# Pricing and Index Considerations in Commercial Real Estate Derivatives

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Until very recently, real estate was the largest class of tradable assets for which no major derivatives markets existed. This began to change around 2005, when swaps based on the Investment Property Databank (IPD) Index began over-the-counter trading in substantial volume in the U.K., growing to over £3 billion of notional value traded in the first quarter of 2007. This is more than one-third of the current total rate of quarterly trading volume in the cash market for IPD-tracked properties. An equivalent number in the U.S., based on properties tracked by Real Capital Analytics Inc. (RCA), would be over $120 billion of derivatives trading volume per year.[1] However, real estate derivatives trading in the U.S. has been slow to take off. While the NCREIF Property Index (NPI) has been available for trading since 2005, few deals had been done by early 2007 when the license was opened to multiple investment banks.[2]

One reason for the slow start may be that the U.S. has no single commercial property index that captures as clearly and completely the relevant commercial property markets with as long a track record as the IPD Index in Britain. Another problem may be a knowledge gap in the U.S. between real estate investors who lack understanding of derivatives and derivatives traders who lack understanding of real estate, so that neither group has an understanding of the subtleties of commercial property price indexes in the U.S. These problems cause potential derivatives trading partners to lack confidence about proper pricing of the derivatives which poses a barrier to getting the market started. A 2006 survey, conducted at the MIT Center for Real Estate, of 37 U.S. real estate investment managers and other likely participants in a derivatives market identified a lack of confidence in how

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the derivatives should be priced as one of the two most important perceived barriers to the use of the derivatives, with 75% of respondents indicating this as either an "important" or "very important" concern.[3]

Such concerns are understandable because commercial property price derivatives differ from the major traditional derivatives products, such as commodities and financial or foreign exchange futures, in that the underlying reference asset (the real estate index) cannot be traded in a cash, or spot, market. This renders the traditional futures–spot arbitrage impossible to execute, undercutting the classic formula for the fair price of the derivative, and raising the need to consider in depth the nature of the dynamics of the underlying index.

This article aims to address this knowledge gap. First, we will discuss key aspects of the major types of real estate indexes that underlie the derivatives. Then, we will present the fundamental equilibrium pricing rules for the derivatives in view of the index characteristics. Finally, we will discuss the implications of these pricing rules, in particular, the dangers and opportunities for the nascent derivatives market in the U.S., especially as they regard the need for information and knowledge about the indexes and the property markets.

## COMMERCIAL PROPERTY PRICE INDEXES

A commercial property price index supporting derivatives aims to track the percentage change in commercial property market prices in each consecutive, relatively short period of time (e.g., monthly, quarterly, or annually). Labeling this periodic capital return in period $t$ as $g_{Pt}$ (continuously compounded), and the natural log of the market price as of the end of period $t$ as $P_t$, the index strives to represent the relationship the following equation:

$$g_{Pt} = P_t - P_{t-1} \tag{1}$$

### Noise and Lag in the Indexes

In the above context, the real estate index can suffer from two major types of issues—noise and lag. Noise refers to random deviation between the index value level and the actual market price, and lag refers to a systematic tendency of the index to only partially reflect the true current return in any given period.

Labeling the log of the index level at the end of period $t$ as $S_t$, and the index-computed return as $g_{St}$, we have the following equation:

$$\begin{aligned} g_{St} = S_t - S_{t-1} &= (P_t + \varepsilon_t) - (P_{t-1} + \varepsilon_{t-1}) \\ &= (P_t - P_{t-1}) + (\varepsilon_t - \varepsilon_{t-1}) = g_{Pt} + \eta_t \end{aligned} \tag{2}$$

where $\varepsilon_t$ is a zero-mean purely random fraction and $\eta_t$ is the noise in the index return, consisting of the first-differences of the random deviations in levels, $\eta_t = \varepsilon_t - \varepsilon_{t-1}$. The pure effect of noise is to add short-run volatility to the index returns and to cause index returns to have negative autocorrelation, in both cases, relative to the actual property market, expressed as

$$COV[\eta_t, \eta_{t-1}] = COV[\varepsilon_t - \varepsilon_{t-1}, \varepsilon_{t-1} - \varepsilon_{t-2}] = -VAR[ \tag{3}$$

where $COV[]$ and $VAR[]$ are the covariance and variance, respectively. Clearly, noise adds basis risk in the use of the index, which can reduce the value of the derivative for purposes of targeted investment, speculation, or hedging.[4]

In terms of index lag, the index value level might be a weighted average of current and past actual market prices, such as[5]

$$\begin{aligned} g_{St} = S_t - S_{t-1} &= (\omega P_t + (1-\omega)P_{t-1}) - (\omega P_{t-1} + (1-\omega)P_{t-2}) \\ &= \omega g_{Pt} + (1-\omega) g_{Pt-1} \end{aligned} \tag{4}$$

where $\omega$ is a fraction greater than zero and less than one. Lag gives the index inertia and predictability, causing the expected future returns in the index to differ from equilibrium property market return expectations as reflected in the true current market values $P_t$ and their returns, $g_{Pt}$. Lag can also cause the index to have different risk characteristics than the average property tracked by the index, as it can result in a smoothing of market volatility and a dampening of apparent correlation with other financial assets (e.g., biasing systematic risk toward zero). Since the index cannot be directly traded, rendering the classic futures–spot arbitrage impossible to execute, these differences between the index and the underlying property market it tracks must be accounted for in the equilibrium price of the derivative. Just as the signature of noise in an index is excess short-run volatility and negative serial correlation, so the signature of lag is dampened volatility and positive serial correlation. The opposite nature of the signatures of noise and lag can cause the two problems to mask one another in an index that contains both, making it at least superficially difficult to detect either problem, even though both problems are, in fact, present.

### Appraisal and Transactions Indexes

There are two major types of commercial property price indexes in the U.S.—appraisal-based indexes and transaction-based indexes. In these two genres, noise tends to be more of a potential problem in transaction-based indexes, and lag tends to be more of a problem in appraisal-based indexes.[6]

In a traditional appraisal-based index all of the properties in the index population are appraised regularly, and the index periodic returns are based on a simple aggregation of those appraised values each period. In the case of the NPI, this has the advantage of similarity to the way many institutional real estate investment funds in the U.S. mark to market their asset values and correspondingly report quarterly returns to their investors.[7] Many U.S. fund managers are benchmarked wholly or partly on the NPI. As a result, the NPI has a particular use in derivatives of interest to the managers of such funds.

Against these advantages, it must be recognized that the dampening of actual property market volatility caused by appraisal-based lag in the NPI may reduce a source of potential profit that might motivate some derivatives traders. In addition to the tendency of the underlying property appraisals to lag market prices, in the case of the NPI in the U.S., not all properties are seriously or independently reappraised every period that the index is reported.[8] This causes a stale-appraisal effect that adds additional lag into the index. Traditionally, in the NPI, there has been a greater frequency of reappraisals in the fourth calendar quarter, and this has imparted an artificial seasonality to the index in that it tends to spike in the fourth quarter. Another consideration is that the NPI represents a relatively narrow segment of the population of U.S. properties. As of 2006, the NCREIF population of properties consisted of less than 10% of the value of investable commercial properties in the U.S., a much smaller percentage than the IPD Index represents in the U.K.[9] For smaller market segments there may be only a few NCREIF properties available in the index, and their specific identities generally cannot be concealed, yet the underlying property appraisal process for each property is a fundamentally subjective evaluation made by a single appraiser or appraisal firm.

While none of these index considerations need to prevent a well-functioning derivatives market, all are considerations that may, and in fact should, affect the proper pricing of derivatives based on appraisal-based indexes and, in particular, on the NPI. Therefore, many (or at least some) derivatives traders in the U.S. may prefer indexes that track a broader population of properties and which tend to lead the appraisal-based indexes in time, without the lagging and smoothing characteristics of the NPI. This makes the other major type of property price index, the transaction-based index, of particular interest for purposes of supporting derivatives trading in the U.S.

In principle, transaction-based indexes can be based on the entire population of commercial investment properties because all such properties potentially transact, providing a random price sample of the population each period. In contrast, only certain specialized portfolios of properties in the U.S. are regularly marked to market using appraisals. Transaction-based indexes can be good bases for derivatives provided the indexes are carefully constructed using sufficient quantity and quality of transactions observation data and state-of-the-art statistical procedures to control for apples-versus-oranges differences in properties trading in different periods and to minimize noise in the index returns.

In the academic literature two major approaches have been developed to calculate transaction-based indexes in a statistically rigorous manner—the repeat-sales regression procedure and the hedonic value model.[10] Both procedures address the fundamental problem in the construction of a transaction-based real estate price index. This problem is that the properties which transact in one period are generally not the same as the properties that transacted in the previous period, making a direct comparison of prices apples versus oranges. The two procedures address this issue in different ways.

The hedonic procedure models property prices as a function of various characteristics of the properties, such as size, age, location, and quality. By regressing property transaction prices onto these hedonic characteristics of the properties that sell, and controlling for or keeping track of the time of the sale, it is possible to construct a constant-quality price-change index, or an index that tracks property market price changes controlling for property differences. In 2006, the MIT Center for Real Estate, in cooperation with NCREIF, began publishing the first regularly produced hedonic index of commercial property based on the prices of the properties sold from the NCREIF database.[11] Because this transaction-based index is based on the same underlying population of properties as the NPI, it can present a good apples-to-apples comparison of the difference between a transaction-based versus an appraisal-based index. Such a comparison over the historical period from 1984 through the first quarter of 2007 is shown in Exhibit 1.

