Can Stricter Bankruptcy Laws Discipline Capital Investment? Evidence from the U.S. Airline Industry

Joe Mazur*

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Abstract

Models of capital investment in industrial organization typically treat bankruptcy as an involuntary and final outcome, yet firms that file under Chapter 11 of the U.S. Bankruptcy Code often do so voluntarily and with the expectation that they will eventually emerge. Moreover, Chapter 11 permits cancellation or renegotiation of long-term contracts for labor and capital, providing otherwise constrained firms an opportunity to downsize, and suggesting a non-financial role for bankruptcy law in investment behavior. This paper is the first to analyze the link between reorganization and investment in a dynamic oligopoly setting. To capture the strategic implications of both decisions, I develop a dynamic game in continuous time that incorporates choices over investment and bankruptcy. I show that strengthening creditors’ bargaining power in bankruptcy proceedings can discipline capital investment behavior outside of bankruptcy, curbing investment in periods of high demand and spurring the sale of capital when demand is low. I test the implications of the model using data on the U.S. passenger airline industry, finding evidence that a recent reform that strengthened creditors’ bargaining power in Chapter 11 may have contributed to the widely acknowledged “capacity discipline” observed in the market since 2006. I then simulate several alternative bankruptcy policies to better understand how the treatment of contracts in bankruptcy affects long-term investment and industry dynamics.

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1 Introduction

Since Arrow (1968), the industrial organization (I.O.) literature has acknowledged that investment irreversibility (a.k.a. “sunkness”) is a key determinant of the capital investment decision. One source of irreversibility is contractual investment, which effectively creates a cost to downsize. Given that reorganization in Chapter 11 bankruptcy is a common setting for rescinding and/or renegotiating contracts, one would expect bankruptcy policy to play a significant role in investment models. Yet studies of capital investment in I.O. typically ignore bankruptcy entirely. Those that do allow for bankruptcy often view it as an involuntary and final outcome, tantamount to exit. However, most Chapter 11 filings are brought forth voluntarily, and two-thirds of public firms filing Chapter 11 eventually emerge from Bankruptcy Court protection. The corporate finance literature has adopted endogenous bankruptcy as the standard, beginning with Leland (1994) and Leland and Toft (1996), yet corporate finance models of capital investment and capital structure focus primarily on single-agent settings, leading one to ask, “What are the implications for jointly modeling investment and bankruptcy in the familiar I.O. context of strategic interaction?”

My paper is the first to show that making bankruptcy policy more creditor-friendly can discipline the investment behavior of non-bankrupt firms. Such “capacity discipline” takes the form of slower investment during periods of high demand coupled with faster disinvestment when demand is low. This new result arises from treating bankruptcy as a potentially non-final decision. Allowing firms to both enter and exit bankruptcy reveals a previously unexplored investment-level impact of bankruptcy policy that is both significant and intuitive. I identify this effect as a potential cause of the recent capacity discipline observed in the airline industry. Using data on airline capacity, bankruptcy, and demand, I find support for the influence of bankruptcy policy on investment and evaluate the consequences of alternative bankruptcy policies.

Modeling bankruptcy as voluntary is reasonable given the appeal of Chapter 11 reorganization as a downsizing option. Chapter 11 gives malleability to many otherwise rigid contractual agreements. For example, financially distressed corporations can often renegotiate substantial portions of debt and other liabilities. On the non-financial side, Chapter 11 offers the potential to rescind or unilaterally alter many types of contracts. These non-financial protections can be especially important for companies with contractual commitments to utilize labor, capital, or materials because they open up cost-cutting options unavailable outside of bankruptcy. Among the more salient examples are pay cuts for unionized employees, renegotiated leasing terms, and pension benefit modifications.

To guide my analysis, I first develop a simple duopoly model that illustrates how bankruptcy can lead to capacity discipline. In my model, the perceived cost of filing Chapter 11 (e.g. legal costs, expected repayments to creditors, risk of liquidation, etc.) increases in the
creditor-friendliness of the bankruptcy regime. Solving for equilibrium, I find that higher bankruptcy costs may tend to reduce firms’ incentive to invest during periods of high demand and increase their likelihood of disinvestment during periods of low demand. In other words, a more creditor-friendly bankruptcy policy may tend to rein in capacity investment behavior overall.

The airline industry presents the ideal context in which to test this link for three main reasons. First, the volatility of air travel demand and the prevalence of contractual labor and capital lease agreements in this industry make Chapter 11 especially appealing for distressed airlines. In other words, airlines satisfy the requirements of an industry that would benefit from Chapter 11: They heavily use long-term contracts, and they face volatile demand that sometimes necessitates breaching those contracts. Second, the prevalence of bankruptcy in the industry suggests it may be strategically used. To the extent that forward-looking firms internalize the reorganization option, they may tend to overcommit to long-term contracts, resulting in rampant bankruptcy when demand falls. The notorious insolvency of U.S. airlines fits this pattern. Third, anecdotal evidence suggests that an airline’s Chapter 11 filing can be strategically timed, indicating that bankruptcy is far from an exogenous event.

To test these implications empirically, I use data from the U.S. airline industry and exploit variation in the expected cost of reorganization due to the Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA) of 2005, which made significant changes to Chapter 11. In particular, BAPCPA reduced the amount of time allowed for a corporation to put forth an exclusive plan of reorganization, increased the amount and priority of wage and benefit claims, tightened the deadlines for accepting certain leases, and raised the priority and amount of a number of other claim categories. Legal scholars and practitioners both agree that the reform served to restrict debtor protection and reduce the likelihood of a successful reorganization, particularly for the largest and most complex corporations. Indeed, under standard economic models of bargaining, such as Merlo and Wilson (1998), limiting the exclusivity period alone is enough to shift bargaining power to creditors.

My empirical approach to studying the link between bankruptcy and investment is three-fold. First, I perform a difference-in-differences analysis on airline industry data to determine whether BAPCPA had any impact on the investment behavior of large airlines. I also test whether the reform’s effects are consistent with the predictions of my theoretical model. Second, I estimate a dynamic oligopoly model of investment and bankruptcy in order to measure BAPCPA’s impact on perceived bankruptcy costs. Third, using the parameters estimated from the structural model, I simulate two counterfactual scenarios. In the first, I simulate equilibrium behavior as though BAPCPA had never been passed, finding an increase

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1See, for example, Iverson (2012); Coelho (2010); Gilson (2010); Ayotte and Morrison (2009); Gottlieb, Klein, and Sussman (2009); Selbst (2008); Herman (2007); Altman and Hotchkiss (2005); and Sprayregen, Cieri, and Wynne (2005).
in industry capacity of about 5%. In the second scenario, I simulate a new equilibrium in which bankruptcy is prohibitively costly, allowing me to measure the overall effect of the Chapter 11 option on industry capacity. I find that eliminating the Chapter 11 option reduces total industry capacity by as much as 20%.

My analysis suggests that BAPCPA may have played a role in the capacity discipline observed in the airline industry since 2005. The phenomenon of capacity discipline has been well documented and discussed in the airline industry since 2006, yet explanations for its persistence have been little more than conjectures. Most observers cite airline consolidation, whereas others point to the disappearing emphasis on market share. Still others say competitors are just more rational nowadays, while most simply take the phenomenon as given. However, my theoretical model suggests a new mechanism: namely, an underlying change in bankruptcy law may have made holding capacity less desirable. My empirical results indicate that BAPCPA may indeed have been a contributing factor in disciplining airline capacity.

The implications of my theoretical model can be extended to other industries. Understanding how airlines react to bankruptcy reform is valuable in its own right, yet my conceptual framework applies to any industry with heavily contractual investment and volatile demand. Steel, auto manufacturing, telecommunications, and even retail conform to this pattern. The capacity discipline engendered by a more creditor-friendly Chapter 11 should correlate positively with an industry’s degree of contract usage and demand volatility, and I hope to test these relationships in future research to better understand BAPCPA’s broader impact on investment.

In sum, this work and its extensions have important and timely implications for bankruptcy lawmakers around the world. Since 2011, the American Bankruptcy Institute’s Commission to Study the Reform of Chapter 11 has heard testimony from legal experts in a variety of fields regarding whether and how the current U.S. Bankruptcy Code should be amended. The Commission is expected to make its final report in December 2014. This multi-year endeavor would greatly benefit from an understanding of how the non-financial provisions of bankruptcy law influence investment behavior outside of bankruptcy. Looking beyond the United States, Halliday and Carruthers (2007), in their study of the globalization of corporate insolvency regimes, document a convergence in bankruptcy law over the past two decades. The authors explain how international institutions, with significant U.S. support, have forged global norms, consequently influencing the lawmaking processes of transitional and developing countries. To the extent that U.S. practitioners and policymakers continue to contribute to global norm making, they must recognize how those norms may impact firm behavior, especially given the crucial role of capital investment for economic growth in developing economies.

The remainder of this paper proceeds as follows: Sections 2 reviews the relevant literature
in industrial organization and corporate finance, while Section 3 provides background on 
bankruptcy law and the airline industry. Section 4 presents a simple theoretical model linking 
reorganization and investment, and Section 5 overviews my three-part empirical strategy for 
analyzing that link. Section 6 describes the capacity, bankruptcy, and profit data I will use. 
Finally, I present and discuss my results in Section 7.

2 Literature Review

A number of studies have combined insights from corporate finance and industrial organi-
zation, yet none has shown how Chapter 11 can influence capital investment in a strategic 
environment. In this section I summarize relevant papers to show how my research combines 
the strategic interaction of industrial organization with the strategic role of bankruptcy in 
corporate finance. My paper also augments the considerable body of work on airline compe-
tition by proposing a new mechanism for capacity discipline.

A rather extensive literature pertains to strategic capacity decisions, and capacity buildup 
is often described as an effective means of deterring entry. Eaton and Lipsey (1979) show 
that anticipated growth leads to buildup of capacity by incumbents that, when compared 
to the decisions of potential entrants, appears premature. Besanko et al. (2010) examines a 
dynamic model of discrete (“lumpy”) capacity investment, in which duopolists precommit to 
soft capacity constraints\(^2\) and then compete in a differentiated products market by setting 
prices subject to their respective constraints. They find that greater product homogeneity 
and capacity reversibility promote capacity preemption races. The authors also link excess 
capacity in the short run to capacity coordination in the long run, and show that capac-
ity preemption races become more intense the more reversible is capital investment. This 
conclusion runs counter to the typical intuition that investment reversibility implies weaker 
commitment, such that the benefits of capacity leadership are transient. On the contrary, 
reversible investment encourages entry into the race to begin with by reducing the cost of 
committing to the race long-term. Hendricks, Piccione, and Tan (1997) demonstrate another 
method of entry deterrence that is more particular to airlines. The authors show that oper-
ating a spoke market at a loss can be a dominant strategy for a hub carrier in response 
to entry by another firm into the spoke market. The network externalities inherent in a 
hub-and-spoke system therefore serve to deter entry. Aguirregabiria and Ho (2012) further 
this notion with their structural model of airline network competition. Takahashi (2011) 
estimates a continuous-time war of attrition among drive-in movie theaters. While the war 
of attrition model seems applicable to airlines’ choice of whether or not to file bankruptcy, 
Chapter 11 is usually filed as means of avoiding exit. The terminal nature of Takahashi’s

\(^2\)“Soft” in this case means that the constraint can be violated at a high cost.
model is therefore inappropriate for examining Chapter 11 reorganization.

Relating price and capacity competition in the airline industry, Snider (2009) develops a dynamic structural model in which cost asymmetries between large and small carriers lead to predatory behavior. He estimates the model to quantify the welfare implications of predation policy in a specific case: the Dallas-Ft. Worth (DFW) - Wichita (ICT) market, one of the four in which the U.S. Department of Justice alleged predatory conduct by American Airlines in 2000. The author’s main goal is to look at the implications of various static cost-based policies used by the courts in determining liability for predatory conduct. Unlike Besanko at al. (2010), Snider’s model treats market-level capacity adjustment as a continuous decision. However, a discrete treatment of capacity may be more appealing, since adding a single seat on a flight may necessitate adding an entire flight. Snider (2009) is one of the few papers I am aware of that combines capacity and price competition in the airline industry. Roller and Sickles (2000) is another, which measures market power using conjectural variation in the European airline industry. The authors employ a two-stage framework in which firms first purchase airplanes, and then compete in prices. Unlike Snider (2009), Roller and Sickles (2000) define capacity in terms of fleet size, as will I.

