Truly Costly Sequential Search and Oligopolistic Pricing*

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Abstract

We modify the paper of Stahl (1989) on oligopolistic pricing and sequential consumer search by relaxing the assumption that consumers obtain the first price quotation for free. When all price quotations are costly to obtain, the unique symmetric equilibrium need not involve full consumer participation. The region of parameters for which non-shoppers do not fully participate in the market becomes larger as the number of shoppers decreases and/or the number of firms increases. The comparative statics properties of this new type of equilibrium are interesting. In particular, expected price increases as search cost decreases and is constant in the number of shoppers and in the number of firms. Welfare falls as firms enter the market. We show that monopoly pricing never obtains with truly costly search.

Keywords: sequential consumer search, oligopoly, price dispersion

JEL Classification: C13, D40, D83, L13

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1 Introduction

A celebrated article by Stahl (1989) studies oligopolistic pricing in the presence of consumer search. There are two types of consumers in the market. Fully informed consumers (referred to as *shoppers* in his article) have no opportunity cost of time and thus search for all prices at no cost; non-shoppers search sequentially, i.e., they first observe one price and then decide whether or not to observe a second price, and so on.[1] Stahl (1989) assumes that consumers observe the first price quotation for free, as do many other papers in the search literature, which implies that every buyer makes at least one search. In this paper, we study the implications of relaxing this assumption.

The optimal sequential search rule implies that a consumer with a price at hand continues searching if, and only if, the observed price is higher than a certain reservation price. Knowing this, no firm will charge prices above consumers' reservation price. Therefore, under the assumption that obtaining the first price observation is costless, buyers 'search' exactly once in equilibrium and buy at the observed price. In this paper, we refer to this type of equilibrium as one with *full consumer participation.* This equilibrium is one of the two possible equilibrium configurations when the first price quotation is *not* for free. The new type of equilibrium that arises with truly costly search is one with *partial consumer participation*, where some buyers decide not to search at all as they rationally expect prices to be so high that they are indifferent between searching and not searching. The existence and characterization of this new type of equilibrium is one of the two main contributions of this paper.

The other main contribution is to provide the comparative statics properties of the equilibrium with partial consumer participation. These comparative statics effects differ from those under full consumer participation in interesting ways. First, the equilibrium distribution of prices with a given search cost dominates in a first-order stochastic sense the price distribution with a lower search cost; as a result expected price increases as search cost decreases. This is due to the fact that a decrease in search cost raises participation of non-shoppers, who happen to search only once in equilibrium. As firms have monopoly power over these consumers, they raise their prices. A second result is that an increase in the number of shoppers does not influence the equilibrium price distribution. This is because more shoppers foster the participation of non-shoppers in such a way that prices remain the same. Finally, we find that firm entry results in a mean-preserving

[1] The functioning of markets in the presence of sequential consumer search is also examined in Anderson and Renault (1999), Reinganum (1979), Rob (1985), Stahl (1996) and Stigtlitz (1987).

spread of prices and in a decrease in welfare because the market participation rate of non-shoppers falls. The last two results imply that, unlike in Stahl's model, expected price does not tend to the monopoly price when the number of shoppers converges to zero, nor when the number of firms goes to infinity. This is because when the fraction of shoppers becomes very small, or the number of firms very large, the economy turns into an equilibrium with partial consumer participation and in such an equilibrium expected price is insensitive to changes in those parameters.

The rest of this paper is organized as follows. Section 2 presents the model. A full characterization and an overview of the two types of equilibrium are given in Section 3. Section 4 presents the different comparative statics results and Section 5 concludes.

2 The Model

We examine the model of oligopolistic competition and sequential consumer search presented in Stahl (1989), but we assume that *all* price quotations are costly to obtain for non-shoppers. The features of the model are as follows. There are N firms that produce a homogeneous good at constant returns to scale. Their identical unit cost can be normalized to zero and prices can be interpreted as price-to-cost margins. There is a unit mass of buyers and we assume that buyers hold inelastic demands.[2] A consumer wishes to purchase at most a single unit of the good and his/her valuation for the item is $v > 0$. A proportion $\mu \in (0, 1)$ of the consumers has zero opportunity cost of time and therefore searches for prices costlessly. These consumers are referred to as *shoppers*. The other $1 - \mu$ percent of the buyers, referred to as *non-shoppers*, must pay search cost $c > 0$ to observe every price quotation they get, including the first one. Non-shoppers search sequentially, i.e., a buyer first decides whether to sample a first firm or not and then, upon observation of the price of the first firm, decides to search for a second price or not, and so on. We assume that $v > c$.