This comparison gives an indication of the typical differences between a transaction-based and an appraisal-based index in the U.S. Note that the transaction-based version of the index is a bit more volatile and tends to slightly lead the NPI in terms of the timing of major turning points in the index history. While excess volatility can be an indication of noise in an index, the transaction-based index in Exhibit 1 does not display the negative autocorrelation that is an indication of noise. Furthermore, the chart in Exhibit 1 indicates that many of the specific short-run movements that show up in the transaction-based index, but not in the appraisal-based index, correspond to actual historical events that might reasonably be expected to have moved the property market in the direction indicated by the transaction-based index.

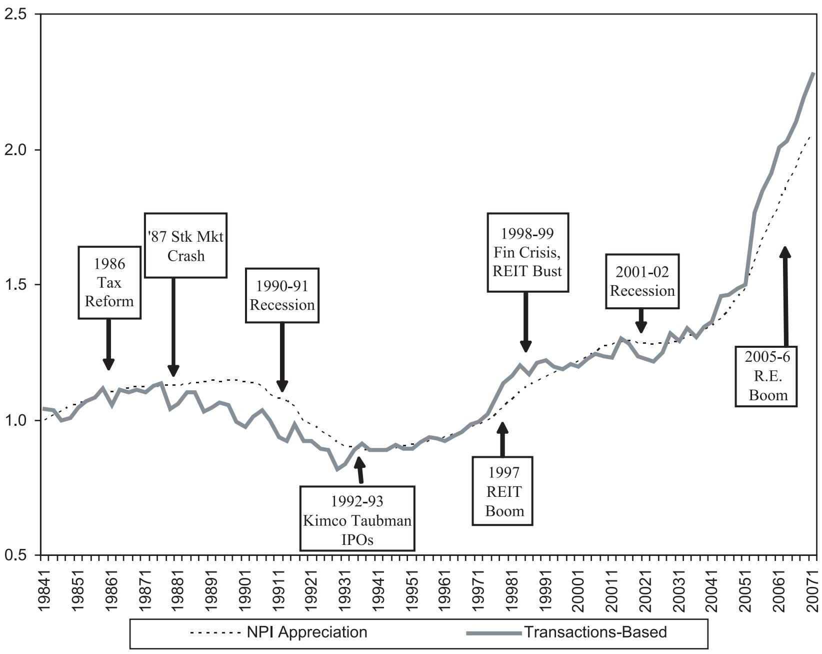
Repeat-sales indexes use a different approach to address the apples-versus-oranges problem. As the name suggests, repeat-sales indexes rely on individual properties selling more than once, so that the change in price between sales provides an indication of how same-property values have changed over time. The index is thus based on the type of price changes that investors in properties actually experience, and the same type of price changes that stock market indexes are based on.[12] It should also be noted that stock share prices reflect the value added by the corporation not paying out all of its cash in dividends, but reinvesting some in the corporation. This is analogous to the effect of capital improvement expenditures in real estate. Thus, repeat-sales indexes aimed at tracking property prices do not generally try to remove the effect of capital improvement expenditures, although data filters are normally applied to eliminate property sale pairs that would reflect major development, redevelopment, or rehabilitation of the properties between the two sale dates. This is in contrast to appraisal-based indexes that may subtract capital expenditures from the appreciation return reported by the index and not remove these expenditures from the income return.

The statistical process used to calculate repeat-sales indexes takes into consideration the time between the same-property sales and appropriately allocates the price change to each period that the index is reported, based on information from other repeat-sales occurring over all of the possible time frames. Repeat-sales regression is the approach used in widely quoted housing price indexes such as the Office of Federal Housing Enterprise Oversight House Price Index and the Standard and Poor's (S&P)/Case-Shiller Home Price Index. In 2006, the Chicago Mercantile Exchange launched futures trading on the S&P/Case-Shiller index.[13]

Derivatives that use both appraisal- and transaction-based indexes are likely to evolve. They may serve different needs for investors. On the one hand, derivatives based on the appraisal-based NPI may be appropriate for investors who are benchmarked against the NPI and want to use its derivative to rebalance their portfolio. On the other hand, investors who want to

## EXHIBIT 1

**NCREIF Index vs Transactions-Based Capital Value Level Index: 1984-2007, Quarterly**




*Source: Fisher, Geltner & Pollakowski (2006), updated.*

hedge a decline in prices or to use a derivative to capture changes in property market prices without any lag should use a derivative supported by a transaction-based index. In either case, investors must understand how each type of derivative should be priced and how well it will perform as a hedge. This is the subject of the rest of this article.

## PRICING REAL ESTATE INDEX-BASED DERIVATIVES

The pricing of derivatives such as forwards and swaps has been well understood for a long time. These derivatives are based on financial indexes such as the S&P 500 stock index. It is easy to understand the pricing of these derivatives because the index underlying the derivative can itself be traded by trading the component stocks. This enables, at least in principle, the construction and execution of arbitrage trading between the derivative and the underlying product which is traded in the so-called cash, or spot, market. This, in turn, provides the classic finance textbook formula for the fair price of futures contracts relative to their underlying assets, known as the futures-spot parity theorem.

In contrast, the real estate indexes which underlie the new real estate equity derivatives cannot themselves be directly traded. In each period, an investor cannot possibly buy and sell all of the properties that compose an appraisal-based index such as the NPI. Even if it were possible, the transactions would not be at prices equal to the appraised values of the properties. Nor can an investor buy and sell all of the properties that were sold in each period that provided the basis for the construction of a transaction-based index. The classic arbitrage that underlies traditional futures pricing cannot be executed with real estate derivatives. Furthermore, since the index itself cannot be directly traded in a well-functioning spot market, there is no guarantee that the real estate index will always represent equilibrium values or equilibrium return expectations going forward from any given point in time. For example, if appraisals tend to lag market values, then an investor will not typically be able to buy and sell properties at their appraised values because those values would differ from the prices that provide investors with equilibrium return expectations. Thus, the expected return in the index does not at any given time necessarily equal the equilibrium expected return in the property market tracked by the index.

Nevertheless, we can derive the fair price of a real estate index derivative in relation to the current underlying index value, where fair is defined as the equilibrium price in the derivatives market. Arbitrage can be viewed as a particularly powerful means of enforcing equilibrium pricing in a market. But just because arbitrage cannot be executed does not mean that the derivatives market cannot function and find equilibrium, or that the theoretical equilibrium price cannot be computed. Most fundamentally, the equilibrium price is that which allocates to each party in the trade an expected return exactly commensurate with the risk that party bears, as the market evaluates such risk. This would seem to be a good basis for defining a fair price even if it cannot be enforced by arbitrage trading, and would be the price that would prevail if arbitrage could be executed.

In this context, arbitrage can be viewed as a means of enforcing equilibrium so that the classic price implied by the futures-spot parity theorem will still apply. It represents an equilibrium price for the derivative in relation to the underlying index, *whenever the underlying index provides expected equilibrium returns in the current real estate asset market tracked by the index*. This will arguably be pretty close to the case for well-constructed contemporaneous transaction-based indexes. It will generally not be the case for typical appraisal-based indexes, at least in countries like the U.S. where appraisals tend to lag market prices and the appraisal-based index contains stale appraisals.[14] But even when the underlying index does not represent the current equilibrium in the property market, the equilibrium analysis can still be used to estimate the price of the appraisal-based-index derivative, including consideration of the lagging and smoothing effects in the index.

### Pricing the Forward Contract

To understand the pricing of real estate derivatives, let us begin with a simple forward contract and the classic arbitrage price derivation. By definition, the forward contract pays the index value at maturity, in return for the payment of the previously agreed upon forward price, also to be paid at that same maturity date (time $t = T$). Note that the price is agreed upon and fixed at the time when the contract is established (time $t = 0$) even though no cash changes hands at that time. A simple arbitrage analysis as described in any introductory investments textbook provides the pricing formula for a forward contract on a total return index—one that includes accumulated dividends or income reinvested—and reflects the futures-spot parity theorem as in the expression

$$F_T = S_0(1+i)^T \tag{5}$$

where

$F_T$ = forward price agreed upon at time 0 to be paid at time $t = T$

$S_0$ = current (spot) price (value) of underlying index (at time $t = 0$)

$i$ = $T$-period interest rate (riskless, with trader bonded via margin or collateral; $i$ could be LIBOR)

One method of building an intuition about this equation is to think of the forward contract in the following way. Purchasing the forward enables an investor to receive at time $T$ the value of the index as of time $T$, or the value $S_T$. If the index includes accumulated reinvested dividends (income), then the present (time 0) market value of the future index value is simply the current value of the index, $S_0$. Normally (that is, in the spot market) the investor would have to pay the full amount $S_0$ in cash at time 0 in order to obtain the claim to the $S_T$ future value in $T$ years (which includes dividends reinvested). However, the forward contract does not require any cash up front. In terms of opportunity value (based on the cash that can be kept and invested for $T$ years), the forward contract enables an investor to earn interest at the rate of $i$ for $T$ years on the $S_0$ amount that has been "saved" for that period of time. This will produce a future cash value in $T$ years of the amount, $S_0(1+i)^T$, available at year $T$ on a riskless basis (that is, with the risk in $i$), because the forward contract allows the investor to keep her cash until time $T$. The commitment to pay $S_0(1+i)^T$ in $T$ years is equivalent to—and can be exchanged in the market for—an up-front payment of $S_0$ at time 0, which equals the claim to $S_T$ in $T$ years and is exactly the same claim provided by the forward contract. Thus, it is fair for the forward price, $F_T$, which is agreed to at time 0, but not paid until time $T$, to equal the amount $S_0(1+i)^T$. The investor should be willing to pay this amount for such a forward contract, and, in equilibrium, will have to pay that price.