Linking the financial structure of the firm to product market competition, Brander and Lewis (1985, 1986) describe two effects. The limited liability effect captures the incentive a firm will have to pursue riskier product market strategies because equity holders do not share in downside risk below the point of bankruptcy. The strategic bankruptcy effect captures the incentive for a firm to pursue product market strategies that will increase the likelihood of competitor bankruptcy, which is contingent upon competitors’ financial structures. To isolate the linkages between financial markets and product markets, Brander and Lewis (1986) treat capital investment as fixed, allowing firms to choose their debt/equity ratios in the first stage of a two-stage duopoly model. The limited liability effect they describe is therefore solely due to short-run competition in output effected through changes in variable inputs. Linking capital structure to input decisions is Matsa (2010), which demonstrates how the presence of collective bargaining agreements can impact the choice of debt levels. This relationship is surely present in the airline industry, but it is beyond the scope of this paper. Abstracting from the capital investment decision allows the aforementioned authors to focus on capital structure decisions and to avoid the additional effects of commitment, studied by Dixit (1980), Eaton and Lipsey (1980), Eaton and Eswaran (1984), Brander and Spencer (1983), and others. Whereas Brander and Lewis (1986) and Matsa (2010) linked the financial structure decision with output market strategies holding investment levels fixed, I will abstract from the capital structure decision and hold financial structure fixed, focusing

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The interested reader in corporate finance should review the citations within Brander and Lewis (1985,1986) for foundational articles on capital structure choice, and in particular, for exceptions to the Modigliani and Miller theorems.
Pindyck (1988) demonstrates that irreversibility of investment reduces optimal capacity relative to an environment where investment decisions are reversible. This seminal paper identified the real option value associated with delaying such an investment when demand is uncertain. Jou and Lee (2008) extend earlier analyses in the real options literature to an oligopolistic industry. Their model incorporates choices over capital structure, investment scale and timing, and bankruptcy filing. By treating investments as fixed and bankruptcy as final, however, the authors necessarily abstract away from both the evolution of capital in the industry and the transient nature of bankruptcy. Beginning with Leland (1994) and Leland and Toft (1996), the corporate finance literature has recognized that the decision to liquidate is an endogenous one. Wulin and Wang (2013) and references therein provide a few examples. Broadie et al (2007) extend these models of optimal capital structure by allowing for reorganization under Chapter 11 in addition to liquidation under Chapter 7. Hamoto and Correia (2012) provide a nice overview of the different models of default, liquidation, and bankruptcy, identifying Broadie et al. (2007) as the only paper to incorporate Chapter 11, although several authors separate the default and liquidation decisions. Even in papers where bankruptcy is endogenous, it is typically treated as a decision rule, optimized before other decisions are made, rather than a repeated choice. Jayanti and Jayanti (2011) show that an airline’s bankruptcy filing or a shutdown is good news for equity-holders of rival airlines, while emergence of a carrier from bankruptcy generally reduces rivals’ firm value. These findings together may suggest that a bankrupt carrier’s strategic changes are profitable for everyone, begging the question of why they weren’t made outside of bankruptcy. However, changes in rival firms’ value could simply reflect the market’s valuation of the expected change in earnings due to a competitor’s potential liquidation.

While many authors have examined market competition and entry in airlines, few have touched on capacity investment at the industry level. On the bankruptcy side, papers discussing the airline industry have tended to look exclusively at product market competition (e.g. Borenstein and Rose (1995), Ciliberto and Schenone (2010), Busse (2002)). One of the papers upon which I have drawn heavily for institutional details is Ciliberto and Schenone (2010). These authors examine the effect of bankruptcy on product market competition, concluding that bankrupt airlines reduce prices under bankruptcy protection and increase them after emerging from bankruptcy, while competitors’ prices do not change significantly. The authors also find that bankrupt airlines permanently prune overall route structures, reduce flight frequency and shed capacity. In particular, relative to pre-bankruptcy figures, routes, frequencies, and capacities fall by about 25% under bankruptcy protection, and by another 25% upon emergence from Chapter 11.

\[4\] Busse (2002) also finds that firms in poor financial condition are more likely to reduce prices.
Regarding estimation, Snider (2009) focuses on Markov Perfect Equilibria (MPE) and employs the forward simulation estimator of Bajari, Benkard, and Levin (2007). Ryan (2011) applies the same estimator to an investment game among regional cement plants. Another recent contribution to the estimation of games in the airline industry is Aguerrigabiria and Ho (2012), who analyze a dynamic model of oligopolistic airline competition to identify factors influencing the adoption of hub-and-spoke networks. They find that the cost of entry on a route declines with the airline’s scale of operation at the endpoints of the route, and for large carriers, strategic entry deterrence is also an important factor. Ciliberto and Tamer (2009) develop a method for estimating payoff functions in static games of complete information and apply this method to the airline industry, examining the role played by heterogeneity in determining market structure. Finally, Roberts and Sweeting (2012) consider selective entry into airline markets in order to more accurately assess the impact of airline mergers.

3 Background

In this section I present three elements of background information that together motivate the link between bankruptcy and capacity. First, I explain some of a firm’s key risks and rewards of filing for bankruptcy in the United States. Second, I describe the 2005 bankruptcy law reform in detail. Third, I demonstrate the appeal of Chapter 11 specific to airlines in the U.S., demonstrating that airline bankruptcy patterns are consistent with strategic use of Chapter 11.

3.1 Bankruptcy

The traditional economic justification\(^5\) for bankruptcy protection is as a solution to a collective action problem, namely, the allocation of an insolvent firm’s assets. In the United States, when a firm defaults\(^6\) on a debt obligation, the creditor whose claim is in default has the right to sue for relief in state court. Secured creditors have the additional right to seize the collateral underlying their claims. A financially distressed firm with many creditors is therefore liable to become a tragedy of the commons. When left to its individual legal rights, each creditor has incentive to secure as big a share of the firm’s assets as possible, as quickly as it can, to the detriment of the other creditors and the company’s chances for success. Much like a bank run, this kind of behavior can turn temporary insolvency into complete financial ruin. Bankruptcy law provides a way of collectivizing creditors’ behavior,

\(^5\) See, for example, Jackson (1986).
\(^6\) Note that default need not be due to failure to make payments. Technical default occurs when one of the provisions of the debt agreement is violated (e.g. working capital, cash on hand, or liquidity ratios fall below pre-specified levels).
with the goal of avoiding inefficient firm failures.

To this end, the United States Bankruptcy Code offers two forms of bankruptcy protection to business entities: liquidation under Chapter 7 and reorganization under Chapter 11. Both processes begin with an “automatic stay” that protects the firm from legal action and asset seizure, but they differ in their subsequent treatment of insolvency. Chapter 7 is pursued (voluntarily or otherwise) when a company is unlikely to return to profitability, even with substantially reduced debt obligations. It provides for an orderly closure of the company, sale of assets, and repayment of claims. Chapter 11 is afforded to companies that have a reasonable chance of remaining a going concern, particularly if they renegotiate their obligations to creditors, vendors, employees, tax authorities, and other stakeholders. Under Chapter 11, a financially distressed corporation can typically negotiate away substantial portions of debt and other liabilities, sometimes on the order of cents on the dollar.

The courtroom is not the only place a firm’s financial distress can be resolved, of course. Litigation is costly, and most secured creditors would prefer to continue receiving debt payments than to own the underlying collateral. Consequently, debt renegotiations (called work-outs) are common in the U.S. However, as White (2007) points out, the negotiation process is imperfect, and work-outs can be easily derailed by hold-out creditor classes. In their study of 169 instances of financial distress among large public corporations in the 1980s, Gilson, John, and Lang (1990) find that slightly less than half (80) of firms successfully restructure their debt outside of bankruptcy. Success was more likely when firms had greater intangible assets, a higher proportion of bank debt, and fewer distinct creditor classes.7 The 89 unsuccessful firms in the study all filed for Chapter 11. In the remainder of this section, I briefly explain the overall process of Chapter 11 and Chapter 7 and describe the history of bankruptcy law in the United States. I then point out the most relevant provisions in the current Bankruptcy Code and describe how these and other rules were changed by BAPCPA.

3.1.1 The Bankruptcy Process

As mentioned above, business entities typically file under one of two chapters in the U.S. Bankruptcy Code: Chapter 7 (liquidation) and Chapter 11 (reorganization). Both procedures begin with an automatic stay to prevent asset seizure and litigation, but they have very different end goals. I now present a rough overview of both processes. For more thorough treatment, see White (2007), LoPucki (2012), and Branch et al. (2007).

Under Chapter 7, a court-appointed or elected trustee manages the orderly shutdown and liquidation of the company. The trustee’s goal is to convert the company’s assets to

7Debt restructuring outside of bankruptcy typically requires unanimous consent of all creditors whose claims are in default, so the likelihood that at least one creditor holds out increases in the number of creditors.
cash as quickly as possible, while seeking to maximize the value received for those assets. Since even distressed companies are typically worth more than the sum of their parts, sale of substantially all of the firm’s assets to a single party is not uncommon. The proceeds are then distributed to claimants according to the Absolute Priority Rule (APR). Also known as liquidation preference, the APR dictates the order in which unsecured claims are paid and stipulates that no class of creditor be paid until all more senior classes have been paid in full. In order of priority, the major divisions are as follows:

1. Administrative Claims (including legal fees)
2. Statutory Claims (including certain unpaid taxes, rents, wages, and benefits)
3. Unsecured Creditors’ Claims (including trade credit, bonds, and legal claims)
4. Post-filing Interest on Paid Claims
5. Equity

Secured creditors are notably absent from the APR ordering because their claims on particular assets remain valid in bankruptcy. Creditors with secured claims are entitled to their collateral or its fair market value (usually replacement value) before any unsecured claims are paid.

Whereas Chapter 7 outlines the orderly paying of creditors’ claims, Chapter 11 provides an orderly way to renegotiate those claims. While the ultimate goal of Chapter 11 reorganization is reemergence from bankruptcy as a going concern, many firms are unsuccessful. Failure can take two forms, conversion or dismissal, each of which results from the bankruptcy judge’s approval of the specified motion. A motion to convert the case to Chapter 7 will, if granted, lead to liquidation. A motion to dismiss the case will, if granted, lift the automatic stay and remove the proceeding from Bankruptcy Court. In the case of dismissal, negotiations with creditors can continue, but as previously mentioned, creditors now have the option to seize collateral or sue the debtor in state court. Iverson (2012) and Morrison (2005) indicate that, in most cases, dismissal is tantamount to liquidation.

Chapter 11 centers on the firm’s reorganization plan, which outlines debt repayment and restructuring. The plan must also estimate firm value as a going concern and show that it exceeds liquidation value. Upon proposal, the judge must first approve the disclosure statement (the plan), before it can be voted on by creditors. If, at each level of seniority, at least 50% of creditors by number and 2/3 of creditors by value accept the plan, then it is deemed accepted by that class. Note that, in order to vote, a creditor must be impaired, in that it will receive less than 100% recovery under the plan. Even after creditors have voted on the plan, the judge still has the ability to approve or reject the plan. Most commonly, the
judge may approve a plan that was voted down if he or she feels that doing so is in the best interest of the firm. Such a decision is known as a “cram-down” and requires that the plan be feasible, filed in good faith, and superior to liquidation in terms of creditors’ recovery.

A reorganization plan need not be approved on the first try (or the second or third, for that matter). The number of attempts is really only limited by the time and patience of the bankruptcy judge. For the first 120 days of bankruptcy, the debtor is given the exclusive right to file a reorganization plan. Often 120 days will be far from enough time to formulate a plan that is agreeable to all parties, so a judge may grant extensions of this exclusivity period if he or she sees fit. The scope of these extensions is perhaps the most substantive change imposed by BAPCPA. Whereas exclusivity was effectively indefinite before 2005, BAPCPA put a hard deadline at 18 months of exclusivity. Once this period expires, any creditor group or case trustee may file an alternative plan and seek approval. The figure below illustrates the overall bankruptcy process.
3.1.2 Bankruptcy Provisions of Interest

A number of sections in the Bankruptcy Code are of particular import in the context of the airline industry. Section 1110 affords special provisions to holders of leases and secured financings of aircraft and aircraft equipment. This section gives bankrupt airlines the right to make any outstanding payments within 60 days in order to keep the aircraft. If the airline fails to make those payments or renegotiate lease terms, the lessor has the exclusive right to repossess the aircraft, similar to a secured creditor’s position outside of bankruptcy. At first glance, this rule appears to favor the lessor. However, lease agreements are often far above market value for aircraft, and if a lessor repossesses the aircraft, it must then find another lessee in what is likely to be a down market. Repossession is therefore not a very attractive option for lessors. Moreover, should the lessor refuse the right to repossess the aircraft, the lease agreement is rescinded and becomes an unsecured claim on the airline, which takes a much lower priority for payment under bankruptcy protection. The lessor is therefore far less likely to be paid. Given the lessor’s grim options in the case of default, renegotiation of lease terms becomes very attractive. Renegotiating leases and secured financings of aircraft is a major source of cost-cutting by airlines in bankruptcy. Benmelech and Bergman (2008) show that renegotiation of aircraft leases is common practice for airlines in financial distress. Moreover, when redeployability of aircraft is low, as in an overall market downturn, lessors are able to negotiate for even greater concessions.