Firms and buyers play the following game. An individual firm chooses its price taking price choices of the rivals as well as consumers' search behavior as given. Likewise, an individual buyer forms conjectures about the distribution of prices in the market and decides on his/her optimal search strategy. We restrict the analysis to symmetric Nash equilibria. The distribution of prices charged by a firm is denoted by $F(p)$, its density by $f(p)$ and the lower and the upper bound of its

[2] Stahl (1989) considers a more general specification of the demand function. The assumption of inelastic demand allows us to compute explicitly the reservation price and give a full characterization of which type of equilibrium exists for which configurations of parameters. Provided that consumer surplus at the monopoly price does not fully cover the search cost, the main qualitative results of our paper do not depend on the assumption of inelastic demand.

support by $\underline{p}$ and $\overline{p}$, respectively.

3 Equilibrium Analysis

We first derive some auxiliary results.

Lemma 1 *An equilibrium where non-shoppers do not search at all does not exist.*

Proof. Suppose non-shoppers did not search. Then, the only consumers left in the market would be the shoppers. Therefore, competition between stores would drive prices down to marginal cost. But then, as $v - c > 0$, the non-shoppers would gain by deviating and searching once. ■

Lemma 1 reveals that existence of equilibrium requires the non-shoppers to be active in the market with strictly positive probability. The next result is provided by Stahl (1989).

Lemma 2 *In equilibrium non-shoppers will not search beyond the first firm.*

Proof. See Lemma 2 of Stahl (1989). ■

The idea behind Lemma 2 is that pricing above consumers' reservation price is never optimal for firms since buyers would continue searching if that were the case; as a result, the price buyers find at the first store they encounter is always accepted and no further search takes place.

Let us introduce the following notation. Let θ_1 be the probability with which a non-shopper searches once. Lemmas 1 and 2 together imply that only two candidates for equilibrium exist: either (a) $\theta_1 = 1$, or (b) $0 < \theta_1 < 1$. The first case is similar to Stahl (1989). We shall refer to this equilibrium as one with *full consumer participation.* This is because if all consumers search once, they will all buy the good. This contrasts with case (b) where consumers mix between not searching at all and searching once so not all consumers enter the market. In this case, we will speak of *partial consumer participation.*

The next remark is that, since $\theta_1 > 0$ in any equilibrium, the equilibrium price distribution must be atomless.

Lemma 3 *Irrespective of the search behavior of non-shoppers, if $F(p)$ is an equilibrium price distribution, then it is atomless. Hence, there is no pure strategy equilibrium.*

Proof. See Lemma 1 of Stahl (1989). The proof extends straightforwardly to the case of partial consumer participation. ■

We note that firms have an incentive to charge low prices in order to attract all the shoppers but at the same time they also have an incentive to charge high prices to extract income from the consumers who do not compare prices. These two forces are balanced when firms randomize their prices. Lemma 3 shows that equilibria must necessarily exhibit price dispersion, and that firm pricing is always characterized by atomless price distributions. In what follows we shall examine the characterization and the existence of the different types of equilibrium.

Case a: Equilibrium with full consumer participation

Suppose that non-shoppers search for one price with probability 1, i.e., $\theta_1 = 1$. This is the case analyzed by Stahl (1989) with two modifications. First, as Stahl considers a more general demand structure, an explicit expression for the reservation price cannot be obtained. Second, as Stahl (1989) assumes the first price quotation to be for free, this full participation equilibrium exists for all values of the parameters in his model, but not in ours. We will explicitly define the parameter space for which an equilibrium with full consumer participation exists when the first price quotation is costly. These two modifications deserve a slightly extended analysis.

Under full consumer participation, the expected payoff to firm i from charging price p_i when its rivals choose a random pricing strategy according to the cumulative distribution $F(\cdot)$ is

$$\pi_i(p_i, F(p_i)) = p_i \left[\frac{1-\mu}{N} + \mu (1 - F(p_i))^{N-1} \right]. \tag{1}$$

This profit expression is easily interpreted. Firm i attracts the μ shoppers when it charges a price that is lower than its rivals' prices, which happens with probability $(1 - F(p_i))^{N-1}$. The firm also serves the $1 - \mu$ non-shoppers whenever they visit its store, which occurs with probability $1/N$.

In equilibrium, a firm must be indifferent between charging any price in the support of $F(\cdot)$. Let us denote the upper bound of $F(\cdot)$ by $\bar{p}$. Any price in the support of $F(\cdot)$ must then satisfy $\pi_i(p_i, F(\cdot)) = \pi_i(\bar{p})$, i.e.,

$$p_i \left[\frac{1-\mu}{N} + \mu (1 - F(p_i))^{N-1} \right] = \frac{(1-\mu)\bar{p}}{N}. \tag{2}$$

Solving this equation for the price distribution yields

$$F(p) = 1 - \left(\frac{(1-\mu)(\bar{p} - p)}{N \mu p} \right)^{\frac{1}{N-1}}. \tag{3}$$

Since $F(\cdot)$ is a distribution function there must be some $\underline{p}$ for which $F(\underline{p}) = 0$. Solving for $\underline{p}$ one obtains the lower bound of the price distribution $\underline{p} = (1-\mu)\bar{p}/(\mu N + (1-\mu))$.