Note that the arbitrage enables Equation (5) to be independent of the expected future index value, $E_0[S_T]$. Fundamentally, this holds because the market's expectation of the future value of the index is already captured in the current value of the index, $S_0$. But this is only true if the index reflects the current equilibrium expected returns going forward in the underlying property market tracked by the index. Assuming such equilibrium pricing, we have the following equation:

$$S_0 = E_0[S_T]/(1+E_0[r_S])^T, \tag{6}$$

where

$E_0[S_T]$ = the expectation as of time 0 of the future index price (value) at time $T$

$E_0[r_S]$ = the market equilibrium-required expected return for the risky index

$E_0[r_S]$ = $i + RP_S$ , where $RPS$ is the market equilibrium-required ex ante risk premium in the index total return going forward (over the riskless rate $i$).

Plugging Equation (6) into Equation (5), we obtain the relationship between the forward price and the expected future value of the index at the time of the forward contract maturity and payoff as expressed by

$$F_T = E_0[S_T]((1+i)/(1+E_0[r_S]))^T \tag{7a}$$

In effect, the forward price equals the time 0 expectation of the time $T$ value of the index discounted at the rate of the risk premium in the index expected total return. A more precise definition of that risk premium is given by

$$1+RP_S = (1+E_0[r_S])/(1+i) \rightarrow RP_S \approx E_0[r_S]-i \tag{7b}$$

Thus, we can rewrite Equation (7a) as

$$F_T = E_0[S_T]/(1+RP_S)^T \tag{7c}$$

The intuition behind the pricing formula above is that the forward price agreed to at time 0 does not have to include the time-value-of-money component (the risk-free interest rate $i$ ) of the equilibrium-required expected total return that would normally be used to discount the future expectation to the present. Neither time nor money is being put up by the investor because the forward price will not be paid until time $T$ when the index value at that time will be received.

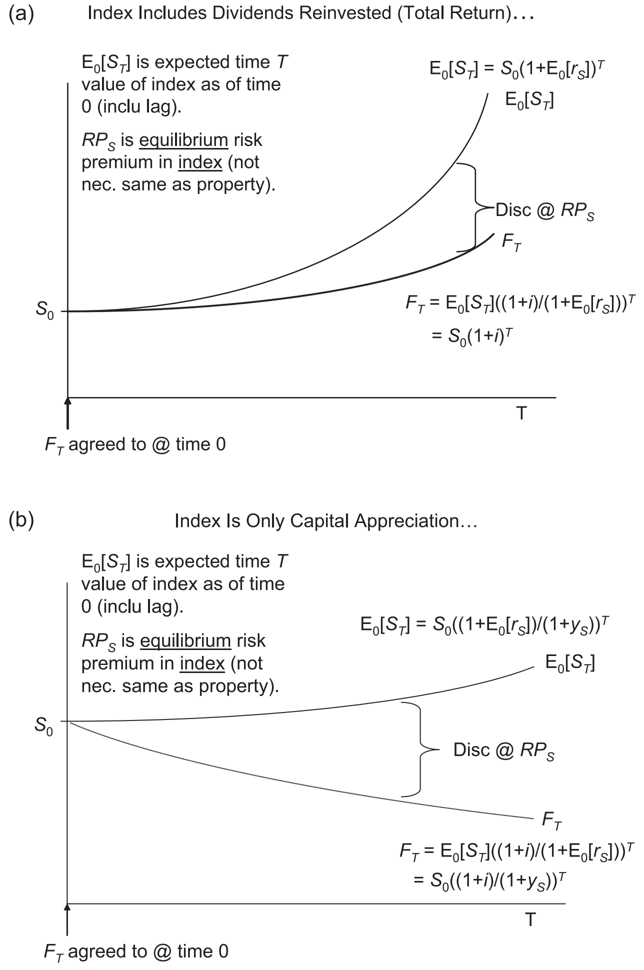
Although we derived Equations (7a) and (7c) from Equation (5), the same result can be obtained by viewing the equations as simply finding the price that gives the investor an equilibrium or fair rate of return on the forward contract. The equilibrium price of any investment is the price that gives the investor an expected rate of return commensurate with the risk of the investment. In this case, the price of the forward contract is the price that gives the investor an expected return that includes a risk premium that reflects the risk of the index. Thus, Equations (7a) and (7c) apply regardless of whether arbitrage is possible, whereas arbitrage enforces Equation (5) to apply as well. That is, in order to get an equilibrium or fair rate of return, the price of a forward contract will be the expected future index value discounted at the risk premium. Arbitrage further enforces that the price of the forward contract will be related to the current index value as expressed in Equation 5.[15] The equilibrium basis of Equations (7a) and (7c), in the absence of arbitrage, will be presented more formally in the section on swap pricing.[16]

Note that it is not necessary to invest in the assets underlying the index to be able to price a derivative based on the index. What is important are the risk and return characteristics of the index that the derivative is based on because the derivative is the investment that is being made.

*Capital Return Index*. If the underlying index is only the capital return component on assets that pay dividends (income), then in the Equations (5), (7a), and (7c), $(1+i)^T$ is replaced with $((1+i)/(1+y_S))^T$, and $(1+\mathrm{E}_0[r_S])^T$ is replaced with $((1+\mathrm{E}_0[r_S])/(1+y_S))^T$, where $y_S$ is the dividend-yield rate or income return (also known as the cash-flow payout rate) of the index assets, and $\mathrm{E}_0[r_S]$ is still the expected total return on index. This makes sense because the forward contract does not pay dividends, while the present value of the index, $S_0$, reflects the present value of the dividends that are expected to be paid by the index, or the assets tracked by the index. The capital return index grows at the rate $g_{St}$ instead of at the rate $r_{St}$. The result is to leave Equations (7a) and (7c) the same but with the recognition that $S_T$ would now reflect only the accumulation of capital growth without any reinvested dividends. These pricing formulae and relationships for the forward contract are depicted in Exhibits 2a and 2b.

Appraisal Based Indexes. It is important to reiterate that the pricing relationships described in Exhibit 2 and the classic Equation (5) apply only when the underlying index presents an equilibrium expected return. This assumption is implicit in the arbitrage derivation of Equation (5). If the current index value does not reflect the current equilibrium in the property market, given that the arbitrage cannot actually be executed on the real estate index, Equation (5) does not generally apply.

## EXHIBIT 2
**Relationship Between Prices**




Equation (5), therefore, will not generally reflect the equilibrium price for the forward contract when the underlying real estate index is appraisal based. Even though Equation (5) does not then apply, Equations (7a) and (7c) will still hold as a representation of the equilibrium price for the forward contract even when the index is appraisal based provided that the

- Actual (possibly disequilibrium) expected future value of the index is used in the formula, with $\mathrm{E}_0[S_T]$ reflecting the lag effect in the index (i.e., including any momentum in the index), and the
- Market equilibrium risk premium for the index, $RP_S$, is used in Equation (7c) and $\mathrm{E}_0[r_S]$ in Equation (7a), where $RP_S$ reflects the amount of risk in the index, including the possibility that $RP_S$ might be

less than the risk premium in the property market if smoothing in the real estate index reduces the risk of that index below the risk of the average property tracked by the index.

By using the expected future value of the index in Equations (7a) and (7c) and by using a risk premium that reflects the risk of the index, investors will expect to earn an equilibrium rate of return on the forward contract.

### The Meaning of Price Changes in the Forward Market

With this understanding, we can now consider the meaning of changes in forward market prices. We can examine either the change over time in the price of a given contract with the same, fixed maturity date (holding the payoff date constant), or we can examine how the prices of new forward contracts compare to previous prices of contracts of the same duration (holding duration constant). Both types of price changes are relevant for marking to market forward contracts that have been executed and are outstanding.[17]

For a given derivatives contract, even if all else remains constant, the forward price of the contract will change as we move along the $F_T$ curve (the lower of the two curves) depicted in either panel of Exhibit 2. The movement will be leftward along the $F_T$ curve, toward the current value of the index, $S_0$, as time $T$, the maturity of the contract, grows shorter. For example, if the current index value $S_0$ is held constant, the price of a total return forward contract maturing at a given date will fall over time at the rate of $i$. The price of a similar capital return forward will evolve at rate $(1+\gamma_S)/(1+i)$-1, which may be either positive or negative depending on the relationship between the property payout rate $\gamma_S$ (assumed to be continuous) and the risk-free interest rate $i$. The result will be a forward price maturity curve, similar to a yield curve in the bond market, in which contract prices, as a function of maturity, are traced out along the $F_T$ curves in Exhibit 2.