Section 1113 of the Bankruptcy Code relates to collective bargaining agreements (CBAs). This section of the Code was enacted in 1984, although bargaining power would have been similar before this time, given the contractual treatment of CBAs. Section 1113 stipulates that a company can unilaterally revise terms of a CBA if attempts to renegotiate with unions have failed. This rule gives airlines significant bargaining power in negotiating more favorable terms with unions, which typically represent half of an airline’s workforce.

Section 1114 of the Bankruptcy Code deals with retiree benefits. Under bankruptcy protection, a carrier can renegotiate or cancel defined benefit pension obligations, thereby requiring the Federal Pension Benefit Guarantee Corporation (PBGC) to foot the bill with taxpayer money. Such a decision must first be approved by the court, which requires 1) that the company first negotiate with representatives of the retirees, and 2) that the decision is necessary for the firm’s survival. Since defined benefit pension programs typically represent a huge burden on financially distressed carriers, renegotiating or cancelling them in Chapter 11 can yield enormous cost savings.
3.1.3 History of Bankruptcy Reform

Several times throughout the 19th century, the U.S. Congress established and repealed bankruptcy legislation, but not until the Bankruptcy Act of 1898 did any set of rules gain permanence.\(^8\) Compared to its creditor-friendly predecessors, the 1898 Act clearly favored debtors, imposing no minimum payment to creditors and establishing debt forgiveness as standard procedure for consumer debtors. The 1898 Act permitted nonbusiness debtors to file voluntarily, while creditors could petition for involuntary bankruptcy against businesses. The 1898 Act also included provisions for an alternative to liquidation for bankrupt corporations, whereby, with the approval of creditors and the court, partial repayment of the corporation’s debt would discharge the entire debt. While bankruptcy law experienced minor changes in subsequent decades, the next major change was the Chandler Act of 1938, which, among other things, rewrote the reorganization provisions into distinct chapters, including chapter X for corporate reorganizations and chapter XI for rearrangements. The Bankruptcy Reform Act of 1978, aimed at streamlining administration and ensuring fairness among classes of creditors, was the last major overhaul of bankruptcy legislation. Among its many changes were the combination of chapter X and chapter XI into chapter 11 for all corporate reorganizations and the removal of long-held caps on attorneys’ fees. The 1978 Act, commonly referred to as the Bankruptcy Code, was such a substantial change that its effect on filing rates has been the subject of considerable research.\(^9\) The Bankruptcy Amendments and Federal Judgeship Act of 1984 (BAFJA) made changes to the bankruptcy judiciary and added provisions to deter bankruptcy abuse by consumers. The Bankruptcy Reform Act of 1994 established a commission to review the Bankruptcy Code, and that commission eventually proposed BAPCPA, widely viewed as the most substantial change since 1978.

3.2 BAPCPA 2005

In order to say whether or not bankruptcy law matters for investment, I need to observe the investment response to an exogenous change in bankruptcy law. To do so I exploit the Bankruptcy Abuse Prevention and Consumer Protection Act of 2005 (BAPCPA). This section summarizes the reform, providing evidence that it increased the expected cost of filing Chapter 11, especially for larger firms.

Although its primary target was consumer bankruptcy abuse, BAPCPA made a number of substantive changes to Chapter 11. A 2005 report by BBC News summarizes the common opinion that the changes were designed to prevent large corporations’ abuse of the

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\(^8\)This brief summary is largely due to Bak, Golmant, and Woods (2008).

\(^9\)See, for example, Bhandari and Weiss (1993), Domowitz and Eovaldi (1993), White (1987), Boyes and Faith (1986), and Nelson (2000)
bankruptcy option by making Chapter 11 filings more difficult. Coelho (2010) finds that the market response to public announcements of bankruptcy filing has been more severe since the reform relative to the pre-BAPCPA period, lending empirical validity to what Gilson (2010) and many other scholars had already agreed upon: the new Bankruptcy Code restricts debtor protection and reduces the likelihood of a successful reorganization. In support hereof, Coelho (2010) cites Altman and Hotchkiss (2005); Gottlieb, Klein and Sussman (2009); and Ayotte and Morrison (2009) as well. Iverson’s (2012) conclusion that busy judges more often leave firms to their own devices agrees with the creditor-friendly perception of BAPCPA. Since bankruptcy court judges see both business and consumer cases, the drastic decline in consumer bankruptcy filings following BAPCPA substantially reduced judges’ overall caseloads. Iverson (2012) identifies this effect and suggests that judges with lighter caseloads are more inclined to dismiss or convert Chapter 11 cases, thereby increasing the probability of liquidation.\footnote{Note that Iverson agrees that BAPCPA likely had an impact on Chapter 11 filings, although the filing rate overall doesn’t appear to have been affected by the reform, at least not in his sample. This seems to conflict with the UCLA Lopucki database of large public filings.}

Even before the reform went into effect, it was commonly expected to shift bargaining power to creditors. While uncertainty surrounded the manner in which BAPCPA would eventually be implemented in the courts, the consensus among legal professionals was that BAPCPA would probably be bad for debtors, especially large ones and those with particular classes of assets. I now detail the reform components most relevant for large companies, relying collectively on Sprayregen, Cieri, and Wynne (2005), Herman (2007), Selbst (2008), and Levin (2005).

### 3.2.1 Changes of Interest

First, and perhaps most important, was the Act’s limitation of the exclusivity period for filing a plan of reorganization. The exclusivity period is the time during which the company has the sole right to put forth a plan of reorganization for consideration by stakeholders. Once the exclusivity period has expired, other parties, such as creditor committees or labor unions, can put forth alternative plans and call for a vote. Under the old regime, large corporations were regularly granted extensions lasting up to several years. United Airlines, for example, required three years before a reorganization plan was confirmed. The 2005 reform set a hard and fast limit of 18 months for exclusivity, and 20 months for acceptance of an exclusive plan. Selbst (2008) explains, “The change was aimed at curbing the perceived abuse of debtors spending too long in Chapter 11 and using exclusivity to coerce concessions from creditors.” This new limit increases the likelihood of losing exclusivity, especially for large companies.
In a 2005 report by law firm Kirkland & Ellis, James Sprayregen and co-authors explained that, “...in many cases, changes in collective bargaining agreements and pension plans...and similar issues cannot be resolved in 20 months.” Before the reform came into effect, airlines were about twice as likely as other firms to exceed the maximum threshold for acceptance of a plan. In other words, the BAPCPA’s change to exclusivity was likely to have a greater impact on airlines than on industry in general.

Coupled with the reduced exclusivity period is a slightly increased scope for dismissal or conversion of a bankruptcy case. By limiting the discretion of the bankruptcy judge, the Act made it more likely for courts to convert a reorganization into a liquidation if procedural requirements are not met. Firms are not only more likely to lose control of the reorganization process by losing exclusivity, but also more likely to lose reorganization as an option in the event of dismissal or conversion. Mitigating these changes was the relaxation of certain procedural requirements for prepackaged plans. However, prepackaging benefits are unlikely to change the net effect of these timing-related reforms.

The second key reform area is employee wages and benefits. One of the more prominent features of the Act was its limitation of key employee retention plans (KERP). This measure was enacted to curb the abuse of such plans as a means of paying out insiders of the company before its coffers were empty. While it likely accomplishes that goal, the limitation is applied broadly to insider payments, which may have made it more difficult for large corporations to retain key employees. Related to the limitation on insider payments is an increase in the required payments to rank-and-file employees. Among other changes, BAPCPA doubled the maximum amount of priority wage and benefit claims per worker and the timeframe for recovery, from about $5,000 to $10,000 and from 90 days to 180 days, respectively. Given that labor costs represent about 1/3 of most airlines’ operating expenses, this change likely moved a large sum of money higher on the priority claims list. Another change to the handling of benefits was the Act’s permission to, at the request of stakeholders, unwind any modification made to retiree benefits in the 180 days prior to filing for Chapter 11, provided that the company was insolvent when the modification was made. This change essentially allows the court to reverse any reduction in benefits made before the company filed for bankruptcy. Important to note is that section 1114 permits unilateral modification (including wholesale cancellation) of retiree benefits if negotiations fall through and the court finds the modification to be necessary for the firm’s survival. BAPCPA essentially grants

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11 Sprayregen’s relevant reorganization expertise includes representation of United Airlines, Japan Airlines, and Trans World Airlines (TWA).

12 Among similarly-sized public companies filing for Chapter 11 between 1980 and 2005 that eventually emerged from bankruptcy, 32% of non-airline companies took longer than 608 days (the new statutory maximum) to confirm an exclusive plan of reorganization, versus 62% of airline companies during this time. Median time spent in reorganization for airlines was also about twice that of non-airline bankruptcies. This qualitative observation is independent of firm size.
employees greater bargaining power under section 1114. An important change outside of BAPCPA in this regard is the Pension Protection Act of 2006 (PPA). While I do not cover it in any detail in this paper, PPA essentially increased the cost to the firm of both carrying and terminating underfunded pensions. The reform may very well have compounded the effects of BAPCPA.

The third major reform category is nonresidential property leases. In particular, the Act limits the timeframe for the assumption or rejection of such leases. Similar to its change in the exclusivity period, BAPCPA overrides the status quo of unlimited extensions by setting a 120-day limit with at most one 90-day extension. Any leases not assumed by the end of this period are deemed rejected. For airlines, this provision applies directly to airport gates or terminals, forcing airlines to decide much sooner whether to remain at certain airports. It should be noted, however, that the Act simultaneously eliminated certain provisions pertaining to airport gate leases in the same section. For instance, the reform deleted the requirement to take all or none of the leased gates at an airport. It is unclear how important these deletions are relative to the overall change in the timeline for accepting leases.

Finally, BAPCPA raised priority for recovery of recently delivered goods, utility costs, and taxes. Both the amount and timeliness of these payments were substantially increased, placing a greater cash burden on companies during the bankruptcy process. Given the prevalence of fuel costs and taxes in the airline industry, it is possible that these changes reduced the likelihood of successfully exiting Chapter 11.

On the whole, the 2005 reform appears to have increased the probability of liquidation, thereby raising the expected cost of filing Chapter 11 from the firm’s perspective. I must point out, however, that sections 1110 and 1113, which represent the two biggest benefits of Chapter 11 to airlines, remain untouched. Nevertheless, the overall strengthening of creditors’ bargaining positions is generally accepted. In fact, even unintended consequences of the reform may have yielded a more creditor-friendly system. As mentioned above, Iverson (2012) associates the decline in consumer bankruptcy filings following BAPCPA with higher probability of dismissal or conversion for Chapter 11 cases. Finally, my own conversations with legal experts confirm that, at least for the largest of firms, BAPCPA’s curtailment of the exclusivity period turned the process of reorganization into almost assured liquidation.

Other, non-legislative changes are also worth noting. Bharath, Panchapegesan, and Werner (2010) identify an overall decline in absolute priority rule (APR) deviations from 10% of firm value to about 2% of firm value, or from 100% of the time to less than 20% of the time. A concomitant rise in the use of debtor-in-possession (DIP) financing and key employee retention plans (KERP) is observed and found to be related to the decline in APR deviations. DIP financing, which came to prominence in the 1990s, tends to impose rigid
restrictions on firm operations, thereby limiting the power of management, while KERPs often align management incentives with creditors. If BAPCPA did indeed enhance the bargaining position of creditors, then DIP financing terms are likely to be even more favorable to creditors. To the extent that KERPs serve as an alternative means of paying out management in reorganization, these two trends could very well have left management’s incentive to reorganize unchanged. Bharath et al. (2010) consider both innovations to have led to more creditor-friendly reorganizations. These authors also note that management turnover in bankruptcy has become more common, especially among managers with significant equity stakes. Yet another trend in Chapter 11 cases has been the increase in Section 363 sales, in which the entire company is sold to an outside party. If we view managers as the ones making investment decisions, this trend coincides with the effects of BAPCPA. A shift of bargaining power toward creditors and an increased likelihood of acquisition under Chapter 11 will both increase a manager’s perceived cost of filing for bankruptcy.

