)},\end{aligned}$$

we obtain that

$$\begin{aligned}c_1'(y)\,c_0(y) - c_1(y)\,c_0'(y) &= 4y^2(1+2y) + 2y(1+y)(2-y)\ln\frac{1-y}{1+y}\\ &\quad + \left(1-y^2\right)(1-y)\ln^2\frac{1-y}{1+y}.\end{aligned}$$

This expression is negative for $0 < y < 1$, so $dh(y)/dy < 0$, and therefore, the symmetric equilibrium is unique. ■

Elsewhere, we have extended this existence result to the case of an arbitrary number of firms N (see Moraga-González, Sándor, and Wildenbeest, forthcoming). General results on uniqueness prove to be very difficult even for the duopoly case because we cannot compute the equilibrium explicitly and the system of equations (A8)–(A9) is non-linear.

The effect of lowering search costs when not all consumers search

The next step in the analysis is to study how a FOSD decrease in search costs affects the equilibrium distribution of prices. As mentioned above, for this it suffices to study how the ratio of "price-comparing to non-price comparing" consumers μ_2/μ_1 is affected by a decrease in search costs. To do so, as in the main body of the article, let $G(c;\beta)$ be a parametrized search cost CDF with $G_\beta'(c;\beta) \equiv \partial G(c;\beta)/\partial\beta < 0$ and denote the equilibrium price distribution corresponding to a given β by $F(p;\beta)$. We now examine how F changes with β.

To understand the effect of a decrease in β on the equilibrium price distribution, we study how the solution to the system of equations that determines c_0, c_1 and c_2 depends on β; this, in turn, determines how μ_1 and μ_2 vary with β and, by implication, the equilibrium price distribution.

Remember the following relationship between the variables we have introduced before:

$$y = x_1/x_0,\ \ x_0 = G(c_0),\ \ x_1 = G(c_1),\ \ \mu_2 = x_1,\ \ \mu_1 = x_0 - x_1.$$

Because

$$\frac{\mu_2}{\mu_1} = \frac{1}{\frac{1}{y} - 1},$$

a decrease in y results in a decrease in μ_2/μ_1 and, correspondingly, in an increase in prices.

Let $y(\beta)$ denote a solution to equation $H(y;\beta) = 0$ in equation (A10). We now study how $y(\beta)$ depends on the shifter β. The Implicit Function Theorem applied to equation (A10) implies that

$$\frac{dy\left(\beta\right)}{d\beta} = -\frac{\frac{\partial H(y;\beta)}{\partial \beta}}{\frac{\partial H(y;\beta)}{\partial y}}. \tag{A14}$$

In order to obtain the sign of this derivative, we first consider its numerator.

$$\begin{aligned}
\frac{\partial H(y;\beta)}{\partial \beta} &= yG'_\beta\left(c_0\left(y\right);\beta\right) - G'_\beta\left(c_1\left(y\right);\beta\right);\\
&= \frac{G\left(c_1\left(y\right);\beta\right)}{G\left(c_0\left(y\right);\beta\right)}G'_\beta\left(c_0\left(y\right);\beta\right) - G'_\beta\left(c_1\left(y\right);\beta\right);\\
&= G\left(c_1\left(y\right);\beta\right)\left[\frac{G'_\beta\left(c_0\left(y\right);\beta\right)}{G\left(c_0\left(y\right);\beta\right)} - \frac{G'_\beta\left(c_1\left(y\right);\beta\right)}{G\left(c_1\left(y\right);\beta\right)}\right],
\end{aligned}$$

where the second equality follows from the equilibrium condition (A10). Therefore, we conclude that

$$\frac{\partial H(y;\beta)}{\partial \beta} > 0 \text{ if and only if } \frac{G'_\beta\left(c_1\left(y\right);\beta\right)}{G\left(c_1\left(y\right);\beta\right)} - \frac{G'_\beta\left(c_0\left(y\right);\beta\right)}{G\left(c_0\left(y\right);\beta\right)} < 0.$$