Now consider the meaning of changes over time in the prices of new forward contracts having the same duration, that is, the price evolution of the derivatives market itself. For example, compare the price of a new 1-year contract today with the price a new 1-year contract launched one month earlier. Suppose interest rates have remained constant at $i$, any change in the prices of the new contracts would reflect either a change in the forward market's expected future value of the index in $T$ periods ($E_0[S_T]$) or a change in the capital market's required risk premium for investments with risk like the index ($RP_S$), or some combination of these two. Suppose that the risk premium has remained constant, so the change in $F_T$ is due solely to a change in $E_0[S_T]$. If the index represents equilibrium expectations, the present value of the index, $S_0$, must also have changed commensurately with the change in $F_T$. In other words, since $F_T = S_0(1+i)^T$ and both $i$ and $T$ are constant, a percentage change in $F_T$ is just a reflection of a change in $S_0$, as the new time 0 evolves forward in time. The same could be said about a change in the market's required risk premium, $RP_S$. Any change in that risk premium, if the index is in equilibrium, will be reflected in a commensurate change in the present value of the index, thereby preserving the $F_T = S_0(1+i)^T$ relationship, even as $F_T$ also equals $E_0[S_T]/(1+RP_S)^T$ with the new changed $RP_S$. Again, this pricing principle can be used for marking to market which suggests that the contract holder will register income equal to the percentage change in the index times the notional amount of the contract.

Now suppose that the price on $T$-period new contracts changes without a commensurate identical percentage change in the current value of the index and $i$ remains constant. This evidences that the index is not in equilibrium. That is, the forward price $F_T$ is changing to reflect changed market predictions or requirements about either $E_0[S_T]$ or $RP_S$, in order to preserve the equilibrium $F_T = E_0[S_T]/(1+RP_S)^T$ relationship. However, the present value of the index, $S_0$, is not changing correspondingly which negates the equilibrium basis of the $S_0$ price, causing $F_T$ not to equal $S_0(1+i)^T$. This could reflect a lag in the index, with the index only subsequently or partially moving in response to the change in the market's $E_0[S_T]$ expectations or $RP_S$ requirements. In these circumstances, the forward market prices would represent a leading indicator for the index. In effect, price discovery would be occurring in the derivatives market, rather than in the index. For example, bad news might cause an immediate downward revision in $E_0[S_T]$ from, say, 100 to 95, resulting in an immediate and proportionate downward revision in the equilibrium forward price from $F_T = 100/(1+RP_S)^T$ to $F_T = 95/(1+RP_S)^T$ even though the current index level might remain near its previous value of, say, $S_0 = 93$. In order for the derivatives market to perform this price discovery function effectively, the derivatives market requires some degree of liquidity and density, with its prices reliably and publicly reported. As of 2007, the

U.S. market has not accomplished this. The U.K. IPD derivatives market, however, does appear to be performing this function because IPD swap prices have fallen dramatically in the past year, even as the IPD index itself has continued to climb. The IPD swap market is signaling a downturn, or at least a flattening, in the U.K. property market.

### Arbitrage Pricing of the Swap Contract

Swaps are essentially just a series of forward contracts stitched together and agreed upon all at once up front. We will, therefore, use the following exploration of swap pricing to further develop the equilibrium, or fair, basis of the pricing that we introduced above, which is applicable even for appraisal-based (disequilibrium) indexes. This will also allow us to explicate the determinants of a feasible price trading range for derivatives. This type of understanding will be useful for negotiating derivatives prices in the OTC market, and for understanding the basis for feasible derivatives markets in which the bid–ask spread can be sufficient to cover the costs of necessary intermediary operations. We will begin with the classic arbitrage-based pricing formula, and then move to a more general analysis that does not require arbitrage.

Consider a swap of LIBOR for the real estate index total return. No cash changes hands up front, but the long party receives the index current return times the notional amount of the trade for each period during the contract term. The swap contract performance is guaranteed by an intermediary, so that the long party is exposed to the exact risk of the index. The short party receives LIBOR times the notional amount in each period of the contract term. This amount is guaranteed as wells, so that the short party is exposed exactly to the risk of LIBOR. Thus, it seems fair that the long party should receive exactly the index total return—assuming that the expected return of the index compensates for its risk, which it will if the index is priced at equilibrium—and that the short party should receive exactly LIBOR, because LIBOR is an equilibrium rate that compensates for the risk in LIBOR. This pricing result is identical to the implication of the classic futures-spot parity theorem which, as we have seen, applies whenever the index price represents equilibrium. In other words, the classic fair price of the swap of an index total return for LIBOR is, literally, that LIBOR is swapped for the index return, straight up (i.e., no spread against LIBOR). Expressing the price of the swap as the fixed-leg rate that the long party must pay to the short party per dollar of the notional value traded, with net cash settlement each period, and no balloon payment at the end of the contract, we can label this price $F$ so that

$$F = i \qquad (8a)$$

Thus, the fixed leg on the swap should be the riskless rate, or LIBOR. Note that this is effectively the same as the arbitrage-based price result that we obtained previously in Equation (5), except that the swap contract lacks a balloon payment of the index value level at the maturity of the contract, so that we do not need the $S_0$ part of the price. The swap is settled each period, paid currently, so it is not necessary to accumulate and compound the price rate, $i$ . Keep in mind that $F$ is paid per dollar of notional value of the trade.[18] The long party is willing to pay a riskfree rate (LIBOR) in order to receive a risky return on the index as long as the expected return on the index includes the appropriate market risk premium. And the short party is willing to accept the LIBOR rate, as their return has exactly the same risk as LIBOR which is a market equilibrium rate.

If the underlying index tracks the capital return only—ignoring the income return component—then the same arbitrage or equilibrium argument as before implies that the expected income return rate must be deducted from the price rate the long party must pay, as follows:

$$F = i - \mathrm{E}[y_S] \qquad (8b)$$

Fundamentally, this is because the income return component is essentially constant and virtually riskless when compared to the capital return component. In other words, virtually all of the risk in the index total return is contained in the capital return component. Therefore, the long party in the swap is fully exposed to all of the index risk, and thus should expect the entire total return in order to be fairly compensated for that risk exposure.

### Equilibrium Pricing of the Swap

Let us now derive a more general way of expressing the fair, or equilibrium, swap price that will not be dependent on either arbitrage execution or the underlying index being priced at equilibrium. To do this, it is convenient to consider the perspectives of covered traders on both sides of the swap. A covered trader is one who holds assets that back his or her obligations under the swap contract. In particular, we will assume that the party taking the long position in the swap will hold, in an amount equal to the notional value of the swap, bonds paying LIBOR. And we will assume that the party taking the short position in the swap will hold real estate assets similar to those tracked by the index and in a value equal to the notional amount of the swap trade. In this model, even though the swap contract itself takes no up-front cash investment, the two parties in the swap trade are effectively making up-front investments of the notional amount of the trade, in the sense that they are incurring the opportunity cost of holding the covering positions. The ability to trade within and across the bond and property markets ensures that this covered-trader perspective is relevant for understanding equilibrium pricing, even when the actual traders themselves may not be fully covered and the backing assets are not directly involved.[19]

From this perspective, both sides in the swap trade need to get a fair ex ante return expectation as if they were making an actual up-front investment of the notional amount of the trade. We will analyze this situation for a derivative on a real estate index with the following characteristics.

Suppose that the equilibrium expected total return on the average property tracked by the index is

$$\mathrm{E}^{E}[r_P] = i + RP_P \tag{9}$$

where $i$ is LIBOR and $RP_P$ is the property market's required equilibrium risk premium (over LIBOR) which property market investors demand for the average unlevered property investment.

Now consider a real estate index whose value, $S_T$, tracks the investment performance of this same population of properties. Over the very long run, the average expected total return on this index would equal the average expected total return on the average property, which is the equilibrium expectation just described, assuming rational expectations, expressed in the equation

$$\mathrm{E}[r_S] = i + RP_P \tag{10}$$

However, even though the real estate index's total return may, on average, and over the very long run, equal that of the average property that the index tracks, it is not necessarily the case that the real estate index will display the same risk as that of the average property it tracks. Specifically, if the index is appraisal-based, it may lag and smooth the actual movements in the property market to which direct investors in the properties themselves are subjected. This could cause the risk in the index to be lower than the actual market-value risk of direct investment in the average property or even a large portfolio of such properties. This would be reflected in the capital market's equilibrium-required total return expectation of the index being lower than that of the average property in the index.

In other words, the market's equilibrium total-return risk premium in the index, $RP_S$, would be lower than $RP_P$.[20] Thus, we would have:

$$\mathrm{E}^{E}[r_S] = i + RP_S < i + RP_P = \mathrm{E}^{E}[r_P] = \mathrm{E}[r_S] \tag{11}$$

In the above expression, the superscripted $\mathrm{E}^{E}[\ldots]$ indicates an equilibrium expectation, whereas the expectation without the superscript, $\mathrm{E}[\ldots]$, indicates an actual realistic expectation over a given span of time; in this case, a very long span of time.

Furthermore, the same lag bias that could underlie the difference between the index risk and the average property risk could cause a momentum lag effect in the index return, such that the realistic expected return on the index at any given time over a short- to medium-term horizon could differ from the long-run average expected return on the index and on its underlying properties. That is, $\mathrm{E}[r_S]$ would differ from $\mathrm{E}^{E}[r_P]$ (and generally also from $\mathrm{E}^{E}[r_S]$ ) at any given time. Over the long run $\mathrm{E}[r_S]$ would cycle around, or mean revert, toward, $\mathrm{E}^{E}[r_P]$.

At any given time, the short- to medium-term expected total return on the real estate index will differ from its equilibrium-required total return due to both this transient and cyclical lag effect, and to the more permanent risk difference, also fundamentally rooted in the lag bias in the index. The sum of these lag effects, $L$, is defined as

$$L = \mathrm{E}[r_S] - \mathrm{E}^{E}[r_S] = RP_P - RP_S + m, \tag{12}$$

where $m$ represents the transient momentum effect, which is positive when the real estate market has been rising at above-average rates, and negative when the real estate market has been falling.