### 3.3 The U.S. Airline Industry

Empirical study of the airline industry is extensive. Air travel is economically critical, data is plentiful, and industry profitability is uniquely terrible. The industry therefore provides fertile ground for asking and answering interesting questions about market behavior. Borenstein and Rose (2008) provide an excellent overview of the domestic commercial airline industry. Much of what follows regarding the history of the industry is largely due to those authors.

Following World War I, military interest in a healthy aviation sector spurred subsidies for fledgeling airlines. Early industry fragmentation sparked government concern over destructive competition, prompting indirect regulation aimed at promoting a network of large national carriers. The U.S. Post Office, by selectively awarding airmail contracts, was in fact the primary seat of indirect control. Direct regulation of the airline industry, including prices, entry, and merger decisions, began in 1938 with the creation of the Civil Aeronautics Board (CAB), which would eventually become the Federal Aviation Administration (FAA). The realized goal of the CAB was to develop and insulate a system of large national (“trunk” or “legacy”) carriers and to regulate entry by smaller airlines offering local service. In pursuit of that goal, prices were set comfortably above marginal cost. Regulated airlines did not enjoy the associated profits, however. Since price competition was restricted, airlines tended to compete on various dimensions of quality, including flight frequency and in-flight service. Moreover, airlines were frequently prohibited from charging lower fares for older planes, speeding industry-wide adoption of new aircraft. Both factors reduced capacity utilization (herein measured by load factor, the number of purchased seats divided by the number of available seats) and increased average cost per seat-mile. In short, airlines competed marginal costs up to the level of prices, dissipating the profits targeted by the CAB.
By the early 1970s, the CAB had developed a sufficiently negative reputation, and public dissatisfaction with regulation in general had grown sufficiently potent, that the U.S. Senate Judiciary Committee began hearings on airline deregulation. Further supported by Senate leaders, economists, and the leadership of the CAB itself, the eventual result was the Airline Deregulation Act of 1978. The Act eliminated price and entry regulation and provided for the eventual closure of the CAB (by 1985), although the FAA continues to regulate operational and safety functions. The Essential Air Service program, which subsidizes and oversees service to small communities, also still exists.

Following deregulation, commercial air travel experienced a wave of entry into the industry by new carriers and expansion by existing regional airlines for several years until the recession of the early 1980s, which prompted a spate of bankruptcies and mergers. Market-level entry flourished as well, reducing concentration and competing down fares, especially on longer distance markets that could be served by many carriers offering a variety of connections. Reiss and Spiller (1989) estimate a static model of entry and fare competition on direct and indirect routes and find that competition from indirect routes can dramatically affect fare determination and entry on direct routes. Entry was shown to be even easier for airlines that already had a foothold at a given airport. Berry (1992) uses a static entry model to show that market share at the origin airport is a strong determinant of entry into other destination markets out of that airport. Along similar lines, Berry (1990) demonstrates the importance of an airline’s presence at the endpoint cities on both the demand and supply side.

As detailed in Borenstein (1992), deregulation also led to widespread adoption of hub-and-spoke operating networks, which allowed carriers to better utilize capacity and increase non-stop flight frequency to and from hub airports. The proportion of connecting service has consequently outpaced the growth in overall traffic. The prevalence of the hub-and-spoke system has prompted an abundance of research on its implications for competition. Borenstein (1989, 1991), among others, shows that carriers with dominant airport-level market shares tend to have increased market power on routes out of those airports, and higher market-level shares are associated with higher markups. More recent work by Borenstein (2011) indicates that the price premium due to strong airport presence has declined in recent years. On the cost side, Brueckner, Dyer, and Spiller (1992) study the impact on airfares of economies of density in hub-and-spoke networks, while Mayer and Sinai (2003) study the effect of hubbing on air traffic congestion. Berry, Carnall, and Spiller (2006) find evidence of economies of density only on longer routes. They also shed further light on the demand side by estimating a model with customer heterogeneity, determining a hub carrier’s markup ability to be largely tied to price-inelastic business travelers. Another prominent outcome of deregulation was the substantial increase in load factors, which hovered around 55% at the
height of regulatory oversight. Initially prodded upwards by cost competition, they continued steadily higher, fueled by advances in computerized ticketing and Internet sales, until reaching nearly 80% for some carriers in the mid-late 2000s. Dana and Orlov (2010) examine Internet penetration as a determinant of airline capacity utilization, hypothesizing that the availability of online information about price and product alternatives reduces friction in the market for air travel.

3.3.1 Airline Bankruptcy

Airline bankruptcy and airline capacity are inextricably linked. Every legacy air carrier has undergone bankruptcy. Just in the past decade, United Airlines (UA), US Airways (US), Delta Air Lines (DL), Northwest Air Lines (NW), and American Airlines (AA) have filed for Chapter 11 protection, each time ranking among the top ten largest bankruptcies of the year by asset value. Ciliberto and Schenone (2010), Benmelech and Bergman (2008), and others demonstrate that bankruptcy is a common time to cut capacity and right-size the labor force. As discussed above, a number of provisions in the Bankruptcy Code make Chapter 11 especially appealing for airlines looking to downsize. If abrogating contracts in Chapter 11 is less costly than breaching them outside of bankruptcy court, then firms will be more willing to sign those contracts in the first place (i.e. invest in capacity) relative to their behavior in a world without Chapter 11. The pattern of rapid investment followed by extensive bankruptcy that we would expect to find is clearly evident in the airline industry.

Not only is bankruptcy a valuable option, but there is evidence to suggest it may be strategically timed. In an interview with broadcast journalist Charlie Rose, former CEO of American Airlines Robert Crandall suggests that the company should have chosen to file for Chapter 11 during the earlier wave of bankruptcies by large legacy carriers. “I would have done it then because I knew that [the other major airlines] would emerge with a huge cost advantage,” he says. More than just a voluntary strategy for managing financial distress, the bankruptcy option can also be misused. Delaney (1992) details Continental Airlines’ 1983 bankruptcy filing, starkly illustrating its strategic intent and abusive nature. The more general case for bankruptcy’s strategic nature is debatable. Flynn and Farid (1991) and Tavakolian (1995) argue that bankruptcy has lost much of its previous stigma and grown into a viable business strategy for turning around failing companies. Moulton and Thomas (1993) provide empirical evidence that, if it is a deliberate strategy, it is not usually a successful one.

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14 http://www.charlierose.com/view/interview/12228
15 The interested reader is referred to Ciliberto and Schenone (2010) for additional evidence of the strategic use of bankruptcy in airlines.
Perhaps the best evidence for both the strategic timing of Chapter 11 filings and the potential impact of BAPCPA on bankruptcy costs is the fact that both Delta Air Lines and Northwest Airlines independently filed for Chapter 11 in September of 2005, just one month before BAPCPA came into effect. Industry experts claim that BAPCPA played a key role in Northwest’s decision, and that Delta’s filing was long expected, suggesting the company had sufficient ability to time the decision.\footnote{See, for example, Maynard (2005) and Corridore (2005).}

4 Theoretical Model

In this section I develop a simple dynamic duopoly model of investment and bankruptcy to show how equilibrium behavior changes with bankruptcy cost. The purpose of this exercise is to provide a transparent framework for thinking about how Chapter 11 influences capital investment in a dynamic, competitive environment. This simple model reveals two key insights. First, an exogenous change that makes bankruptcy more costly will limit capacity expansion when demand is high. Second, the same exogenous change will quicken capacity retraction when demand is low. While the intensity of each effect varies with the relative dominance of each firm, the overall implication lines up nicely with the capacity discipline that has been observed in the airline industry. In other words, as bankruptcy becomes more costly, firms will be less willing to invest when demand is good. At the same time, they’ll be more willing to get rid of capacity when demand is bad.

4.1 Duopoly Setup

Two firms compete for demand, which can be either high or low. The demand state evolves randomly according to two Poisson arrival processes. When demand is high, nature arrives at rate $\psi$ to reverse the demand state. When demand is low, nature arrives at rate $\psi'$ to reverse the demand state. Suppose firm $i$’s profit, conditional upon demand, can be given in reduced form by a function of $i$’s capital level, $n_i$, relative to its competitor. This relative level takes on one of 5 values, that is, $n_i \in N \equiv \{-2, -1, 0, 1, 2\}$. Flow profit is given by

$$\Pi(n_i) \in \{\pi_{-2}, \pi_{-1}, \pi_0, \pi_1, \pi_2\}, \quad \pi_{n+1} > \pi_n \text{ when demand is high, and}$$

$$\Pi'(n_i) \in \{\pi'_{-2}, \pi'_{-1}, \pi'_0, \pi'_1, \pi'_2\}, \quad \pi_n > \pi_{n+1} \text{ when demand is low.}$$

In other words, having more capital relative to your opponent is profitable in high-demand states, but costly in low-demand states.\footnote{Large size could be costly in downturns if, for example, fixed costs are linear in capacity, while variable profits are concave. If the demand state shifts variable profit only, then fixed costs may very well dominate.} Given this ordering, firms will want to increase
their capital stock in good times, and decrease it in bad times. When demand is high each firm can increase its capital level by a Poisson investment process, which yields a unit increment to the capital stock at rate \( x_i \geq 0 \) and costs \( \lambda x_i \). Similarly, when demand is low each firm can decrease its capital level at rate \( y_i \geq 0 \) at a cost of \( \theta y_i \).

In each demand-capital state, default follows two Poisson processes, one yielding a single increment decrease in capital, and another resulting in a two-increment decrease. That is, firms never liquidate but are occasionally forced to downsize by one or two units, where applicable. The overall rate of default is held constant for a given demand and capital state, such that

\[
D_{N_i} \equiv \begin{cases} 
    d_2 &= \gamma_2 d_2 + (1 - \gamma_2) d_2 \\
    d_1 &= \gamma_1 d_1 + (1 - \gamma_1) d_1 \\
    d_0 &= \gamma_0 d_0 + (1 - \gamma_0) d_0 \quad \text{when demand is high, and} \\
    d_{-1} &= d_{-1} \\
    d_{-2} &= 0
\end{cases}
\]

\[
B_{N_i} \equiv \begin{cases} 
    b_2 &= \phi_2 b_2 + (1 - \phi_2) b_2 \\
    b_1 &= \phi_1 b_1 + (1 - \phi_1) b_1 \\
    b_0 &= \phi_0 b_0 + (1 - \phi_0) b_0 \quad \text{when demand is low,} \\
    b_{-1} &= b_{-1} \\
    b_{-2} &= 0
\end{cases}
\]

where \( \gamma_n \) and \( \phi_n \) describe the probability that default will be of the two-increment type. Upon default, firms must pay a capital-dependent, one-time fee reflecting the cost of bankruptcy to equity holders. These restructuring costs, \( R(n_i) \in \{R_{-1}, R_0, R_1, R_2\} \) are independent of both the demand state and the size of default, and they are not paid when firms transition to lower states of their own accord.