The cumulative distribution (3) represents optimal firm pricing. We now turn to discuss optimal consumer behavior. Consider a buyer who has observed a given price p. This consumer will continue to search if the expected benefits from searching further exceed the search cost. We can define the reservation price ρ as the price that makes a consumer indifferent between searching once more and accepting the price at hand; this price satisfies:

$$\int_{\underline{p}}^{\rho} (\rho - p) f(p) dp = c. \tag{4}$$

No firm will charge a price above ρ since this will lead to continued search (Stahl, 1989). As a result the upper bound $\bar{p} = \rho$. We now derive an expression for ρ. First rewrite (4) as

$$\rho - E[p] - c = 0. \tag{5}$$

To calculate $E[p]$ we solve equation (3) for p, which gives

$$p = \frac{\rho}{1 + bN(1-F)^{N-1}}, \tag{6}$$

where $b = \mu/(1-\mu) > 0$. Note that $E[p] = \rho - \int_{\underline{p}}^{\rho} F(p)dp$. By changing variables we can write $E[p] = \int_0^1 p dy$. Plugging p from equation (6) gives, after rearranging,

$$E[p] = \rho \int_0^1 \frac{dy}{1 + bNy^{N-1}}. \tag{7}$$

Equation (7) can be plugged into equation (5) to solve for ρ:

$$\rho = \frac{c}{1 - \int_0^1 \frac{dy}{1+bNy^{N-1}}}, \tag{8}$$

It can be shown that the reservation price ρ increases in c and in N, decreases in μ and is insensitive to v.

It must be the case that $\rho \leq v$. In addition, non-shoppers must find it profitable to search once, rather than not searching at all, i.e.,

$$v - E[p] - c \geq 0. \tag{9}$$

Inspection of (5) and (9) implies that $\rho \leq v$. It is useful to rewrite condition (9) as

$$1 - \int_0^1 \frac{dy}{1 + bNy^{N-1}} \geq \frac{c}{v}. \tag{10}$$

This equation gives the set of parameters for which an equilibrium where buyers search once for sure exists. For future reference, let us denote the left-hand-side of equation (10) as $\Phi(1;\mu;N)$.[3] We note that $0 < \Phi(1;\mu;N) < 1$ for all values of the parameters.

Proposition 1 *Let $0 < \frac{c}{v} \leq \Phi(1;\mu,N)$. Then a market equilibrium with full consumer participation exists where firms prices are distributed according to (3) on the set $[(1-\mu)\rho/(\mu N + (1-\mu)), \rho]$ and all non-shoppers search once, where $\overline{p} = \rho$ and ρ solves equation (8).*

From (8) it follows that the reservation price ρ converges to v when c/v approaches $\Phi(1;\mu,N)$. The question that arises is: What happens when search cost is high, in particular when $\frac{c}{v} > \Phi(1;\mu,N)$? In what follows we show that the symmetric equilibrium involves partial consumer participation. This type of equilibrium is new in the sequential search literature and, as we shall see later, its properties are quite interesting.

Case b: Equilibrium with partial consumer participation

Now suppose that non-shoppers randomize between searching once and not searching at all, i.e., $0 < \theta_1 < 1$. The expected payoff to firm i is

$$\pi_i(p_i, F(p_i)) = p_i \left[\frac{(1-\mu)\theta_1}{N} + \mu(1 - F(p_i))^{N-1} \right]. \tag{11}$$

The economic interpretation of this profit function is analogous to that of equation (1), except that now there are only $(1-\mu)\theta_1$ non-shoppers active, rather than $1-\mu$. A similar analysis as above yields the following equilibrium price distribution:

$$F(p) = 1 - \left(\frac{\theta_1(1-\mu)(\bar{p} - p)}{N\mu p} \right)^{\frac{1}{N-1}}, \tag{12}$$

with support $[\underline{p}, \bar{p}]$ where $\underline{p} = (1-\mu)\theta_1\bar{p}/(\mu N + (1-\mu)\theta_1)$. We now notice that the upper bound is no longer equal to ρ, but equal to v. To see this, note that non-shoppers optimal behavior requires

[3] The number 1 in the arguments of $\Phi(\cdot)$ stands for θ_1=1.

that they are indifferent between searching once and not searching all, i.e., it must be the case that

$$v - E[p] - c = 0 \tag{13}$$

It is obvious that conditions (5) and (13) can only hold together if $\bar{p} = v$. Condition (13) can be rewritten as:

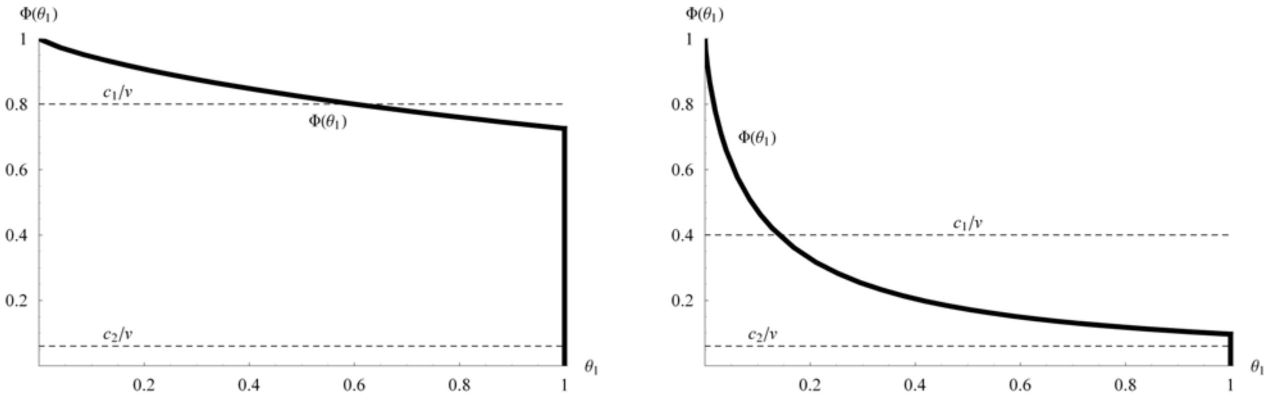
$$1 - \int_0^1 \frac{dy}{1 + \frac{1}{\theta_1} bNy^{N-1}} = \frac{c}{v}. \tag{14}$$

where $b = \mu/(1-\mu)$. For future reference, denote the left-hand-side of equation (14) as $\Phi(\theta_1, \mu, N)$. Inspection of this function reveals that $\Phi(0, \mu, N) = 1$ and that $\Phi(\theta_1, \mu, N)$ is monotonically decreasing in θ_1 and increasing in μ. It can also be shown that $\Phi(\theta_1, \mu, N)$ decreases in N.

Proposition 2 *Let $\Phi(1, \mu, N) < \frac{c}{v} < 1$. Then a market equilibrium with partial consumer participation exists where firms prices are distributed according to (12) on the set $[(1-\mu)\theta_1 v/(\mu N + (1-\mu)\theta_1), v]$ and non-shoppers search once with probability θ_1, which is the solution to (14) (with the remaining probability, non-shoppers stay out of the market).*

In summary, the game outlined above has a unique symmetric equilibrium where consumers may either search once surely, or mix between searching and not searching, depending on parameters. Inspection of the equations above immediately reveals that whether consumers participate in the market fully or partially depends on three critical parameters: (i) the value of the purchase compared to the search cost c/v, (ii) the number of consumers with zero opportunity cost of time μ, and (iii) the number of firms N. To illustrate this issue we have represented the regions of parameters for which the equilibrium exhibits full or partial consumer participation in Figure 1. In these graphs we set $N = 2$ and vary μ. The left graph exhibits a market with many shoppers while the right one illustrates a market with just a few of them. The decreasing curve represents $\Phi(\theta_1; \mu, N)$ as a function of θ_1. For large search cost parameters, say c_1, non-shoppers participate in the market with probability less than one. This probability is given by the point at which the curve $\Phi(\cdot)$ and the line c_1/v intersect. When search cost is low enough, e.g. c_2, non-shoppers search for one price with probability one.

The region of parameters for which there is partial consumer participation is larger the lower the parameter μ. This is because, as mentioned above, the function $\Phi(\cdot)$ falls as μ decreases (see equation (14)). A similar remark can be made when N increases. Indeed, when μ approaches 0 or N becomes very large, $\Phi(\theta_1; \mu, N)$ approaches 0 for all values of θ_1. In that case the region for which consumers participate partially in equilibrium covers almost the entire parameter space. This indicates that partial consumer participation is relevant when the number of firms in the market is large and/or there are few shoppers.



(a) $\mu = 0.8$ (b) $\mu = 0.1$

Figure 1: Parameter regions for which distinct types of equilibrium exist ($N = 2$)

4 Comparative Statics

In this section we study the influence of changes in the parameters of the model on search intensity θ_1, on the equilibrium price distribution, on the average price charged in the market $E[p]$, and on welfare, denoted W. The main results are summarized in Table 1. The results for the case of full consumer participation are similar to Stahl (1989); the others are new. An upwards (downwards) arrow means that the variable under consideration increases (falls); the symbol '$-$' means that the variable remains constant. Our discussion shall concentrate on the most striking and interesting observations concerning the equilibrium with partial consumer participation.