To analyze the equilibrium price and the feasible trading range for derivatives based on this real estate index, let us consider a swap between the real estate index capital return and LIBOR. And suppose that in a given swap trade the long position is being taken by a Mr. Bull, *B*, matched to a short position being taken by a Ms. Bear, *b*.

### Feasible Trading Condition for the Long Position

Consider first the perspective of Mr. Bull's long position. We will label the return Mr. Bull achieves on his overall position (including the swap and the cover) as $r^B$. The long position is fully exposed to the risk of the real estate index, but only to that risk because his covering LIBOR investment will offset his fixed payment on the swap. Thus, he faces an expected return given by the following condition:

$$E^B[r^B] \geq E^E[r_S] = i + RP_S \tag{13a}$$

That is, Mr. Bull's expected total return, based on his private expectations as indicated by $E^B[]$, must at least equal the market's required expected return on investments with risk equal to the risk in the index which, of course, is also the equilibrium expected return on the index.

While the condition specified in Equation (13a) is Mr. Bull's required expectation, what he will actually receive is LIBOR on his covering bond investment plus the index capital return less the fixed-leg price of the swap that he must pay to Ms. Bear, resulting in an actual expected return of

$$E^B[r^B] = i + E^B[g_S] - F \tag{13b}$$

Mr. Bull's expectation about the growth rate of the index, $E^B[g_S]$, is based on the actual expected total return on the index during the term of the swap contract, $E[r_S]$, reduced by the income return component, including any momentum effect in the index as well as Mr. Bull's own private expectations about the direction of the underlying real estate market during the term of the swap contract. Let us label Mr. Bull's private expectations in this regard as $B^L$, and to reflect his relatively bullish outlook, we will assume that $B^L$ is positive. Thus, employing relationship expressed in Equation (12), we can expand Mr. Bull's private growth expectations for the index to

$$E^B[g_S] = E[r_S] - E[y_S] + B^L = E^E[r_S] + L + B^L - E[y_S] \tag{13c}$$

Combining (13b) and (13c), we obtain an expanded version of Mr. Bull's expectations such that

$$E^B[r^B] = i + E^E[r_S] + L + B^L - E[y_S] - F \tag{13d}$$

And now finally combining Equation (13d) with Mr. Bull's trading feasibility condition expressed in Equation (13a), we obtain Mr. Bull's feasibility or trading criterion for the price of the swap, as follows:

$$\begin{aligned} E^B[r^B] = i + E^E[r_S] + L + B^L - E[y_S] - F \geq E^E[r_S] \\ \rightarrow F \leq i + L + B^L - E[y_S] \end{aligned} \tag{13e}$$

That is, Mr. Bull will be willing to pay any price rate (fixed leg) less than or equal to LIBOR plus the index lag effect plus his own private bullish expectation minus the expected income return on the index. The expected income return is subtracted because the swap is a capital-return swap not a total-return swap.

Notice that if the index were always in equilibrium such that there was no lag or risk differential effect ($L = 0$), and if Mr. Bull had neutral rather than bullish expectations about the property market ($B^L = 0$), then this feasibility or trading condition would just exactly equal the $F = i - E[y_S]$ fair-price condition given by the classic futures-spot parity theorem and the arbitrage analysis described previously. Of course, if the index is a total-return index, then the $E[y_S]$ term in the condition expressed in Equation (13) drops out.

### Feasible Trading Condition for the Short Position

Now, consider the other side of the swap trade and examine the situation of Ms. Bear. As posited, Ms. Bear's short position in the swap is covered by property holdings of similar, although not necessarily identical, risk to that of the average property tracked by the index. This covering position exposes Ms. Bear to the risk of her property portfolio, but also provides her with the full total return on that portfolio. Her short position in the swap which requires her to pay the return on the index, reduces her risk exposure by the magnitude of the risk in the index and gains her the fixed-leg payment of *F* in each period, but also obligates her to pay the capital return on the index in each period. Because virtually all of the risk in both her property portfolio and in the real estate index is in the capital return, and since the fixed-leg payment $F$ is riskless, the result is that Ms. Bear's risk exposure is the difference between her own property portfolio's risk minus the risk in the index. Thus, Ms. Bear faces an expected return, given by the following condition, where $r_B$ refers to the total return on her overall position (the swap plus the cover):

$$\mathrm{E}^b[r^b] \geq i + RP_{bRE} - RP_S = i + \mathrm{E}^E[r_{bRE}] - \mathrm{E}^E[r_S], \qquad (14a)$$

where $\mathrm{E}^E[r_{bRE}]$ is the equilibrium expected total return on Ms. Bear's property portfolio, which equals LIBOR plus the equilibrium risk premium, $RP_{bRE}$, or $\mathrm{E}^E[r_{bRE}] = i + RP_{bRE}$.

The swap will give Ms. Bear an actual expected total return, including her covering property holdings, of

$$\mathrm{E}^b[r^b] = \mathrm{E}^b[r_{bRE}] + F - \mathrm{E}^b[g_S] \qquad (14b)$$

where $\mathrm{E}^b[r_{bRE}]$ represents Ms. Bear's private expectations about the total return she will generate in her own real estate portfolio during the term of the swap contract, and $\mathrm{E}^b[g_S]$ represents Ms. Bear's private expectations about the growth rate in the real estate index. Both of these private expectations may differ form the market's equilibrium requirements.

Consider, first, that Ms. Bear's private expectations about the index capital return over the period of the swap contract, $\mathrm{E}^b[g_S]$, may differ not only from equilibrium expectations, but also from the private expectations of Mr. Bull; that is, $\mathrm{E}^b[gS] \neq \mathrm{E}^b[g_S]$. In particular, assume that Ms. Bear is, well, bearish on the outlook for the underlying real estate market. That is, Ms. Bear believes that the underlying real estate market over the swap contract term will provide total returns less than its equilibrium requirement. In other words, she expects the index return over the swap contract term to be less than it otherwise would be if it simply reflected the normal index lag effect in addition to the equilibrium returns in the underlying property market. If we label the absolute magnitude of Ms. Bear's bearish expectations as $bS$ (in percent per annum), her private growth expectations about the index can be expanded by employing the relationship in Equation (12) as follows:

$$\mathrm{E}^b[g_S] = \mathrm{E}[r_S] - \mathrm{E}[y_S] - b_S = \mathrm{E}^E[r_S] + L - b_S - \mathrm{E}[y_S] \qquad (14c)$$

Although Ms. Bear is bearish about the real estate market, she is assumed to be confident in her own ability to generate positive alpha from her own property portfolio over the period of the swap. That is, Ms. Bear believes that her own property portfolio, considering its risk level, will provide a total return in excess of what the market requires in equilibrium. Ms. Bear's positive alpha expectations, $\alpha$, expand her private expectations as follows:

$$\mathrm{E}^b[r_{bRE}] = \mathrm{E}^E[r_{bRE}] + \alpha = i + RP_{bRE} + \alpha \qquad (14d)$$

Combining Equations (14b), (14c), and (14d), we can define Ms. Bear's expectations about her return on the swap trade, including her covering portfolio, as

$$\begin{aligned}\mathrm{E}^b[r^b] &= \mathrm{E}^b[r_{bRE}] + F - \mathrm{E}^b[g_S] \\ &= \mathrm{E}^E[r_{bRE}] + \alpha + F - (\mathrm{E}^E[r_S] + L - b_S - \mathrm{E}[y_S]) \\ &= \mathrm{E}^E[r_{bRE}] - \mathrm{E}^E[r_S] + \alpha - L + b_S + \mathrm{E}[y_S] + F \qquad (14e)\end{aligned}$$

Combining (14e) with Ms. Bear's trading feasibility condition in Equation (14a), we obtain Ms. Bear's feasibility or trading criterion for the price of the swap to be

$$\begin{aligned}\mathrm{E}^b[r^b] &= \mathrm{E}^E[r_{bRE}] - \mathrm{E}^E[r_S] + \alpha - L + b_S + \mathrm{E}[y_S] \\ &\quad + F \geq i + \mathrm{E}^E[r_{bRE}] - \mathrm{E}^E[r_S] \\ &\quad \rightarrow F \geq i + L - \alpha - b_S - \mathrm{E}[y_S] \qquad (14f)\end{aligned}$$

That is, Ms. Bear will be willing to enter into the swap trade as long as the price (fixed leg) is at least equal to LIBOR plus the index lag effect less Ms. Bear's expected alpha on her properties and the magnitude of her bearish expectations about the real estate market and also the income return component of the index.[21]

Notice once again that if the index were always in equilibrium such that there was no lag or risk differential effect ($L = 0$), and if Ms. Bear had neutral rather than bearish expectations about the property market ($b_S = 0$), and if Ms. Bear did not feel she could add any alpha to her property portfolio's investment performance ($\alpha = 0$), then this feasibility or trading condition would just exactly equal the $F = i - \mathrm{E}[y_S]$ fair-price condition given by the classic futures-spot parity theorem and the arbitrage analysis described previously. Of course, if it is a total return index, then the $\mathrm{E}[y_S]$ term in the condition in Equation (14f) drops out.