Finally, suppose the common rate of time preference is given by \( r > 0 \). Given this set of incentives and processes, let \( V \) represent value functions in good states and \( W \) represent when demand is low.
value functions in bad states. We can then define firm values recursively as follows

\[
\begin{align*}
\text{rV}_2 &= \pi_2 + x_{-2} [V_1 - V_2] + (1 - \gamma_2) d_2 [V_1 - V_2 - R_2] + \gamma_2 d_2 [V_0 - V_2 - R_2] + \psi [W_2 - V_2] \\
\text{rV}_1 &= \max_{x_1 \geq 0} \left\{ \pi_1 - \lambda x_1 + [x_1 + d_{-1}][V_2 - V_1] + x_{-1} [V_0 - V_1] + ... \right. \\
&\quad \left. + (1 - \gamma_1) d_1 [V_0 - V_1 - R_1] + \gamma_1 d_1 [V_{-1} - V_1 - R_1] + \psi [W_1 - V_1] \right\} \\
\text{rV}_0 &= \max_{x_0 \geq 0} \left\{ \pi_0 - \lambda x_0 + [x_0 + (1 - \gamma_0) d_0] [V_1 - V_0] + x'_0 [V_{-1} - V_0] + ... \right. \\
&\quad \left. + (1 - \gamma_0) d_0 [V_{-1} - V_0 - R_0] + \gamma_0 d_0 [(V_2 - V_0) + (V_{-2} - V_0 - R_0)] + \psi [W_0 - V_0] \right\} \\
\text{rV}_{-1} &= \max_{x_{-1} \geq 0} \left\{ \pi_{-1} - \lambda x_{-1} + [x_{-1} + (1 - \gamma_{-1}) d_1] [V_{-1} - V_{-1}] + x_1 [V_{-2} - V_{-1}] + ... \right. \\
&\quad \left. + d_{-1} [V_{-2} - V_{-1} - R_{-1}] + \gamma_{-1} d_1 [V_{-1} - V_{-1}] + \psi [W_{-1} - V_{-1}] \right\} \\
\text{rV}_{-2} &= \max_{x_{-2} \geq 0} \left\{ \pi_{-2} - \lambda x_{-2} + [x_{-2} + (1 - \gamma_{-2}) d_2] [V_{-2} - V_{-2}] + \gamma_{-2} d_2 [V_{-2} - V_{-2}] + \psi [W_{-2} - V_{-2}] \right\}
\end{align*}
\]

The left-hand side of each equation represents the rate of appreciation of the firm’s value. On the right-hand side of each equation, the first term is flow profit. For equations with maximization, the second term is the cost of (dis)investment. The remaining terms give the probabilities of each possible state change multiplied by their associated changes in continuation value. Note that restructuring costs are one-time values, which is why they appear only when continuation values change due to default. We assume that default rates are not so large as to make (dis)investment unappealing. Solving for equilibrium investment and disinvestment intensities yields the following:18

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18See Appendix for additional details on the solution.
4.2 Duopoly Implications

Solving for equilibrium investment and disinvestment strategies reveals the two key features of capacity discipline at work: Higher bankruptcy costs slow investment in high-demand states and speed disinvestment in low-demand states. Intuitively, higher bankruptcy costs make disinvestment more expensive overall, increasing the risk of being large in a down market, thereby reducing the incentive to invest. At the same time, disinvestment outside of bankruptcy becomes less expensive relative to bankruptcy, leading to quicker retraction outside of bankruptcy. The magnitude of each effect depends on the nature of competition between the duopolists. In particular, the disinvestment effect is stronger for more dominant firms, while the investment effect is stronger for weaker firms.

The equations above give explicit expressions for optimal investment/disinvestment, which we can analyze to determine the impact of a change in bankruptcy policy. I view BAPCPA as increasing the cost of reorganization conditional upon filing, which is best proxied by an

\[ x^* \] maximizes
\[ \max \left\{ 0, \frac{\pi_2 - \pi_2 - R_2 d_2 - 4\theta \psi}{\lambda} - (4(r + \psi) + 2(1 + \gamma_2)d_2) \right\} \]
\[ x^* \] maximizes
\[ \max \left\{ 0, \frac{\pi_1 - \pi_2 - R_1 d_1 - 3\theta \psi}{\lambda} - (3(r + \psi) + (1 + \gamma_2)d_2 + (1 + \gamma_1)d_1 - d_{-1}) \right\} \]
\[ x^* \] maximizes
\[ \max \left\{ 0, \frac{\pi_0 - \pi_2 - R_0 d_0 - 2\theta \psi}{\lambda} - (2(r + \psi) + (1 + \gamma_2)d_2) \right\} \]
\[ x^* \] maximizes
\[ \max \left\{ 0, \frac{\pi_{-1} - \pi_2 - R_{-1} d_{-1} - \theta \psi}{\lambda} - ((r + \psi) + (1 + \gamma_2)d_2 - (1 + \gamma_1)d_1 + d_{-1}) \right\} \]
\[ x^* \] equals 0

\[ y^* \] equals 0
\[ y^* \] maximizes
\[ \max \left\{ 0, \frac{\pi_1' - \pi_2' + R_2 b_2 - R_1 b_1 - \lambda \psi'}{\theta} - ((r + \psi') + (1 + \phi_2)b_2 - (1 + \phi_1)b_1 + b_{-1}) \right\} \]
\[ y^* \] maximizes
\[ \max \left\{ 0, \frac{\pi_0' - \pi_2' + R_0 b_2 - 2\lambda \psi'}{\theta} - (2(r + \psi') + (1 + \phi_2)b_2) \right\} \]
\[ y^* \] maximizes
\[ \max \left\{ 0, \frac{\pi_{-1}' - \pi_2' + R_{-1} b_{-1} - 3\lambda \psi'}{\theta} - (3(r + \psi') + (1 + \phi_2)b_2 + (1 + \phi_1)b_1 - b_{-1}) \right\} \]
\[ y^* \] maximizes
\[ \max \left\{ 0, \frac{\pi_{-2}' - \pi_2' + R_{-2} b_{-2} - 4\lambda \psi'}{\theta} - (4(r + \psi') + 2(1 + \phi_2)b_2) \right\} \]

While these effects do not account for BAPCPA’s impact on steady-state equilibrium industry structure, the Appendix shows that the qualitative implications of this section continue to hold when we weight intensities by long-run probabilities.
increase in the one-time restructuring costs \{R_n\}. The first and most intuitive effect of such a change is to reduce investment intensity during high-demand periods, as seen by \(\frac{\partial x^*}{\partial R_{n}} < 0\). A greater reluctance to invest in upturns is one component of the capacity discipline observed in the market since 2005. This effect is stronger when investment costs are smaller and when the arrival rate of default is higher. Based on the changes it makes to the Bankruptcy Code, BAPCPA is expected to have greater impact on the expected restructuring costs of the largest firms. If we further suppose that BAPCPA has a larger impact on larger firms, such that \(\Delta R_n > \Delta R_{n-1}\), we should expect the investment effect to be strongest for small firms and weakest for large firms.

The other component of capacity discipline is greater eagerness to disinvest during downturns, which we find in \(\frac{\partial y^*}{\partial R_2} > 0\). However, this effect is tempered by the restructuring cost change at lower levels. If we again assume that \(\Delta R_n > \Delta R_{n-1}\), then the overall effect of BAPCPA will indeed be faster disinvestment. Moreover, the effect will be stronger the larger is the firm. A few more intuitive observations:

- Investment in good times decreases with the arrival rate of bad times, while disinvestment in bad times falls with the arrival rate of good times.
- Investment in good times falls with the price of investment, while disinvestment in bad times falls with the cost of disinvestment.
- Disinvestment in bad times falls with the arrival rate of default for the largest firm, as well as with the probability of “big” default for the largest firm.

Finally, capacity discipline could be further amplified through the ancillary effects of restructuring cost on negotiation with labor groups. That is, if the bankruptcy change leads to greater likelihood of liquidation, union members may be more inclined to agree to pay cuts to avoid losing their jobs. BAPCPA certainly did plenty to enhance bargaining power of employees relative to equity holders, which suggests the opposite effect. However, the reform may have done so much to help creditors that the pie split amongst equity holders and employees is much smaller.

## 5 Empirical Strategy

My empirical approach to studying the link between bankruptcy and investment is three-fold. First, I perform a difference-in-differences analysis of airline data to test whether investment behavior changed following a 2005 bankruptcy law reform. Second, I estimate a dynamic, structural model of investment, competition, and bankruptcy to measure the incremental firm-level cost due to that reform. Finally, I use the estimated parameters to simulate two
counterfactual scenarios in which 1) bankruptcy is effectively prohibited, and 2) BAPCPA was never enacted.

5.1 Difference-in-Differences Model

The comparative statics of my theoretical model suggest that an increase in bankruptcy cost will reduce overall investment. The figures on the previous pages show that investment has fallen since BAPCPA was enacted. While this pattern is consistent with the theoretical model’s predictions, further analysis is necessary if we are to attribute the decline to an increase in bankruptcy cost. To separate the effect of a bankruptcy cost change from the effects of time, demand, or other macroeconomic variables, we would like to compare BAPCPA’s effect on investment behavior across two groups of airlines - one that was affected by the change, and one that was not. Here I describe my preliminary difference-in-differences approach to test that implication by comparing large and small airlines before and after BAPCPA.

The specifics of the BAPCPA reform suggest that its effects will have been felt most by highly complex firms. Legal experts agree that the new limit on the exclusivity period makes successful reorganization virtually impossible for the largest and most complex corporations. Intuitively, the more parties with which a firm must negotiate, the slower it will expect to gain consensus, the more likely is the exclusivity period restriction to bind. Given the shift of bargaining power to creditors upon termination of exclusivity, firms will expect to face a harsher bankruptcy regime if the new restriction binds. I use firm size\textsuperscript{20} as a proxy for complexity, based on the observation that larger entities tend to have more creditors, more bankruptcy committees, more entities filing joint bankruptcy petitions, and so forth.\textsuperscript{21} I verify that firm size is correlated with bankruptcy duration using Lynn LoPucki’s database of public firm filings and outcomes. In section 7, I present the results of my difference-in-differences analysis. After controlling for demand, seasonality, and firm type, I find evidence that larger firms reduced investment more than smaller firms during the post-BAPCPA era.

5.2 Structural Model

In this section I describe the structural model that will be used to perform counterfactual simulations. The model benefits my analysis in three critical ways. First, the continuous-time approach is both intuitive and computationally tractable to solve. Second, the model produces numerical comparative statics that line up with the theoretical model of Section 4. Finally, the model lends itself well to estimation using conditional choice probability (CCP)

\textsuperscript{20}I measure size as the number of available aircraft seats in the fourth quarter of 2004.

\textsuperscript{21}One might also consider the number of unions, the number of outstanding debt classes, etc. as proxies.
methods, which greatly expedite computation while also resolving some equilibrium selection issues.

To empirically analyze the relationship between BAPCPA and airline investment behavior, only a dynamic model is suitable. Most structural dynamic models in the airline literature describe market-level decisions, which are complicated in their own right, but in this case I must look at the industry as a whole. The number of players in my model is therefore necessarily large, making the computation of Markov-perfect equilibria (MPE) for a traditional discrete-time, simultaneous-move model (i.e. an Ericson and Pakes (1995)-style (EP) model) somewhat difficult. One way to ease the computational burden is to assume that firms make decisions based on less (or less precise) information. For example, Aguirregabiria and Ho (2011) examine industry-wide route network decisions by making assumptions to simplify the set of payoff-relevant variables for each of 22 airlines. A similar concept is used more generally by Weintraub, Benkard, and Van Roy (2008), who introduce the concept of oblivious equilibrium to approximate EP models when many firms are involved. The more popular approach, pioneered by Hotz and Miller (1993) and Hotz, Miller, Sanders, and Smith (1994) and adapted to the I.O. context by Bajari, Benkard, and Levin (2006) and others,\textsuperscript{22} has been to estimate players’ actual choice probabilities from the data, incorporating them into a single-agent dynamic programming framework. The model I employ combines this second approach with a continuous-time model, further expediting computation.

5.2.1 Setup: Discrete Choices in Continuous Time

A continuous-time, discrete-choice model is an intuitive and computationally tractable way to model interaction among a relatively large number of firms. I now lay down the foundations of this model, following Arcidiacono, Bayer, Blevins, and Ellickson (2013), henceforth referred to as ABBE (2013).

Consider a continuous-time, infinite-horizon game following ABBE (2013), in which \( N \) firms compete in capacity levels with the option to file for bankruptcy. At any given time, a firm is fully represented by a capacity level \( q_i \in Q \) and a bankruptcy state \( b_i \), which equals 1 if the firm is under Chapter 11 protection and 0 otherwise. The state of the game is characterized by the set of all players’ states as well as the demand state, \( \alpha \in \{ \alpha_{lo}, \alpha_{hi} \} \), and a state governing the bankruptcy regime, \( \phi \), equal to 0 before the BAPCPA reform and 1 after the reform takes effect on 10/17/2005. Let \( \theta \in \Theta \) represent the vector of economic states and \( x \in X \) represent the vector of firms’ states. Flow profit for firm \( i \) is \( u_i = u(x_i, x_{-i}; \theta) \).

As in ABBE (2013), the state evolves according to a number of independent, continuous-time processes governing the arrival of move opportunities for nature and for all \( N \) players.