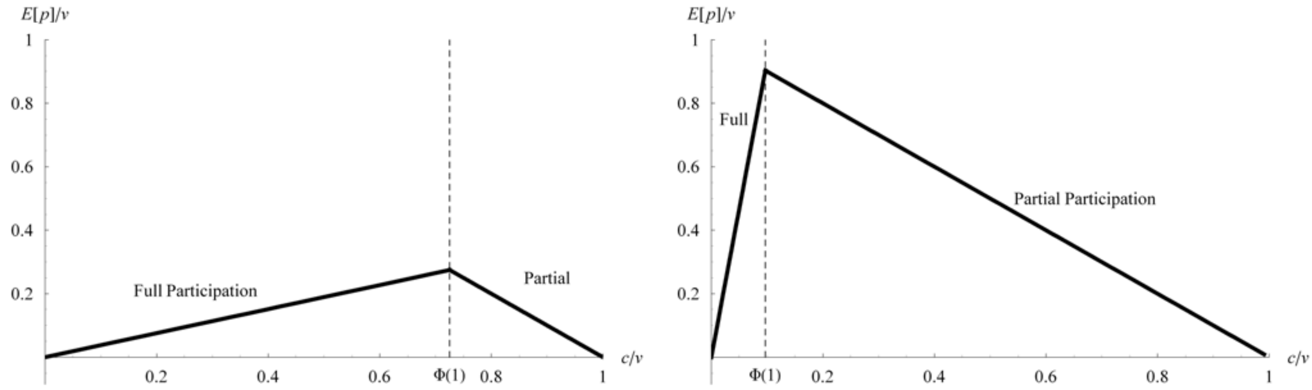
	Partial consumer participation			Full consumer participation		
	θ_1	$E[p]$	W	ρ	$E[p]$	W
$\downarrow c$	$\uparrow$	$\uparrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$
$\uparrow \mu$	$\uparrow$	$-$	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$
$\uparrow v$	$\uparrow$	$\uparrow$	$\uparrow$	$-$	$-$	$\uparrow$
$\uparrow N$	$\downarrow$	$-$	$\downarrow$	$\uparrow$	$\uparrow$	$-$

Table 1: Summary of comparative statics results

a. The effects of a reduction in search cost c

The first result we want to emphasize is that, under partial consumer participation, a reduction in search cost leads to an increase in expected price. This follows immediately from the equilibrium condition $v - E[p] - c = 0$. The intuition behind this result is simple. As Figure 1 shows, the intensity with which non-shoppers search in this type of equilibrium rises as c falls. Note further that these consumers are precisely those who do not exercise price comparisons, and thus they are prepared to accept higher prices. Consequently, a fall in c increases sellers' incentives to charge higher prices. Indeed, inspection of equation (12) reveals that the distribution of prices with a high θ_1 (lower c) dominates in a first-order stochastic sense the price distribution with a low θ_1 (higher c).

As c decreases further, non-shoppers eventually start participating fully. In such a case, a decline in c results in a fall in the reservation price ρ (equation (8)). Inspection of the equilibrium price distribution in Proposition 1 reveals that F increases as ρ decreases. As a result, expected price decreases in c under full consumer participation.



(a) $\mu = 0.8$ (b) $\mu = 0.1$

Figure 2: The influence of lower search cost c on expected price ($N = 2$)

These observations are illustrated in Figure 2. In Figure 2(a) we have simulated an economy where the number of informed consumers is large ($\mu = 0.8$). This graph shows that expected price is non-monotonic in relative search cost c/v. When there are few shoppers in the market ($\mu = 0.1$) expected price decreases in search cost for almost the entire parameter region (cf., Figure 2(b)). This is because for this parameter constellation the economy is most likely in an equilibrium with partial consumer participation.

Welfare is given by $W = \mu v + \theta_1(1-\mu)(v-c)$ in this market, with $\theta_1 = 1$ in the full consumer participation case. A decrease in search cost c increases the surplus of the non-shoppers as well as their participation rate; as a result, welfare increases as search cost falls. Proposition 3 summarizes these findings. For this purpose, let $\varepsilon > 0$ be a small enough number so that if non-shoppers participate fully when the search cost is c, they also do it when the search cost increases to $c + \varepsilon$.