### The Equilibrium Price and the Feasible Trading Price Window

When we combine Mr. Bull's and Ms. Bear's trading feasibility conditions regarding the price of the swap, $F$, we obtain the following window of feasible trading prices:

$$i + L - \alpha - b_S - \mathrm{E}[y_S] \leq F \leq i + L + B^L - \mathrm{E}[y_S] \qquad (15)$$

Notice that the terms $i + L - \mathrm{E}[y_S]$ are common to both sides of the condition state in Equation (15). Thus, if both parties have neutral expectations about the market, and the short party expects no alpha from their covering property portfolio ($B^L = b_S = \alpha = 0$), then the only feasible trading price is the single point $F = i + L - \mathrm{E}[y_S]$, which includes the index lag effect, $L$. Note further that if the index is always priced at equilibrium, so that there is no momentum effect and no risk differential between the index and the underlying properties ($L = 0$), then the trading feasibility window in Equation (15) collapses to the classic arbitrage-based formula $F = i - \mathrm{E}[y_S]$. Finally, given the definition of $L$, note that an alternative and equivalent way to express the neutral-expectations value ($B^L = b_S = \alpha = 0$) of $F$ is

$$F = E[g_S] - RP_S \qquad (16)$$

This method of expressing the equilibrium price condition demonstrates its equivalence to the forward price formula in Equation (7c).[22]

### Additional Pricing Considerations

The equilibrium and expectational considerations incorporated in the preceding pricing and trading feasibility analysis are the basics that must underlie a successful derivatives market in the long run. However, other considerations can also be quite important, and can affect both the feasible trading window as well as the empirically observable equilibrium price indicated by the midpoint of the bid–ask spread around that price. In fact, a trading window in which the long position is willing to pay a fixed-leg rate $F$ greater than what the short position is willing to accept is vital to the practical feasibility of derivatives trading, because intermediaries, such as exchanges, clearinghouses, servicers, investment banks, and broker–dealers, are necessary for the functioning of the market. These intermediaries need to cover their costs and make a sufficient profit for themselves, and this can only be done by the existence of a spread between what the long and short parties are willing to pay and accept. While such a bid–ask spread must be based fundamentally on the kind of trading window described in the condition in Equation (15), it can be broader than the $B^L+b_S+\alpha$ breadth indicated.

Other important pricing considerations related to the practical use of derivatives can exist from the perspective of either the long or short position. For example, the long party may perceive other benefits from the swap, such as a lower-cost, preferable method, as compared to direct investment, for taking a more diversified investment position in real estate, an objective the long party may have for their overall investment portfolio. The long party may be willing to pay a fixed-leg price, $F$, greater than the maximum value indicated on the right side of the condition in Equation (15) in order to achieve such transaction/management cost savings and diversification. And the short party may want real estate market-value insurance, or to hedge their real estate exposure. The short party may be willing to accept a fixed-leg price payment, $F$, that is less than the minimum value indicated on the left side of the condition in Equation (15), in effect being willing to pay for such insurance or hedge, because it enables them to accomplish portfolio or other risk management objectives.[23] Of course, the value to the short party of such hedging will be reduced to the extent that the derivative does not provide a perfect hedge against the market exposure that the short position wants to be protected against. Thus, the extent to which the short position will take a lower swap price in consideration of hedge value will be reduced if the index underlying the derivative contains much noise, as described earlier in the discussion of indexes.

To see how such additional considerations can affect the equilibrium price and the bid–ask spread around that price, consider the following illustrative scenario. Suppose that the long position is generally willing to pay up to 100 bps per year to trade the derivative, over and above the equilibrium considerations in the condition expressed in Equation (15), out of consideration of transaction/management cost savings as well as real estate investment diversification benefits. And suppose the short position is generally willing to accept up to 100 bps less than the equilibrium considerations of the same condition. Then a bid–ask spread of at least 200 bps could open up around an equilibrium price that might be near the midpoint of the range specified in the condition. For simplicity, assume neutral expectations on both sides, so that the condition collapses to $F = i - \mathrm{E}[y_S]$. Then we could have an ask, or offer, price of $F^A = i - \mathrm{E}[y_S] + 100$ bps; and a bid price of $F^B = i - \mathrm{E}[y_S] - 100$ bps. Long parties would pay the ask price of $F^A$, short parties would receive the bid price of $F^B$, and the 200 bps in between would be retained by the intermediaries.

Now suppose that parties in the short position are concerned about noise in the index that adds to the basis risk of their hedges against real estate market risk relevant to their particular property exposure.[24] Suppose this concern leads the short position to be willing to pay only 50 bps, instead of our previous assumption of 100 bps, to use the derivative to effect the hedge (i.e., they deduct 50 bps from the price they are willing to pay for the hedge). Now, because we have not changed what the long party is willing to pay, the minimum tradable bid price is $F^B = i - \mathrm{E}[y_S] - 50$ bps, while the maximum tradable ask price is still $F^A = i - \mathrm{E}[y_S] + 100$ bps. The resulting midpoint between the bid and ask prices is now 25 bps above the neutral equilibrium condition at $F = i - \mathrm{E}[y_S] + 25$ bps, with the pricing range being +/- 75bps around that midpoint, giving the intermediaries 150 bps, instead of the previous 200 bps.

Finally, suppose the intermediaries who make the derivatives market by quoting bid and ask prices do not really need 150 bps in order to cover their costs and make sufficient profit. Suppose they actually only need 100 bps. And suppose competition among potential intermediaries, such as brokers, dealers, and exchanges, is sufficient to compete away the bid–ask spread down to just the amount necessary to cover costs and necessary intermediary profit. Assuming this process is symmetrical on the long and short sides, we would see a market in which the observed ask price was $F^A = i - \mathrm{E}[y_S] + 75$ bps, and the bid price was $F^B = i - \mathrm{E}[y_S] - 25$ bps, with the midpoint at $F = i - \mathrm{E}[y_S] + 25$ bps. This provides the necessary +/- 50 bps spread (or 100 bps total range) for the intermediaries and provides both the long and short positions with an improved price—25 bps more than they would be willing to trade, if they had to.

In this example, considerations other than fundamental equilibrium and expectational factors led to an empirical equilibrium price that, in the sense of an observable bid–ask midpoint price in a well-functioning market, is 25 bps above the theoretical equilibrium value. In this case, it is the short position's concern about basis risk in the derivative that tipped the balance of supply and demand to a price slightly above the theoretical equilibrium.

## IMPLICATIONS OF THE EQUILIBRIUM PRICE ANALYSIS

The more general equilibrium analysis presented in the previous section shows two things that are important for real estate derivatives that the classic formula and arbitrage derivation does not show. First, we see how a lagged index, $L$, must be accounted for in the equilibrium, or fair, price of the derivative which includes both the transient, or cyclical, momentum effect, $m$, included in Equation (12), plus the permanent risk-difference effect, $RP_P - RP_S$. The equilibrium price of the swap is also shown to be the expected return on the index during the swap contract minus the equilibrium risk premium applicable to the index. Second, our analysis shows how a feasible trading window can open up in the fixed-leg price condition, $F$, on the swap, of a magnitude equal to $B^L + b_S + \alpha$, reflecting the complementary private expectations of the parties—long party bullish, short party bearish, and both parties expecting positive alpha. Such heterogeneous and private expectations are likely to be necessary as a fundamental source of liquidity in the derivatives market, although additional pricing considerations noted in the last section may also be able to play that role.

An important point highlighted by the feasible pricing result in Equation (15) is that equilibrium expected returns on either the underlying real estate market or the index do not figure directly in the feasible pricing condition. Neither $\mathrm{E}^E[r_P]$ nor $\mathrm{E}^E[r_S]$ appear in Equation (15). However, both of these expectations do figure indirectly in the feasible swap price, because they underlie the value of the index lag effect, $L$. Recall from Equation (12) that the lag effect equals the difference between the expected index return and the equilibrium index return $L = \mathrm{E}[r_S] - \mathrm{E}^E[r_S] = \mathrm{E}^E[r_P] - \mathrm{E}^E[r_S] + m$. This clarifies explicitly the importance of derivatives market participants becoming familiar, and comfortable, with the nature of the real estate index that underpins a specific derivative product, including how the risk and return in the index may differ from that of the underlying property market, as well as the nature of the dynamics of the index which is possibly distinct from that of the property market. We see that $L$ includes not only the more permanent difference in equilibrium risk premium between the property market and the index, $RP_P - RP_S$, but also the transient momentum effect, $m$. And we see that this concern exists on both sides of the derivatives market—the long position as well as the short.