\textsuperscript{22}Pakes, Ostrovsky, and Berry (2007), Pesendorfer and Schmidt-Dengler (2003), Ryan (2011), Dunne, Klimek, Roberts, and Xu (2006), and Aguirregabiria and Mira (2006), to name a few.
Nature flips the demand state whenever the opportunity arises, and those opportunities follow a Poisson process with parameter $\gamma$. Firm capacity and bankruptcy adjustment opportunities follow separate Poisson processes with parameters $\lambda_a$ and $\lambda_b$, respectively. When a capacity adjustment opportunity arrives, a firm may choose to remain in its current state, increase capacity by one increment, decrease capacity by one increment, or exit. Exit and entry are accounted for by adjustment to and from a level of zero capacity. If the firm changes capacity levels, it incurs a potentially asymmetric adjustment cost that depends on whether or not the firm is currently in bankruptcy. When a bankruptcy adjustment opportunity arrives, the firm may choose to remain in its current state or change its bankruptcy status. A firm filing for Chapter 11 incurs no explicit cost to transition into bankruptcy, but a firm exiting bankruptcy incurs an explicit cost to adjust its capital structure via court approval of a plan of reorganization. This cost reflects the bargaining power of creditors and is therefore conditional upon the bankruptcy regime. For example, if bankruptcy is more creditor-friendly, then the firm must sacrifice more of its equity upon exit, making reemergence from Chapter 11 more costly.

The structural parameters I will estimate are the capacity adjustment costs and bankruptcy exit costs, which together make up the set of state transition costs, $\psi_{j,k}$, to transition to state $j$ from state $k$. Firms maximize expected lifetime profits, discounting at continuous rate of time preference $\rho$ and taking their opponents’ strategies (conditional choice probabilities) as given. The value to player $i$ of being in state $k$ can be written as

$$ V_{i,k} = \frac{1}{\rho + N\lambda_a + N\lambda_b + \gamma} \left\{ u_{i,k} + \gamma V_{i,l(demand,k)} + \lambda_a E\left[ V_{i,l(i,k;a)} \right] + \lambda_b E\left[ V_{i,l(i,k;b)} \right] + \sum_{i' \neq i} \lambda_a E\left[ V_{i,l(i',k;a)} \right] + \sum_{i' \neq i} \lambda_b E\left[ V_{i,l(i',k;b)} \right] \right\} $$

where

$$ E\left[ V_{i,l(i,k,r)} \right] = E \max_{j \in J_{r,k}} \left\{ V_{i,l(i,j,k)} + \psi_{j,k} + \epsilon_{ij} \right\} $$

and

$$ E\left[ V_{i,l(i',k,r)} \right] = \sum_{j \in J_{r,k}} \sigma_{i',j,k} E\left[ V_{i,l(i',j,k)} \right] $$

and where $r \in \{a, b\}$ is the type of move opportunity, and $J_{r,k}$ is the corresponding choice set. When a move opportunity arrives, agents receive a Type I Extreme Value shock, $\epsilon_{ij}$, to
the value of each possible choice, such that the probability of player $i$ making a particular choice $j$ from state $k$ when the move arrival type is $r$ takes the familiar logit form:

$$
\sigma_{ijkr} = \frac{\exp(V_{i,l(i,j,k)} + \psi_{jk})}{\sum_{j' \in J_r} \exp(V_{i,l(i,j',k)} + \psi_{j'k})}
$$

### 5.2.2 Empirical Model Implications

While the theoretical model provided a framework simple enough to generate comparative statics, its simplicity came at the expense of realism. The empirical model is by no means an accurate depiction of reality, but it incorporates many features the theoretical model could not. To verify that both models generate the same set of predictions, I now present some numerical comparative statics from the empirical model.

I calibrate the empirical model as a duopoly with three possible capacity levels: exit (0), low (1), and high (2). Low demand is set to make a player indifferent between exiting and remaining in the market when he is the only incumbent and has low capacity. High demand is set to ensure that both firms earn a profit even when both are at high capacity. Nature’s move arrival rate is set to 0.25, implying a change every 4 years on average and roughly reflecting the macroeconomic cycle. Players’ move arrival rates are 2, implying 2 choices per year on average, for each choice type. I set the flow cost of holding a unit of capacity to 2, and the flow cost of being in bankruptcy is set to 0.25.\footnote{According to AMR Corporation’s 2010 10-K filing, the company’s aircraft rental expense of $580 million was distributed over a fleet of 241 aircraft under operating lease, suggesting a cost of $2.4 million per aircraft. If we interpret the flow cost of bankruptcy along the same lines, a $250k figure per year seems overly reasonable. However, this conservatism is meant to highlight the role of bankruptcy emergence cost, which I allow to vary from 0 to 10.}

Increasing capacity outside of bankruptcy or decreasing capacity under bankruptcy protection are costless, while the cost of decreasing capacity outside of bankruptcy or increasing capacity within bankruptcy is set to 4.\footnote{I interpret time in annual units, so a flow cost of $c$ generalizes to a rate of $c$ per year in the absence of discounting. Firms discount at continuous rate of time preference $\rho$, such that a flow cost of $c$ yields an annualized cost of \(\int_0^1 c e^{-\rho t} dt = c \frac{1-e^{-\rho}}{\rho}\). A flow cost of 1 for one year when $\rho = 0.1$ therefore has a present value of about 0.95.}

The continuous rate of time preference is set to 10%.

Below I present conditional probabilities of increasing and decreasing capacity based on this intuitive calibration. The probability of increasing capacity is conditional on having low capacity when demand is good, while the probability of decreasing capacity is conditional on...

\footnote{Again using AMR Corporation’s 2010 10-K filing, we can estimate an early lease termination fee of about $10 million per aircraft, based on a $94 million charge for grounding 9 Airbus A300s prior to lease expiration. This value roughly amounts to completion of a 5-year lease term, and more than half of AMR’s leased fleet had remaining lease terms of 5 years or more. However, a given airline is unlikely to rent more than 50% of its fleet. Setting the adjustment cost to twice the annual lease term is therefore a somewhat conservative figure. Setting the upward adjustment cost under bankruptcy protection to be the same value simplifies explanation of the calibration.}
on having high capacity when demand is bad. These two probabilities represent the two sides of capacity discipline: caution on the upswing and quickness in downturns. Since the probability of filing for bankruptcy falls with bankruptcy emergence cost, the probabilities of both investment and disinvestment rise. Therefore, to compare the relative appeal of investment and disinvestment, the numbers presented below are conditional on choosing not to file for bankruptcy. Finally, the first panel of the figure presents equilibrium strategies when the firm is the only incumbent, while the second panel shows how the effect of bankruptcy emergence cost is amplified upon the entry of a low-capacity opponent.

5.2.3 Estimation Using Conditional Choice Probabilities

As previously mentioned, CCP (or “two-step”) methods begin by estimating players’ state-specific choice probabilities directly from the data. The Type I Extreme Value assumption for the distribution of the choice-specific error term then allows me to combine the estimated CCPs with a guess of the structural parameters to construct the value function. Representing the value function in this way eliminates the computationally costly value function iteration loop characteristic of full-solution methods, speeding estimation by orders of magnitude. Moreover, CCPs provide a reasonable equilibrium selection criterion by assuming the relevant equilibrium is the one played in the data. In what follows I explain how to implement this method.

The normal algorithm for nested-fixed-point estimation is

1. guess parameters
2. converge to value function
3. compute likelihood using the CCPs associated with that value function and the parameter guess
4. maximize the likelihood by changing the guess

CCP estimation allows me to skip step 2 of the process above, replacing it with a step 0, in which I estimate the empirical CCPs from the data. This step is performed only once, outside the maximum likelihood loop. Armed with empirical CCPs, the new algorithm is

1. guess parameters
2. converge to value function
3. compute likelihood using estimated CCPs and the parameter guess
4. maximize likelihood function by changing the guess

The empirical CCPs are estimated as flexibly as possible and can be thought of as a kind of interpolation in which we use the data to tell us the probabilities with which agents will make every relevant choice at every observed node in the state space, even if such choices or their resultant states never occur in the data. I estimate conditional choice probabilities using a linear-in-parameters multinomial logit specification. For instance, the contribution to the log likelihood of capacity choice $j' \in J_a$ from state $k$ is

$$ l_{kj'} = \log \left( \frac{\exp(X_k \beta_{j'_a})}{\sum_{j \in J_a} \exp(X_k \beta_j)} \right) - \tau N \lambda_a \sum_{j_a \neq 0} \left( \frac{\exp(X_k \beta_{j_a})}{\sum_{j \in J_a} \exp(X_k \beta_j)} \right) $$

where $\lambda_a$ is a chosen value of the capacity move arrival rate, $\tau$ is the duration in state $k$, and $N$ is the number of firms. $X_k$ is a matrix of regressors specific to state $k$, and $\beta_j$ is the $j$th column of $\beta$, a matrix of coefficients for which the column associated with the continuation choice is normalized to zero. The set of regressors includes own capacity and its square, the sum of opponents’ capacities and its interaction with own capacity, own bankruptcy state and its interaction with all of the above, the demand state and its interaction with all of the above, and an indicator for the implementation of BAPCPA and its interaction with all of the above. I also assume that I know the common, continuous rate of time preference, $\rho$.

---

26 This expression is not actually the log likelihood, but its argmax in $\beta$ is the same as the argmax of the underlying log likelihood.
5.2.4 Constructing the Likelihood

In order to write the likelihood of the data as a function of only these elements requires the presence of either a terminal state or some sort of finite dependence. I use firm exit as a terminal state in order to take advantage of CCP methods. Suppose I observe every non-continuation choice \( j > 0 \) for each player, and that I observe all changes in market demand (high or low). I now derive the likelihood, using ABBE (2013) as a guide. What we observe is a series of events and their points in time. Let the state space be indexed by \( k = \{1, ..., K\} \), and let \( Q_0 \) be the \( K \times K \) intensity matrix governing exogenous state transitions. Let \( Q_N \) be the intensity matrix governing agent-related state transitions. An intensity matrix characterizes a finite-state Markov jump process, in which the elements of \( Q \) represent the rates at which the possible transitions occur. For example, if \( K = 3 \), we have intensity matrix

\[
Q = \begin{bmatrix}
q_{11} & q_{12} & q_{13} \\
q_{21} & q_{22} & q_{23} \\
q_{31} & q_{32} & q_{33}
\end{bmatrix}
\]

For \( l \neq k \), \( q_{kl} \) is the hazard rate for transitions from state \( k \) to state \( l \), that is,

\[
q_{kl} = \lim_{h \to 0} \frac{\mathbb{P}[X_{t+h} = l | X_t = k]}{h}
\]

For \( l = k \), \( q_{kk} \) is the overall rate at which the process leaves state \( k \) and is defined as a negative number

\[
q_{kk} = -\sum_{l \neq k} q_{kl}
\]

such that the sum across any given row is always zero. The intensity matrix tells us everything we need to know about the transition process. In particular, we know that the duration in state \( k \) has an exponential distribution with parameter \(-q_{kk}\). That is,

\[
F_k(t) = 1 - \exp(-t \sum_{l \neq k} q_{kl})
\]

and

\[
f_k(t) = \left(\sum_{l \neq k} q_{kl}\right) \exp(-t \sum_{l \neq k} q_{kl})
\]

Conditional on a jump occurring, the probability of transitioning to state \( l \) from state \( k \) is \( \frac{q_{kl}}{\sum_{l' \neq k} q_{kl'}} \). Therefore, the joint likelihood of a jump occurring at time \( \tau \) from state \( k \) to state
Putting this back into the terms of the model, where $Q_N$ governs players and $Q_0$ governs nature, we can write the likelihood separately for nature’s moves and players’ moves. Let choice $j = 0$ be a player’s continuation choice, such that the state does not change. Then the likelihood that the next state change occurs after time $\tau$ and is the result of player $i$ making capacity choice $j > 0$ is given by

$$
\lambda_a \sigma_{ijk} \exp \left[ -\tau \left( \sum_{l \neq k} q^0_{kl} + \sum_i \lambda_a \sum_{j \neq 0} \sigma_{aijk} + \sum_i \lambda_b \sum_{j \neq 0} \sigma_{bjlk} \right) \right]
$$

which can be written

$$
\lambda_a \sigma_{ijk} \exp \left[ -\tau \left( \sum_{l \neq k} q^0_{kl} + \sum_{r \in \{a,b\}} \lambda_r \sum_i (1 - \sigma_{ri0kt}) \right) \right]
$$