Proposition 3 *If non-shoppers participate partially (fully), the equilibrium distribution of prices with search cost c dominates (is dominated by) the price distribution with a higher search cost $c+\varepsilon$ in a first-order stochastic sense. As a result, expected price is non-monotonic in search cost. Moreover, welfare increases as search cost decreases.*

b. The effects of an increase in the fraction of shoppers μ

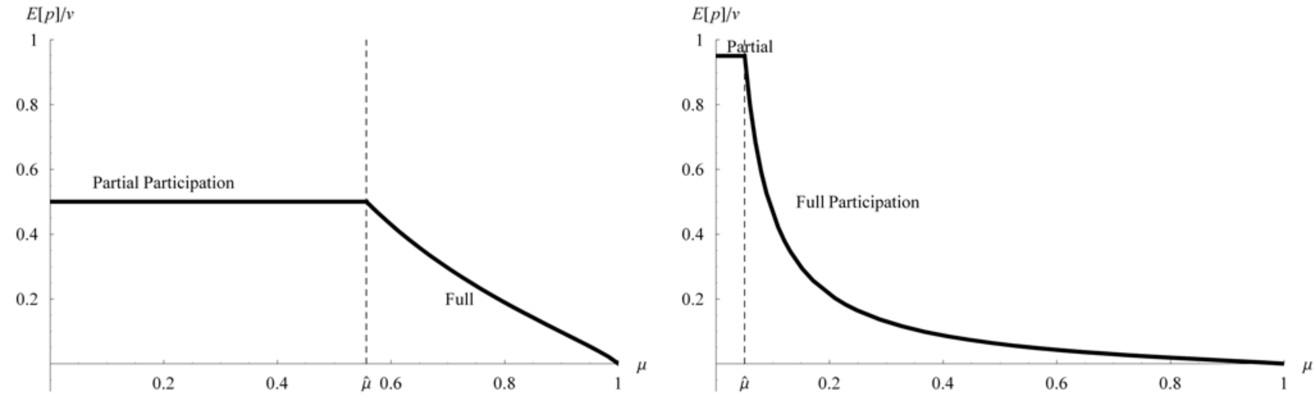
We next consider the effects of an increase in μ. Under partial consumer participation, a change in the number of shoppers does not influence expected price-to-cost margins as nothing changes in the equilibrium condition $v - E[p] - c = 0$. To understand the economic forces underlying this result, we first note that an increase in μ has in principle a pro-competitive effect. Keeping the search intensity of non-shoppers constant, firms would tend to charge lower prices as the number of shoppers in the market becomes higher. However, a change in μ also affects θ_1. To see how, one can apply the implicit function theorem to equation (14) to obtain

$$\frac{d\theta_1}{d\mu} = -\frac{\Phi'_\mu}{\Phi'_{\theta_1}} = \frac{\theta_1}{\mu(1-\mu)} > 0, \tag{15}$$

which means that an increase in μ results in an increase in the search intensity of the non-shoppers. This is because more informed consumers in the market makes searching more attractive for the non-shoppers, as the former buyers put pressure on firms to cut prices. A higher participation rate of non-shoppers in turn gives firms incentives to increase prices, since non-shoppers do not compare prices. Interestingly, these two opposite forces offset each other so that expected price remains constant. Indeed, it is easy to see that the entire distribution of prices (12) does not change in μ. The reason is that, using (15), the ratio of non-shoppers to shoppers $\theta_1(1-\mu)/\mu$ is constant in μ.

If non-shoppers search for one price for sure, the pro-competitive effects of an increase in μ mentioned above are strengthened by the fact that the reservation price ρ decreases in μ and thus expected prices fall (cf., Stahl (1989)). These remarks are illustrated in Figure 3. Figure

3(a) simulates an economy where product's valuation is relatively low compared to search cost ($c/v = 0.5$). The figure depicts expected price-to-cost margins as a function of μ. As $\Phi(1;\mu;N)$ is decreasing in μ, it easily follows that for a given N and c/v, there is a unique $\widehat{\mu}$ such that (10) holds with equality. Therefore, the market equilibrium exhibits partial consumer participation when μ lies in the interval $(0, \widehat{\mu})$ while full participation arises when μ lies in the interval $[\widehat{\mu}, 1)$. Starting from full consumer participation, expected price increases as μ falls. As μ decreases further, the economy eventually moves into an equilibrium with only partial consumer participation. Even if $\mu \to 0$, expected price remains below v, so a Diamond type of result does not arise in a setting with truly costly search. In Figure 3(b) we have simulated an economy where search cost is relatively low ($c/v = 0.05$). The only difference is that the region of parameters for which consumers search once surely is much larger than before.



(a) $c/v = 0.5$

(b) $c/v = 0.05$

Figure 3: The influence of μ on expected price ($N = 2$).

Welfare increases in μ in the full consumer participation case simply because consumers who incur search cost are replaced by zero search cost consumers. If non-shoppers participate only partially, an increase in μ raises the participation rate of non-shoppers, which implies that welfare also goes up in this case. These findings are summarized in the following result. As above, let $\varepsilon > 0$ be a small enough number so that if non-shoppers participate partially when the fraction of shoppers is μ they also do it when the fraction of shoppers increases to $\mu + \varepsilon$.