Consider now the denominator of equation (A14). For a given β, $\partial H(y;\beta)/\partial y$ is the derivative of H at the solution y. We note that for $y = 0$ and $y = 1$ we have

$$\begin{aligned}
H(0;\beta) &= 0 \cdot G\left(c_0\left(0\right);\beta\right) - G\left(c_1\left(0\right);\beta\right) = -G\left(0;\beta\right) = 0,\\
H(1;\beta) &= G\left(c_0\left(1\right);\beta\right) - G\left(c_1\left(1\right);\beta\right) = G\left(1;\beta\right) - G\left(0;\beta\right) = G\left(1;\beta\right) > 0.
\end{aligned}$$

Consider now the value of $\partial H\left(y;\beta\right)/\partial y$ at $y = 0$. Because $0 = c_1\left(0\right) = c_0\left(0\right)$ and $c'_1\left(0\right) > 0$ we have

$$\frac{\partial H(0;\beta)}{\partial y} = G\left(0;\beta\right) - g\left(0;\beta\right)c'_1\left(0\right) = -g\left(0;\beta\right)c'_1\left(0\right) < 0.$$

Given these three observations (i.e. $H(0,\beta)=0, H(1,\beta)>0$ and $\partial H(0,\beta)/\partial y<0$), we conclude that there exists at least one equilibrium at which H is increasing in y.[5] We then obtain the following result:

Proposition 3 *Consider the non-sequential search duopoly model with homogeneous products and let $G(c;\beta)$ be a parametrized search cost CDF with positive density on $[0,\overline{c}]$ and with derivative $G'_\beta(c;\beta)<0$ for all c. Assume that $\overline{c}$ is sufficiently large so that c_0 defined in equation (A8) satisfies $c_0<\overline{c}$. Then, if there exists a unique equilibrium, a FOSD decrease in search costs results in a FOSD increase (decrease) in prices if and only if*

$$\frac{G'_\beta(c_1(y);\beta)}{G(c_1(y);\beta)}-\frac{G'_\beta(c_0(y);\beta)}{G(c_0(y);\beta)}>(<)0,$$

where $c_0(y)$ and $c_1(y)$ are given by equations (A11) and (A12), respectively.

Corollary 1 *When G'_β/G is monotone decreasing (increasing) in c, then prices rise (fall) as search costs decrease.*

This result is similar to that in the main body of the article. Prices can increase or decrease after search costs go down for all consumers provided that not all consumers search. What is needed for prices to increase is that the impact of higher search costs on the *intensive search margin* is weaker than that on the *extensive search margin*, and this is guaranteed when the ratio G'_β/G decreases in c.

We finish this subsection by relating Corollary 1 to the *reversed hazard rate* stochastic ordering (see Shaked and Shanthikumar, 2007).

Definition 1. *The distribution $G(c;\beta)$ has the increasing reversed hazard rate property (IRHRP) if and only if for any $\beta'<\beta$,*

$$G(c,\beta)G(d,\beta')\leq G(c,\beta')G(d;\beta)$$

for any $c\leq d$ in the union of the supports of $G(c,\beta')$ and $G(c;\beta)$.

It is straightforward to check that with distributions that satisfy the IRHRP, the ratio G'_β/G increases in c, in which case a FOSD fall in search costs results in a FOSD decrease in prices.

[5] We ignore ill-behaved situations where at the solutions of equation (A10), $H(\cdot;\beta)$ is tangent to the horizontal axes, that is, we assume that $\partial H(y,\beta)/\partial y\neq 0$ at any solution y. Moreover, if there are multiple equilibria, the number of equilibria is odd. In such situation each odd-numbered solution $y(\beta)$ satisfies $\partial H(y,\beta)/\partial y>0$, whereas each even-numbered solution $y(\beta)$ satisfies $\partial H(y,\beta)/\partial y<0$.

Like in the main body of the article we can define the reverse property, namely, *decreasing reversed hazard rate property* (DRHRP) as follows:

Definition 2. *The distribution $G(c;\beta)$ has the decreasing reversed hazard rate property (DRHRP) if and only if for any $\beta' < \beta$,*

$$G(c,\beta)G(d,\beta') \geq G(c,\beta')G(d;\beta)$$

for any $c \leq d$ in $[0, \min\{\bar{c}(\beta), \bar{c}(\beta')\}]$.