Similarly, the likelihood that the next state change occurs after time $\tau$ and is the result of nature changing the state from $k$ to $l$ is

$$
q^0_{kl} \exp \left[ -\tau \left( \sum_{l \neq k} q^0_{kl} + \sum_{r \in \{a,b\}} \lambda_r \sum_i (1 - \sigma_{ri0kt}) \right) \right]
$$

Given data on $T$ observations of a change in the state and associated length of time, $\tau$, since the last state change, we construct the likelihood of the data as follows:

$$
L(Q_0, \lambda_a, \lambda_b, \theta) = \prod_{t=1}^{T} \left\{ q^0_{kl} \exp \left[ -\tau_t \left( \sum_{l \neq k} q^0_{kl} + \sum_{r \in \{a,b\}} \lambda_x \sum_i (1 - \sigma_{ri0kt}) \right) \right] \right\}^{d_t}
\times \left\{ \lambda_b \sigma_{bijkl} \exp \left[ -\tau_t \left( \sum_{l \neq k} q^0_{kl} + \sum_{r \in \{a,b\}} \lambda_r \sum_i (1 - \sigma_{ri0kt}) \right) \right] \right\}^{(1-d_t)b_t}
\times \left\{ \lambda_a \sigma_{aijkt} \exp \left[ -\tau_t \left( \sum_{l \neq k} q^0_{kl} + \sum_{r \in \{a,b\}} \lambda_x \sum_i (1 - \sigma_{ri0kt}) \right) \right] \right\}^{(1-d_t)(1-b_t)}
$$
where $d_t$ indicates a demand move, and $b_t$ indicates a bankruptcy move. Noting that $q^0_{kt} = \gamma$ and taking logs, we can write the log-likelihood function as follows:

$$l(\psi; \gamma, \lambda_a, \lambda_b, \rho, \hat{\sigma}, \hat{\beta}, \hat{\alpha}) = \sum_{t=1}^{T} \{ d_t \log (\gamma) $$

$$+ (1 - d_t) b_t \log (\lambda_b \hat{\sigma}_{bkt}) $$

$$+ (1 - d_t) (1 - b_t) \log (\lambda_a \hat{\sigma}_{ajkt}) $$

$$- \tau_t \left( \gamma + \sum_{r \in \{a,b\}} \lambda_r \sum_{i} (1 - \hat{\sigma}_{ri0kt}) \right) \}$$

### 5.2.5 Flow Profit Estimation

A key element of the state-specific value function is the flow profit, $u_{ik}$, in that state. Given the highly complex nature of network-level competition in the airline industry, I refrain from explicitly modeling network choice. Instead, I model flow profit in the domestic U.S. market as a reduced-form function of state variables such as the carrier’s capacity, the aggregate capacity in the market, and consumer demand. One of the many advantages of analyzing the U.S. airline industry is the abundance of data, including quarterly line-item-level accounting data. I proxy for flow profits using a carrier’s inflation-adjusted EBITDA. To estimate flow profit as a function of the state, I regress the set of carrier-quarter EBITDA values on the associated time-weighted average values of each state variable for each carrier-quarter. Estimating flow profit in this way allows me to abstract away from modeling utilization or network effects.

### 5.3 Counterfactual Equilibria

Armed with structural parameter estimates, I can solve for equilibria under alternative assumptions and measure the corresponding industry statistics. To examine just how much the bankruptcy option influences overall capacity levels in the industry, my counterfactual equilibrium of interest makes bankruptcy prohibitively costly. This section explains how to solve for such an equilibrium.

For a given set of parameters $\theta$, I can solve for a symmetric, anonymous MPE using value function iteration. The solution process can take some time, especially for large games, which is why full-solution estimation can be extremely time-consuming. As previously discussed, the interested reader will find a host of articles tackling that challenge, beginning with the basic entry model of Berry (1992) and stretching to the more complex models of Aguirregabiria and Ho (2012) and Ciliberto and Tamer (2011).
CCP methods allow me to avoid solving for equilibrium during estimation, but doing so is necessary for simulating data from the model.

The number of possible states for each bankruptcy regime is $2(2Q)^N$, representing a severe curse of dimensionality. To make the state space more manageable, I take advantage of exchangeability (a.k.a. anonymity) to reduce the number of payoff-relevant states over which the value function must be computed.\(^{28}\) This approach results in a much smaller state space of size $S = 4Q(2^{Q+N-2})$. To understand how this helps, consider that a 7-player game with 5 capacity choices has 20 million basic states, but only 100,100 anonymous states. The value function iteration program proceeds as follows:

1. Guess $V$, an $S \times 1$ vector
2. Compute firm 1’s expected value of a move arrival
   (a) Compute the normalized choice-specific values (including adjustment costs)
   (b) Expected value of moving is the inclusive value term (the log-sum)
3. Compute conditional choice probabilities (CCPs) for other players
4. Compute firm 1’s expected value of each opponent’s move arrival\(^{29}\)
5. Update $V$ according to the updating equation below

We repeat this process until $V$ converges.\(^{30}\) The updating equation is

$$ V_{i,k} = \frac{1}{\rho + N\lambda_a + N\lambda_b + \gamma} \left\{ u_{i,k} + \gamma V_{i,l(demand,k)} + \lambda_a \mathbb{E} \left[ V_{i,l(i,k;a)} \right] + \lambda_b \mathbb{E} \left[ V_{i,l(i,k;b)} \right] + \sum_{i' \neq i} \lambda_a \mathbb{E} \left[ V_{i,l(i',k;a)} \right] + \sum_{i' \neq i} \lambda_b \mathbb{E} \left[ V_{i,l(i',k;b)} \right] \right\} $$

where $i$ indexes the firm, $k$ indexes the current state, and $l$ indexes the future state. The value function can be described more intuitively using asset pricing terms. Let us first re-write it

---

\(^{28}\)An alternative is the Pakes/McGuire algorithm, which iterates over the value of recurrent states only. The algorithm also conserves memory in computing the value function at each iteration because it considers only the set of states that can be visited in the next move, rather than the full set of possible states, some of which will not be reachable for at least two moves.

\(^{29}\)This is just the sum of the values (from firm 1’s perspective) associated with each possible choice for each possible opponent, weighted by the corresponding CCP.

\(^{30}\)Convergence is not guaranteed for a multi-player game, but when estimating the model, opponents’ CCPs are fixed, reducing the process to a single-player dynamic programming problem, which is guaranteed to converge.
this way

\[
\rho V_{i,k} = u_{i,k} + \gamma (V_{i,l(demand,k)} - V_{i,k}) + \lambda_a \left( \mathbb{E} [V_{i,l(i,k;a)}] - V_{i,k} \right) + \lambda_b \left( \mathbb{E} [V_{i,l(i,k;b)}] - V_{i,k} \right) \\
+ \sum_{i' \neq i} \lambda_a \left( \mathbb{E} [V_{i,l(i',k;a)}] - V_{i,k} \right) + \sum_{i' \neq i} \lambda_b \left( \mathbb{E} [V_{i,l(i',k;b)}] - V_{i,k} \right)
\]

The formulation above indicates that the instantaneous opportunity cost of holding an asset \((\rho V)\), should be equal to the dividend flow received from that asset \((u)\) plus the capital gain realized when a change in value occurs \((V' - V)\), weighted by the chance of that gain being realized \((\lambda_a, \lambda_b, \text{ or } \gamma)\). We can simplify the value function expression by substituting the following:

\[
\mathbb{E} [V_{i,l(i,k;r)}] = \mathbb{E} \max_{j \in X_{i,k;r}} \{V_{i,l(i,j,k)} + \psi_{j,k} + \epsilon_{ij}\}
\]

\[
\mathbb{E} [V_{i,l(i',k;r)}] = \sum_{j \in X_{i',k;r}} \sigma_{i',j,k} \mathbb{E} [V_{i,l(i',j,k)}]
\]

where \(r \in \{a, b\}\) is the type of move opportunity, and \(X_{r,k}\) is the corresponding choice set. Firms’ strategies/CCPs are given in \(\sigma\), and instantaneous payoffs are given in \(\psi_{j,k}\). Instantaneous payoffs are the capacity adjustment costs and bankruptcy exit costs. The key benefit of continuous-time modeling is that only one event can occur at a time. Firms’ state transitions are therefore deterministic conditional upon their choices, such that \(\mathbb{E} [V_{i,l(i',j,k)}] = V_{i,l(i',j,k)}\). Finally, our assumption on the error structure allows us to write the inclusive value term, \(\mathbb{E} \max_{j \in X_{i,k,r}} \{V_{i,l(i,j,k)} + \psi_{j,k} + \epsilon_{ij}\}\), as

\[
\gamma_{\text{eul}} + \log \sum_{j \in X_{i,k,r}} \exp (V_{i,l(i,j,k)} + \psi_{j,k})
\]

where \(\gamma_{\text{eul}}\) is Euler’s constant.

6 Data

I employ three data sets, which together allow me to match capacity and bankruptcy decisions with firm profitability over time. The first is the Ascend Online Fleets database, maintained by Ascend Advisory,\textsuperscript{31} which contains ownership and technical data on over 200,000 aircraft worldwide. I aggregate Ascend’s daily aircraft-level data to measure airlines’ fleet size. The

\textsuperscript{31}I am grateful to the Duke Economics Department and Andrew Sweeting for helping me purchase this data. More details can be found at the company’s Website: http://www.ascendworldwide.com
second data set includes the timing and outcome of all bankruptcy filings in the U.S. airline industry. The third is a set of publicly available databases maintained by the U.S. Department of Transportation (DOT). Data on quantities and prices for commercial passenger air travel come primarily from the Airline Origin & Destination Survey, known as Data Bank 1B (DB1B). I supplement the DB1B data with the Form 41 Traffic database (T100) and the Form 41 Financial database. Below I describe each data set in further detail.

### 6.1 Daily Fleet Data

Capacity is defined as the number of seats in a carrier’s aircraft fleet, grouped into a number of bins. Daily fleet data comes from the Ascend Online Fleets data base, which was purchased from Flightglobal, a division of U.K.-based Reed Business Information. This data set works well with a continuous-time modeling approach because it provides a daily snapshot of aircraft ownership and usage. Each observation covers all passenger aircraft operated in North America, including their registration and serial numbers, owners and operators (indicating leased vs. owned), aircraft and engine types and manufacturers, and status (in storage, on order, in service, etc.), among other details. I aggregate this data into daily fleet snapshots for all domestic passenger air carriers. Fluctuations in operating fleet serve as a key indicator of capacity investment. However, I must account for the fact that many aircraft are purchased years ahead of time. The fleet database provides each aircraft’s build year, order date, and delivery date, so I know when each aircraft was ordered, at least for brand new planes. Another concern is that the aircraft fleet is not partitioned into regional subcategories, posing a challenge when analyzing domestic data only. Following Severin Borenstein’s lead, I can restrict the analysis to narrow-body jets, since wide-body jets are more often used to fly over-ocean routes. Another key piece of data is the financier, if present, for each aircraft, which allows me to measure how many parties (either lessors or secured creditors) with which a given carrier is contracted. The two figures on the following page demonstrate that fleet investment has fallen since BAPCPA was enacted, and demand for passenger air travel fails to explain the trend. Table 1 on page 38 summarizes daily aircraft fleets for several large carriers.