Proposition 4 *If non-shoppers participate partially, an increase in the proportion of shoppers from μ to $\mu + \varepsilon$ leaves the equilibrium price distribution unchanged; by contrast, if non-shoppers participate fully, the price distribution with a higher proportion of shoppers $\mu + \varepsilon$ is dominated by* *the distribution with a fraction of shoppers μ in a first-order stochastic sense. As a result, expected price is weakly decreasing in μ. Moreover, welfare increases in the proportion of non-shoppers for all parameters.*

c. The effects of an increase in the number of firms N

Under partial consumer participation, it immediately follows from the equilibrium condition $v - E[p] - c = 0$ that an increase in N does not affect expected price. The economic forces underlying this result are, however, less straightforward. We have noted before that $\Phi(\theta_1; \mu; N)$ is decreasing in N. This means that without a change in θ_1 expected price would rise: the idea is that as N increases it becomes more and more unlikely that an individual firm sells to the shoppers and, consequently, it concentrates more and more on selling to the non-shoppers. A higher expected price means, however, that a larger fraction of non-shoppers prefers not to participate in the market and this effect exactly offsets the first effect.

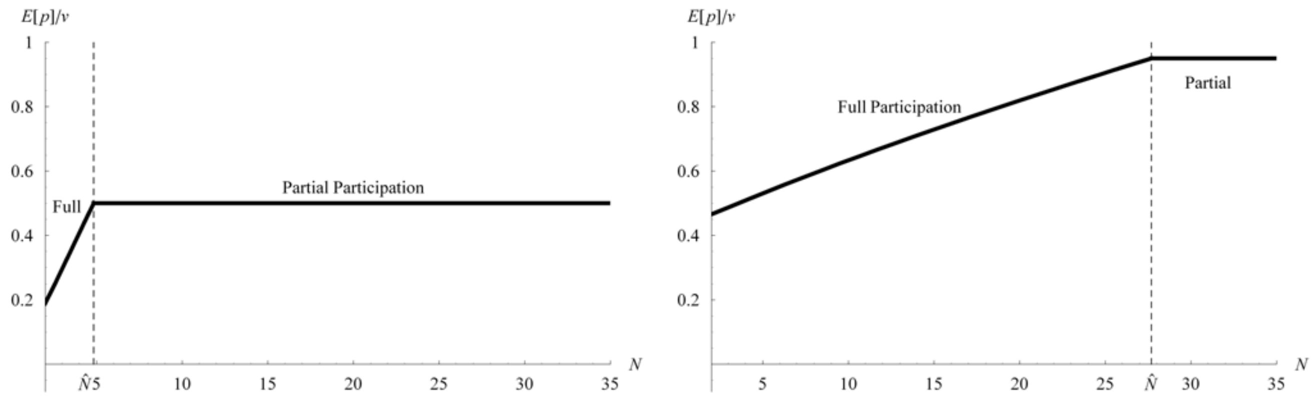
We now notice that, under partial consumer participation the distribution of prices with $N+1$ firms is a mean-preserving spread of the price distribution with N firms. This follows from the following two remarks. First, since the equilibrium θ_1 falls as a result of an increase in N, the lower bound of the price distribution in Proposition 2 decreases; the upper bound does not change. Second, it is easy to see that there exists a unique value of p, denoted by $\widetilde{p}$, such that $F(\widetilde{p}; N, \theta_1^*(N)) = F(\widetilde{p}; N+1, \theta_1^*(N+1))$, where $\theta_1^*(N)$ and $\theta_1^*(N+1)$ denote the non-shopper participation rates when there are N and $N+1$ firms in the market, respectively. From equation (12), this value is defined by the equality

$$\left(\frac{1-\mu}{\mu}\frac{v-\widetilde{p}}{\widetilde{p}}\right)^{\frac{1}{N(N-1)}} = \frac{\left(\frac{\theta_1^*(N+1)}{N+1}\right)^{\frac{1}{N}}}{\left(\frac{\theta_1^*(N)}{N}\right)^{\frac{1}{N-1}}}. \tag{16}$$

As expected price remains constant, these two observations imply that $F(p; N+1, \theta_1^*(N+1))$ is a mean-preserving spread of $F(p; N, \theta_1^*(N))$.

Under full consumer participation Stahl (1989) shows that expected price rises in N. Intuitively, only the first effect discussed above is relevant here as θ_1 is fixed to be equal to 1. Stahl (1989) also shows that expected price converges to the monopoly price as $N \to \infty$ (Diamond type of result).

It is then interesting to see which type of equilibrium arises for which values of N. We find that for any given c/v and μ, the equilibrium is characterized by partial consumer participation when N is sufficiently large. What happens as N increases is that, if the non-shoppers keep searching once with probability one, expected price tends to the monopoly price and eventually, the condition $v - E[p] - c > 0$ is violated. This is easily seen upon inspection of (10) and noting that $\Phi(1; \mu, N)$ declines monotonically in N and converges to zero as N approaches infinity. This implies that starting from an equilibrium with full consumer participation, our model does not yield the Diamond result in the limit when $N \to \infty$ since at some point the economy turns to a situation of partial consumer participation. Figure 4 below illustrates these comparative statics results.