Note that, as in the definition of DLRP that we give in the main body of the article, we define DRHRP up to the minimum of the upper bounds of the supports of the search cost distributions $G(c,\beta)$ and $G(c,\beta')$. This is less restrictive and is needed for compatibility of DRHRP with FOSD.

It is straightforward to check that with distributions that satisfy the DRHRP, the ratio G'_β/G decreases in c, in which case a FOSD fall in search costs results in a FOSD increase in prices.

We finally note that IRHRP is weaker than ILRP; likewise DRHRP is weaker than DLRP. Therefore IRHRP and DRHRP are also sufficient for our results in the main body of the article.

B.3 Equilibrium and comparative statics for sufficiently low search costs

We finish this appendix by discussing the case in which the upper bound of the search cost distribution is sufficiently low so that $c_0 = \bar{c}$. This implies that $\mu_0 = 0$. In this case higher search costs only have an effect at the intensive search margin and thereby we should obtain the standard result that higher search costs lead to higher prices. For this case,

$$\frac{\mu_2}{\mu_1} = \frac{1}{\mu_1} - 1.$$

As a result, the equilibrium price distribution is uniquely determined by μ_1, which in turn depends on

$$c_1 = \theta \int_0^1 \frac{[1 - G(c_1;\beta)]\,(1-2u)}{1 - G(c_1;\beta)(1-2u)} du. \tag{A15}$$

Using the change of variables $y \equiv G(c_1)$ we can write equation (A15) as $y - G\left(c_1(y)\right) = 0$ where $c_1\left(y\right)$ is given in equation (A12). Rewriting gives:

$$H(y;\beta) \equiv y - G\left(-\frac{\theta\,(1-y)}{2y^2}\left[2y + \ln\left(\frac{1-y}{1+y}\right)\right];\beta\right) = 0 \tag{A16}$$

As above, an equilibrium of the model is given by a solution to equation (A16).

Proposition 4 *In the non-sequential search duopoly model with homogeneous products, assume that* $\bar{c}$ *is sufficiently small so that* c_0 *defined in equation (A8) satisfies* $c_0 = \bar{c}$*. Then: (A) For any search cost distribution function* $G(c)$ *with support* $(0, \bar{c})$ *such that* $g(0) > 3$*, an SNE exists. (B) For concave search cost distributions* G*, if an SNE exists, it is unique.*

Proof. Existence follows from making sure that equation (A16) has a solution. Note that $c_1(y)$ is strictly concave and $c_1(0) = c_1(1) = 0$. For an equilibrium to exist, then, it suffices that the derivative of $G(c_1(y))$ with respect to y at $y = 0$ is greater than 1. The condition $g(0) > 3$ ensures this.

Regarding uniqueness, what we need is that G preserves the strict concavity of $c_1(y)$. Concavity of G ensures this. ■

The effect of lower search costs when all consumers search

If we consider the parametrized search cost distribution above $G(c; \beta)$ and compute the derivative of H with respect to β, we get $\partial H(y; \beta)/\partial\beta = -G'_{\beta}(c_1(y); \beta) > 0$. This implies that the sign of equation (A14) is negative. As a result:

Proposition 5 *Consider the non-sequential search duopoly model with homogeneous products and let* $G(c; \beta)$ *be a parametrized search cost CDF with positive density on* $[0, \bar{c}]$ *and with derivative* $G'_{\beta}(c; \beta) < 0$ *for all* c*. Assume that* $\bar{c}$ *is sufficiently low so that* c_0 *defined in equation (A8) is equal to* $\bar{c}$*. Then,* $F(p; \beta)$ *decreases in* β *for all* p *(so prices unambiguously decrease if search costs fall).*

Conclusions

This supplementary appendix has illustrated that the main results in the article remain valid in alternative settings. First, we have studied the case of duopoly and consumer search for differentiated products. Secondly, we have examined a setting in which firms sell homogeneous products, consumers search non-sequentially for lower prices, and firm equilibrium is characterized by a distribution of prices.

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