Indeed, the presence of both the index lag effect, $L$, and the expected income return component of the index, $E[y_S]$, in the equilibrium price of the swap raises informational questions that could either motivate or frustrate trading of the derivative. These informational issues raise both a danger and an opportunity for starting and facilitating the derivatives market. Divergent perceptions across the parties about what the values of $L$ and $E[y_S]$ are could either eliminate any feasible trading price between a given pair of traders (i.e., if the short party believes $L$ to be bigger and $E[y_S]$ to be smaller than the long party believes them to be), or open up a positive spread of feasible trading prices if the differences in perception are compatible (i.e., if the short party believes $L$ to be smaller and $E[y_S]$ to be larger than the long party believes them to be). Heterogeneity in these perceptions across potential trading parties could either facilitate or curtail liquidity in the derivatives market. The danger is that lack of understanding or a comfort level about what are reasonable values for $L$ and $E[y_S]$ could make parties hesitate to trade a capital return swap. Our analysis shows that these two variables may be crucial to resolving the lack-of-pricing-confidence barrier indicated in the MIT survey cited at the outset of this article. Of these two variables, it seems likely that $L$ , the index lag effect, will be most problematic for derivatives based on appraisal-based indexes, because well-constructed transaction-based indexes should be able to more closely approximate the actual price dynamics of the underlying property market ($S_t$ more similar to $P_t$ ). Also, the $E[y_S]$ term can be eliminated altogether in a derivative based on an appraisal-based index, because such indexes can normally include the income return, enabling trading on a total return index instead of just on a capital return index. However, the expectation of the index income return component, $E[y_S]$, may well be more problematical with derivatives based on transaction-based indexes, because such indexes usually present only the capital return.[25]

## The Meaning of Price Changes in the Swap Market

Let us now consider as we did previously with the forward contract the meaning of a change in the equilibrium price of the swap contract. Ignoring alpha and non-neutral market expectations and considering a total return swap, the previous analysis tells us that the equilibrium price is just the risk-free interest rate plus the index lag effect as in the equation[26]

$$F = i + L \tag{17}$$

If a sufficiently dense and liquid swap market develops, changes in this price over time might be observed to be a good approximation of the changes in the midpoint of the bid–ask spread in contracts of similar duration traded at different points across time. Clearly, a change in this equilibrium swap price has only two components—a change in interest rates and a change in the index lag effect—as indicated in the equation

$$\Delta F = \Delta i + \Delta L \tag{17a}$$

Assuming interest rates are constant, an observed change in the swap price, observed as a change in the quoted spread to LIBOR, indicates a change in the index lag effect, $\Delta L$. As $L$ consists of the difference between the index and property equilibrium risk premia, $RP_P - RP_S$, plus the momentum effect, $m$, any change in $L$ must simply reflect a change in these terms. As the risk premia difference is likely to remain relatively constant over time, changes in swap prices are likely to largely reflect changes in the index momentum effect, $\Delta F \approx \Delta m$.

This change can be quite revealing and interesting as an indicator of what is happening in the property market, more rapidly and precisely than is revealed by contemporary returns to the index itself. Even though the index on which the swap is based is lagged (or the $L$ term would not be part of the swap price), the change in the swap price can reflect contemporaneous changes in the underlying property market without a lag. Consider an example. Suppose the property market has recently been rising rapidly, faster than its equilibrium return can support. This will impart a positive momentum into a lagged index, making the $L$ term positive in the swap price, reflecting the overhang, or positive inertia, in the index caused by the recent upsurge in property prices. Now suppose the property market suddenly levels off or plateaus. This will not yet be fully reflected in the lagged index, which will continue to rise for a while, catching up with the past surge. But the swap price will consider the average lag in the index throughout the future period of the swap contract. And during this contract period the positive overhang in the index will gradually be working its way out, given that the actual property market has now flattened. This will cause the lag effect in the swap price to be smaller than it was when the property market was still rising. In other words, $\Delta m$ will be negative, so $\Delta L$ will be negative, which causes $\Delta F$ to be negative, holding interest rates constant or holding $m$ constant relative to interest rates, even though $m$ may still be positive. The actual flattening of the property market will be observed as a drop in the swap price (a reduction in the quoted spread to LIBOR), possibly earlier than it can be observed in the lagged index itself. This will be especially true if the swap market becomes more liquid, with more trading volume, than the cash property market, thereby facilitating the occurrence of price discovery in the derivatives market ahead of that in the cash property market. As noted earlier, an effect similar to this may have recently been observed in the U.K. derivatives market.

### Hedging Real Estate Exposure Using a Lagged Index

Finally, consider the effect of the index lag on the use of the derivatives to hedge changes in the underlying property market. The change in the price of the derivative in response to the change in contemporaneous values in the property market reflects only the portion of the property market value change that will filter through the index by the end of the derivative contract's term.

For example, suppose that an unexpected event suddenly drops property values by 10%, but that the index will only reflect half of that amount, a 500-bp drop, within the duration of a swap contract expiring in one year. Therefore, the change in the swap contract equilibrium price will only be 500 bps, even though the property market has lost 1,000 bps of value. The implication is that a swap or forward contract hedge needs to have a notional value in the derivatives market equal to twice the value of the real estate exposure being covered. In other words, the necessary hedge ratio will be greater than unity due to the index lag relative to the duration of the swap contract. The hedge ratio is defined as the notional value of the derivative contract that must be traded per dollar of underlying property exposure that is covered or hedged by the trade. In the previous example, the hedge ratio is two.

For any given swap contract with a fixed maturity date, the hedge ratio will increase with the passage of time if the index underlying the swap contract is characterized by lagging and smoothing. At the outset, a three-year contract may have little or no lag effect in hedging property price movements that occur shortly after the contract is first traded, because three years is likely sufficient time for the lagged realization of the price movement to work its way through the index.[27] But as the maturity of the contract approaches and the remaining duration of the contract is reduced, the hedge ratio will increase and the value of real estate effectively covered by the hedge will diminish. This occurs because the remaining maturity in the contract becomes too short to allow underlying property market price changes to fully work their way through the lag in the index. To recall our previous example, if the swap contract with one year remaining allows half of a 10% change in property market value to be reflected in the contract price, a similar contract with only six months left might only allow a quarter of the 10% price change to filter through; that is, the swap contract would show only a 2.5% equilibrium price change and the hedge ratio would have grown from two to four.

The implication of this for an effective hedging strategy using derivatives based on a lagged index is that the hedger must trade repetitively and frequently in the derivatives market, keeping a portfolio of hedges with staggered maturity dates. In the absence of a fully liquid secondary market for the swap contracts, fully contemporaneous hedging will require over-coverage with short-term swap contracts; that is, swapping short-term notional value that exceeds the value of the real estate covered by the hedge. For example, if the hedge ratio is two, then the hedger must maintain a notional value in derivatives twice that of the value of real estate covered by the hedge. Alternatively, full-contract hedging can be achieved with a smaller hedge ratio by regularly updating a portfolio of smaller longer-term swap contracts with staggered maturity dates in order to maintain a swap portfolio duration long enough to include the full lag in the index. A trader will have to trade contracts extending well beyond (perhaps at least a year or two) the point in time until which he wishes to have his position hedged, and this will partially expose the trader to risk beyond that target date. The requirement for excess trading of the derivative will tend to magnify the transaction costs, including the noise and resulting basis risk, in the hedge. For example, suppose the bid–ask spread in the derivative contract is 100 bps per dollar of notional value traded. If half of that spread is borne by each side of the trade, the hedger would only face 50 bps of transaction cost if the hedge ratio were unity. But with a hedge ratio of two, the transaction cost effectively becomes 100 bps per dollar of real estate covered by the hedge—even though it is only 50 bps per dollar of notional value traded. Similarly, if there is 50 bps of noise (basis risk) in the index, the hedge ratio of two will result in twice that much effective noise or basis risk per dollar of real estate covered by the hedge. And for some traders the basis risk will also be affected by the difficulty of precisely targeting the hedge in time because a contract ending at time $t$ does not fully hedge against all events prior to $t$, but a contract ending beyond time $t$ partially exposes the trader to events occurring after time $t$.

## CONCLUSION

This article began with a review of the key and fundamental characteristics of the major types of property price indexes as they relate to real estate equity derivatives in the U.S., including the important differences between the two major types of indexes—appraisal-based and transaction-based indexes. Then, we presented in detail the basic principles and implied formulas for the equilibrium pricing of real estate index derivatives, focusing on the two basic types of derivatives—forwards and swaps—including consideration of the implications of changes in derivatives' prices over time. We examined pricing from both a classic arbitrage-based perspective, and from a broader equilibrium perspective that enables a more complete treatment of issues related to the underlying real estate indexes, such as how the lag in appraisal-based indexes impacts the price of a derivative. We noted that even if it is not possible to invest in the assets underlying the index and even if property markets are not in equilibrium or there is lag in the index, a fair or equilibrium price for the derivative exists that accurately reflects its risk and return characteristics. The equilibrium treatment also allows an explication of the feasible trading window of derivatives pricing, which can facilitate the parties better understanding of how to negotiate appropriate prices in an illiquid OTC market or the position they should take in a liquid market or public exchange when faced with a specific price. Finally, we considered the implications of the pricing and index analysis for the informational requirements necessary to underpin a healthy property derivatives market in the U.S. We saw how these informational and knowledge requirements focused particular attention on understanding the nature of the difference between the real estate index and the underlying property market. Particularly in the nascent phase of the derivatives market, lack of such pricing and index knowledge can present a serious danger. The concomitant opportunity is for the U.S. real estate investment and derivatives industry to take steps to educate the potential end users of property derivatives—the real estate investors, owners, and fund managers—who must provide the necessary liquidity for the market.

## ENDNOTES

[1]If just half of all commercial developers in the U.S. hedged their market risk exposure during the construction phase of their development projects (much as farmers sell short in the commodities futures markets when they plant their crops), the result would be over $150 billion of derivatives trading per year.

[2]Fisher [2005] provides an introduction to and overview of NCREIF-based swap products and Clayton [2007] examines more recent market developments, including a discussion of the various indexes being developed for derivatives trading.

[3]The other very important barrier noted by respondents was lack of liquidity or a secondary market for the derivatives. See Lim & Zhang [2006] and Fisher [2005].

[4]Note that noise does not accumulate over time, whereas the true returns do accumulate. Thus, the relative magnitude of noise compared to either the volatility or the expectation of the true return accumulation tends to diminish over longer return horizons.