(a) $c/v = 0.5$ and $\mu = 0.8$

(b) $c/v = 0.05$ and $\mu = 0.1$

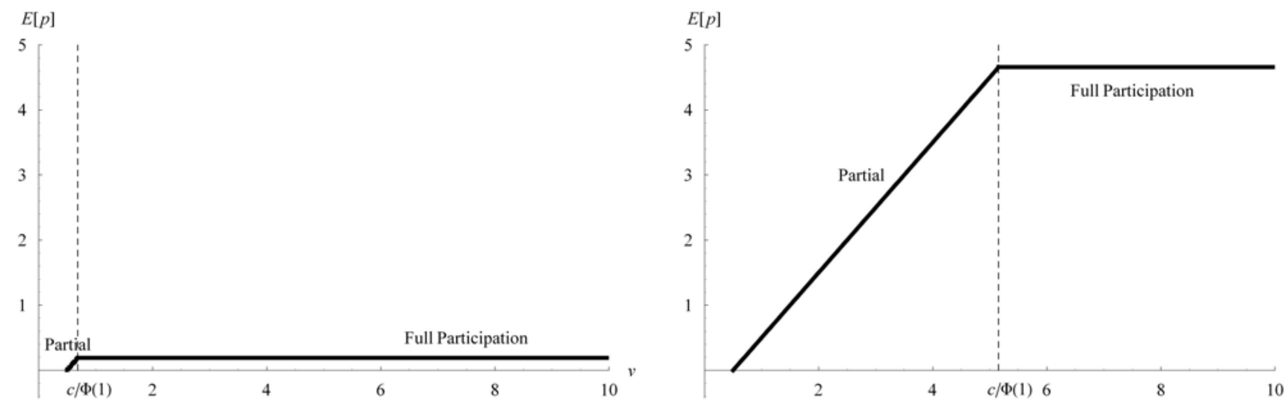
Figure 4: The impact of entry of firms on expected price

The effects of firm entry on welfare are straightforward. If non-shoppers participate fully, firm entry has no bearing on welfare. If non-shoppers participate only partially, an increase in the number of firms reduces the participation rate of these consumers, which reduces welfare. These findings are summarized in the following Proposition.

Proposition 5 *An increase in the number of firms results in a mean-preserving spread of the price distribution if non-shoppers participate partially, while if they participate fully the price distribution shifts downwards; as a result, expected price is weakly increasing in N. Welfare decreases with firm entry under partial consumer participation while it remains constant if non-shoppers search once surely; as a result, welfare is weakly decreasing in N.*

d. The effects of an increase in the value of the purchase v

We next briefly discuss the effects of changes in v. The main difference with the effects of a change in c is that, under full participation, c affects ρ whereas v does not affect ρ. As a result, the only difference with the discussion on the comparative statics effects of changes in search cost c is that now when buyers search for one price for sure, an increase in v does not alter price-to-cost margins.



(a) $c = 0.5$ and $\mu = 0.8$ (b) $c = 0.5$ and $\mu = 0.1$

Figure 5: The impact of v on expected price ($N = 2$)

Figure 5 shows the influence of an increase in v on expected prices. In Figure 5(a) the number of informed consumers is relatively high ($c = 0.5$ and $\mu = 0.8$). When v lies in the interval $(c, c/\Phi(1))$, non-shoppers participate in the market with probability less than one. In this parameter area, expected price rises as v increases. When v is above $c/\Phi(1)$ non-shoppers search once surely. For this region of parameters expected price is unaffected by a change in v. Figure 5(b) shows the case of an economy with relatively few shoppers.

5 Conclusion

In this paper we have taken the seminal model of Stahl (1989) on sequential consumer search and oligopolistic pricing and studied the implications of relaxing the assumption that consumers obtain the first price quotation for free. When also the first price quotation is costly, the Nash equilibrium need not entail full consumer participation. Partial consumer participation arises when search cost is above a certain threshold value which depends on the other parameters. This threshold value becomes arbitrarily low as the number of firms becomes large enough and/or the number of shoppers sufficiently small. Therefore, especially in markets with many firms and/or with few shoppers, this situation of partial consumer participation should be seriously considered.

This new type of equilibrium exhibits interesting comparative statics properties. In particular, expected price increases as search cost decreases, and is constant in the number of shoppers and in the number of firms; welfare decreases as firms enter the market. Finally, the paper shows that, starting from an equilibrium with full consumer participation, monopolistic pricing never obtains when the number of shoppers goes to zero and/or the number of firms goes to infinity because with truly costly search the economy eventually turns into an equilibrium with partial consumer participation.

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