[5]A lagged index may ultimately fully reflect the market value increment that actually occurs in any given period. For example, in Equation (4), the true value increment in period $t$, the gain percentage $g_{Pt}$, will have $\omega$ fraction reflected in the index in period $t$, and the remaining $(1 - \omega)$ fraction picked up by the index in period $t + 1$. However, for a derivatives trader whose contract expires at the end of period $t$, this may be too late.

[6]See Fisher and Geltner [1994] and Chapter 25 of Geltner, Miller, Clayton, and Eichholtz [2007]. This is not to say that either type of index is immune from the problem most associated with the other type. In transaction-based indexes care must be taken with the regression specification to avoid temporal aggregation (effectively averaging the transaction prices within each time period), which will induce a lag (See, e.g., Geltner [1993, 1997].). In appraisal-based indexes, if the number of properties covered is too small, the index will likely contain noise as well as lag. Clinical and other evidence from both the U.K. and the U.S. indicate that there is a substantial random component to individual property appraisals, perhaps as large as that in individual transaction prices when the appraisals are truly independent. For example, in an experiment in which two appraisers are given the task of independently appraising the same property as of the same point in time, their resulting value estimations will typically differ by up to 10% or even more. (See, e.g., Crosby et al. [1998] and Diaz and Wolverton [1998].)

[7]Of course, the NPI reflects property-level returns, unlevered and before any fund-level or management expenses and fees to which investors are subject. For more information on the NPI, see the National Council of Real Estate Investment Fiduciaries web site at: www.ncreif.org.

[8]This is evidenced by the fact that many properties' reported values do not change at all from one quarter to the

next, or change only in exactly the amount of their capital improvement expenditures.

[9]For example, in 2006 the NPI included less than $30 billion of property sales, whereas the Real Capital Analytics Inc. (RCA) database recorded over $300 billion of commercial property sales tracking only sales greater than $2.5 million. As of the end of 2006, the NPI consisted of approximately 5,000 properties worth a total of about $250 billion, whereas JP Morgan Asset Management's *Real Estate Universe* report estimated the total value of U.S. commercial real estate at that time to be some $6.7 trillion, or over 25 times the NCREIF population value (although this included corporate or non-traded real estate and small "mom and pop" properties as well as the larger properties tracked by the RCA database). In contrast, the IPD Annual Index in the U.K. is estimated by IPD to cover almost 50% of British commercial property.

[10]The hedonic procedure traces to Court [1936], Griliches [1961], and Rosen [1974]. The repeat-sales approach originated with Bailey, Muth, and Nourse [1963].

[11]This index uses the recent appraised values of the sold properties as a composite indicator of the hedonic characteristics of the properties, thereby controlling for cross-sectional differences in the sold properties.

[12]Stock market indexes are also based on comparing the transaction prices of stock shares sold in one period with the transaction prices of similar shares sold in the previous period. As stock shares are homogenous (a share of IBM that traded this month is the same as a share of IBM that traded last month), the result is comparable to a same-property price change index such as the repeat-sales transaction-based indexes.

[13]In 2006, the MIT Center for Real Estate developed the methodology for a repeat-sales index of U.S. commercial property prices based on the Real Capital Analytics Inc. (RCA) database. (See Geltner and Pollakowski [2006].) As of 2007, this index suite includes a monthly frequency national aggregate index and 28 other quarterly or annual indexes by property type and geographical location. The index methodology has been licensed to Real Estate Analytics LLC (REAL).

[14]Of course, it is possible for the underlying property market to have some sluggishness or momentum even in its equilibrium; that is, prices can be persistent in the real property market. If so, then a good transaction-based index should also reflect any such sluggishness or momentum that is actually in the property market. In this case, the derivatives market will try to incorporate any resulting property market predictability into the pricing of the derivatives, enabling the derivatives market pricing to help inform the property market where prices may be headed.

[15]These pricing relationships also generally hold for forward contracts on commodities. In the case of physical commodities (e.g., oil, soybeans, and pork bellies) the expected future price of the commodity, $E_0[S_T]$, reflects the expected physical supply/demand balance in the market for the physical commodity at time $T$ See any good graduate investments text for an explication, such as Bodie, Kane, and Marcus [1999] Chapter 23.

[16]Hull [1997] also discusses a capital asset pricing model approach to obtaining the price of a forward contract by discounting the expected future spot price by the difference between the discount rate and the risk-free rate (i.e., the risk premium).

[17]In general, changes in the marked-to-market value of outstanding derivatives contracts must be accounted for in the current income statement of the entity holding the contracts. Where a sufficiently liquid secondary market for the derivatives does not exist, they may be marked-to-model (instead of to market). The pricing principles described here would apply to such modeled valuation. It is also important to note that accounting rules may allow users of derivatives contracts to apply hedge accounting rules, which allow them to also mark to market the underlying assets owned by the entity using the derivatives to hedge those assets, and pass such value changes through the income statement.

[18]In terms of net cash flow, the original index level, or notional amount, of the trade, $S_0$, cancels out in the payoff of a forward, as each party owes that amount to the other. The structure of the swap contract enables this bookkeeping value to drop out of the quoted price of the derivative.

[19]Positions that are only partially covered (or in the extreme, completely uncovered) are simply linear extrapolations of the fully covered positions in risk–return space. So-called linear pricing, also known as the law of one price, in which each party faces an equal expected return risk premium per unit of risk exposure, is the mathematical representation of the economist's definition of equilibrium pricing. For example, the long position with 50% coverage by bonds is effectively a 50% loan-to-value leveraged equity position in the underlying property index that is exposed to twice the risk per dollar of equity invested compared to a fully covered position. With no coverage at all, it might appear as though derivatives enable real estate investment with a 100% loan-to-value ratio. But, in reality, the parties to derivatives trades must post bonds or margin cash or collateral requirements, or otherwise guarantee, or pay for the guarantee of, their performance under the contract. Nevertheless, it is true that one benefit of derivatives can be a more efficient way to effectively leverage real estate investment—and indeed investment that is highly diversified (the index)—and with effectively synthetic debt that is off balance sheet. Beware, however, that the leverage is very real indeed in that the risk of the position is concomitantly magnified.

[20]The amount of this difference depends on how much the index is lagged, and how the capital market prices risk.

Suppose the lag in the index can be well represented by a simple moving-average model as in Equation (4)

$$g_{St} = \omega g_{Pt} + (1-\omega) g_{Pt-1}`$$

Under the classic capital asset pricing model, and assuming that neither the market portfolio nor the underlying actual property market is lagged, the index risk premium equals the property market risk premium multiplied by the index return's fractional weight on the contemporary market return, $RP_S = \omega RP_P$. (See Geltner [1991].) For example, if $\omega = 2/3$ (giving the index an average lag of one-third of a return period; for example, a four-month average lag in an annual-frequency index), then if the property market risk premium is 300 bps, the index risk premium would be 200 bps, and the $RP_P - RP_S$ component of $L$ would equal $(1-\omega)RP_P = 100$ bps.

[21]Note that we have defined Ms. Bear's alpha relative to the equilibrium expected returns on her properties. If we defined her alpha relative to her bearish expectations about the market, then the $b_S$ term would cancel out in the above derivation and not appear in Equation (14f). However, in that case Ms. Bear's bearish expectations, $b_S$, would be included in the $\alpha$ term, so the numerical value of $F$ would be the same as what is indicated in the way we have labeled Equation (14f). Notice also that the criterion in Equation (14f) assumes that Ms. Bear would be willing, if necessary, to trade away all of her anticipated positive alpha. If she is not willing to trade away any of that alpha, then we must increase her minimum feasible trading price by the amount of $\alpha$.

[22]The equilibrium pricing perspective presented here can be implemented and applied to the economic valuation of any swap or forward contract using the certainty equivalence approach to discounted cash flow (DCF) analysis. The traditional risk-adjusted discount rate approach to DCF valuation cannot be applied to futures contracts, because no finite discount rate will discount non-zero expected future cash flows to a zero present value, which must be the equilibrium value of a futures contract in which no cash changes hands up front. Certainty-equivalence discounting provides a solution to this problem. (See Geltner, Miller, Clayton, and Eichholtz [2007] Chapter 26.) The certainty-equivalence discounting model provides a valid framework for marking to model the value of outstanding real estate swap contracts.

[23]As an example, consider the value to the real estate developer of laying off exposure to market risk during the time of project construction (akin to the commercial farmer selling his corn in the futures market when he plants it rather than when he harvests it).

[24]Basis risk refers to differences between the payoff of the derivative contract and the market return against which the short position is trying to hedge.

[25]A notable exception is the NCREIF-based transaction-based index developed by MIT and discussed in Exhibit 1. Of course, any capital return index can establish a corresponding income return component that may well be a reasonable approximation; for example, based on cap-rate data from the same property market tracked by the index. The cap rate is the property net operating income divided by its market value. This ignores the cash flow impact of property capital improvement expenditures, and cap rate information is often not as solid or reliable as actual transaction price information, but it may serve as an acceptable approximation for the derivatives market.

[26]Note that the fair, or equilibrium, swap price is clearly not the expected return on the index, $F \neq E[r_S] = i + L + RP_S$. To derive the implied expected return on the index from the swap price, the equilibrium risk premium required for investment in the index must be added. Only if this risk premium is zero will the swap price equal the expected return on the index.

[27]In the absence of a liquid secondary market for the swaps, this assumes either that the hedger can wait for the duration of the derivatives contract to receive their hedge payments, or that the contract is marked to model per the equilibrium pricing framework described here, and the hedger is content with such marked, as opposed to current cash, hedge payments

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