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32 A wealth of air traffic information is publicly available for download from the Bureau of Transportation Statistics (http://www.transtats.bts.gov/). The DB1B data since 1993 are freely available here, and earlier years are available for purchase in hard copy.
Table 1: Capacity Statistics
(in thousands of seats)

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<th>Mean</th>
<th>Median</th>
<th>Min</th>
<th>Max</th>
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</thead>
<tbody>
<tr>
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<td>99.8</td>
<td>105.4</td>
<td>23.8</td>
<td>186.1</td>
</tr>
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<td>Continental (CO)</td>
<td>46.8</td>
<td>47.1</td>
<td>5.0</td>
<td>77.6</td>
</tr>
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<td>92.0</td>
<td>32.5</td>
<td>160.8</td>
</tr>
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<td>Northwest (NW)</td>
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<td>69.5</td>
<td>20.9</td>
<td>97.2</td>
</tr>
<tr>
<td>United (UA)</td>
<td>85.0</td>
<td>88.2</td>
<td>28.9</td>
<td>188.3</td>
</tr>
<tr>
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<td>52.3</td>
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</tr>
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<td>124.2</td>
</tr>
<tr>
<td>America West (HP)</td>
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<td>0.1</td>
<td>141.5</td>
</tr>
<tr>
<td>Total</td>
<td>66.8</td>
<td>66.4</td>
<td>0.1</td>
<td>188.3</td>
</tr>
<tr>
<td>Observations</td>
<td></td>
<td></td>
<td></td>
<td>98,448 carrier-days</td>
</tr>
</tbody>
</table>

6.2 Bankruptcy Events

Evaluating firms’ decisions to enter and exit bankruptcy requires data on the timing and circumstances of these decisions. I extend and cross-check Ciliberto and Schenone’s (2012) list of pre-2008 bankruptcies using news and trade journal reports, court dockets, Lynn LoPucki’s (UCLA) Bankruptcy Research Database, and data from Airlines for America (A4A), the U.S. airline industry’s primary trade organization. While nearly 200 airline cases have been filed since 1978, many of those involved small and/or cargo carriers. I focus on the filings of passenger airlines with at least 20 aircraft who provide service on their own routes, as opposed to regional carriers who primarily operate as feeder airlines to larger companies. After imposing those restrictions and combining mutually owned companies, I end up with 19 bankruptcy filings matched to capacity data. If that figure seems a bit small, recall that the model is not just estimated off of transitions between states. Every daily observation of a firm’s bankruptcy state provides information on the hazard of state transition, so knowing that American Airlines was bankrupt on January 1, 2013 is just as important as knowing that the firm was not bankrupt on March 3, 2005, for example. The two figures on the following page show no discernible trend in overall bankruptcy filings but suggest that firms nearing insolvency chose to file under the pre-BAPCPA rules.\footnote{Data include all airline filings, not just those that are matched with capacity data.}
Annual Airline Bankruptcy Filings

Monthly Airline Bankruptcy Filings

Source: Airlines for America

The spike on 9/2005 includes Chapter 11 filings by Delta, Northwest, and Comair, a wholly-owned Delta subsidiary, as well as a Chapter 7 filing by TransMeridian.

The 10/2005 observation is Mesaba, a regional airline primarily serving Northwest, which filed before BAPCPA took effect.

The subsequent 3 observations are Chapter 11 filings by Independence Air, Era Aviation, and Florida Coastal Airlines.

6.3 Demand and Flow Profit

Airline demand is typically estimated using publicly available price and quantity data. I used quarterly data from the Department of Transportation to construct such a measure and found that it was no better at predicting profit or investment than a measure of real GDP growth. Moreover, using real GDP allows me to credibly treat my demand measure as exogenous, while also sidestepping the need to account for demand estimation error when reporting final results. Therefore, I measure industry demand using year-over-year quarterly
growth in real GDP. I define the demand state as good if growth was above the linear trend, and bad otherwise. This definition amounts to about 20 demand changes over the course of my data, consistent in both number and direction with results from estimating demand with price and quantity data.

As mentioned in Section 5.2.5, the abundance of airline data is a boon for estimation. The Department of Transportation’s Form 41 Financial Data, Schedule P-1.2 provides an abundant source of financial information, including operating revenue and expense data for reporting carriers in the U.S. Several expense categories are even broken into detailed subcategories. More importantly, this data set breaks down each accounting category by region, which is not always done in SEC filings for publicly traded airlines. As such, I am able to link domestic operating profits to domestic demand. In order to convert accounting data into economic profits, I assume that operating cash flow is proportional to economic profit. I measure operating cash flow as earning before interest, taxes, depreciation, and amortization, or EBITDA. Table 2 summarizes this value for several large carriers.

<table>
<thead>
<tr>
<th>Player</th>
<th>Mean</th>
<th>Median</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>American (AA)</td>
<td>1274</td>
<td>1551</td>
<td>-4509</td>
<td>4267</td>
</tr>
<tr>
<td>Continental (CO)</td>
<td>548</td>
<td>538</td>
<td>-1467</td>
<td>2104</td>
</tr>
<tr>
<td>Delta (DL)</td>
<td>1486</td>
<td>1169</td>
<td>-1475</td>
<td>6111</td>
</tr>
<tr>
<td>Northwest (NW)</td>
<td>801</td>
<td>879</td>
<td>-1348</td>
<td>3132</td>
</tr>
<tr>
<td>United (UA)</td>
<td>1019</td>
<td>1186</td>
<td>-4307</td>
<td>4638</td>
</tr>
<tr>
<td>US Airways (US)</td>
<td>255</td>
<td>255</td>
<td>-3646</td>
<td>2349</td>
</tr>
<tr>
<td>Southwest (WN)</td>
<td>1000</td>
<td>960</td>
<td>62</td>
<td>2253</td>
</tr>
<tr>
<td>America West (HP)</td>
<td>377</td>
<td>393</td>
<td>-2499</td>
<td>1529</td>
</tr>
<tr>
<td>Total</td>
<td>846</td>
<td>737</td>
<td>-4509</td>
<td>6111</td>
</tr>
<tr>
<td>Observations</td>
<td>696 carrier-quarters</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 2: Profit Statistics (in millions of $2009)
7 Results

I now present the results of each empirical analysis: My difference-in-differences test shows that an increase in bankruptcy cost tends to discipline capital investment; my structural estimation quantifies the effect of BAPCPA; and my counterfactual simulation demonstrates that completely restricting bankruptcy as an option reduces industry capacity levels by nearly 20%, while rescinding BAPCPA would increase industry capacity by about 5%.

7.1 Difference-in-Differences Results

<table>
<thead>
<tr>
<th>Table 3: Difference-in-Differences Results</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dependent Variable = Year-over-Year % Change in Fleet Size</strong></td>
</tr>
<tr>
<td>Large</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Post BAPCPA</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Large X Post BAPCPA</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Moving Average of Demand Growth</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Seasonal Fixed Effects</td>
</tr>
<tr>
<td>Linear Time Trend</td>
</tr>
<tr>
<td>Type Fixed Effects (LEG vs. LCC)</td>
</tr>
</tbody>
</table>

Number of Observations 4,192

Robust standard errors in parentheses.

*** p<0.01, ** p<0.05, * p<0.10

As mentioned previously, the changes made by BAPCPA were more likely to affect the behavior of the most complex firms, for which Chapter 11 typically represents a multi-year process. Following the empirical literature on bankruptcy, I proxy for complexity using firm size. I measure firm size as the average number of seats available in the fleet during the quarter, and I split the sample in half by size as of the fourth quarter of 2004, using 5,000 seats as the cutoff. Investment is the percent change in fleet size from the same quarter of the previous year. Table 3 shows that BAPCPA reduced overall investment by 40% for sufficiently large firms, while it had little to no effect on small airlines.
7.2 Structural Model Estimates: Flow Profit Estimates

<table>
<thead>
<tr>
<th>Table 4: Flow Profit Estimation</th>
<th>Dependent Variable = Annualized Quarterly EBITDA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(in millions of $2009)</td>
</tr>
<tr>
<td>High Demand</td>
<td>427.60 *</td>
</tr>
<tr>
<td>Sum of Opponents’ Capacity</td>
<td>1.22</td>
</tr>
<tr>
<td>Own Capacity</td>
<td>41.79 **</td>
</tr>
<tr>
<td>OwnCap X OppCap</td>
<td>-0.13</td>
</tr>
<tr>
<td>HighDem X OwnCap</td>
<td>10.02</td>
</tr>
<tr>
<td>HighDem X OppCap</td>
<td>-1.94 *</td>
</tr>
<tr>
<td>HighDem X OppCap X OwnCap</td>
<td>-0.02</td>
</tr>
<tr>
<td>Seasonal Fixed Effects</td>
<td>Yes</td>
</tr>
<tr>
<td>Number of Observations</td>
<td>1120</td>
</tr>
</tbody>
</table>

Robust standard errors in parentheses, clustered at the player level.

*** p<0.01, ** p<0.05, * p<0.10

Table 4 presents results from an ordinary least squares regression of annualized free cash flows on capacity, demand, and a number of other state variables. As one might expect, demand and own capacity tends to improve performance, while opponents’ capacity reduces profitability. These results provide the basis for the flow profit function used in the second stage of the estimation.
7.3 Structural Model Estimates: Second Stage

Table 5 illustrates that BAPCPA more than doubled the cost of emerging from Chapter 11, raising it from $508 million to $1.1 billion. Upward adjustment costs are estimated to be around $500 million both in and out of bankruptcy. Downward adjustment costs are about $1.2 billion outside of bankruptcy and $730 million under bankruptcy protection. Put in context, the median annualized cashflow across carriers is about $700 million. Adjustment costs are based on a change of between 20 and 40 aircraft. The ballpark value for a new Boeing 737 is $50 million,\footnote{http://blog.seattlepi.com/aerospace/2009/07/01/how-much-is-a-shiny-new-boeing-737-worth-not-72-million/} while a rough estimate\footnote{Based on AMR Corp’s 2010 10-K} of early lease cancellation fees amounts to $10 million per aircraft.

<table>
<thead>
<tr>
<th>Table 5: Structural Parameter Estimates</th>
</tr>
</thead>
<tbody>
<tr>
<td>(costs in millions of $2009)</td>
</tr>
<tr>
<td>Baseline Bankruptcy Emergence Cost</td>
</tr>
<tr>
<td>BAPCPA Incremental Bankruptcy Emergence Cost</td>
</tr>
<tr>
<td>Upward Adjustment Cost, Non-Bankruptcy</td>
</tr>
<tr>
<td>Downward Adjustment Cost, Non-Bankruptcy</td>
</tr>
<tr>
<td>Upward Adjustment Cost, Bankruptcy</td>
</tr>
<tr>
<td>Downward Adjustment Cost, Bankruptcy</td>
</tr>
<tr>
<td>Scale Parameter</td>
</tr>
<tr>
<td>Number of Observed Events</td>
</tr>
<tr>
<td>Bootstrapped standard errors in progress</td>
</tr>
</tbody>
</table>

7.4 Counterfactual Simulation

Using the structural model estimates, I now solve for two counterfactual equilibria. In the first, I simulate industry evolution as though the expected cost of filing bankruptcy is infinite, effectively precluding bankruptcy as a downsizing option. As a result, I find that eliminating the bankruptcy option reduces overall capacity by as much as 20% relative to its actual level, suggesting that the malleability of contracts in Chapter 11 has a significant inflationary effect on capacity. This result is not entirely surprising given the steep adjustment cost associated with downsizing outside of bankruptcy.

The second counterfactual simulates what would have happened had BAPCPA never been passed. In that scenario, I find a modest increase in industry capacity of about 5%, representing an estimate of the contribution of BAPCPA to observed capacity discipline. Though small in magnitude, the presence of any effect at all should give pause to bankruptcy law
reformers. In his recent testimony before the American Bankruptcy Institute’s Commission to Study the Reform of Chapter 11, bankruptcy expert and law professor Daniel Keating stressed policymakers to respect the potential for unintended consequences from tweaking the U.S. Bankruptcy Code.\textsuperscript{36} As the Commission prepares to make its final report to Congress, the influence of Chapter 11’s non-financial provisions on investment behavior should be considered.

7.5 Conclusion

The key takeaway of this paper is that bankruptcy law, specifically Chapter 11, can influence oligopoly investment behavior. I have presented a straightforward theoretical model that predicts capacity discipline as an outcome of stricter bankruptcy policy, and I have provided support for that prediction with a rigorous, multi-faceted empirical approach. As the first paper to link the investment and bankruptcy decisions in a strategic setting, this work has a number of interesting extensions in both the corporate finance and industrial organization literatures, all of which I am excited to tackle. Moreover, the predictions of the theoretical model generalize to any industry with heavy contractual investment and volatile demand, suggesting an important and heretofore undiscussed consideration for bankruptcy law reform, both in the United States and abroad.

\textsuperscript{36}Wiese Niemeyer, Kelly. Kansas City infoZine, June 24, 2013.  
8 Appendix

8.1 Overview of Distributions

The following is meant as a refresher on the PDFs and CDFs of Poisson and exponential distributions. Poisson is a discrete distribution with PMF

\[
P(x) = \frac{\exp(-\mu)\mu^x}{x!}
\]

where \( x = 0, 1, 2, \ldots \) and the mean and variance are both \( \mu \). Exponential is a continuous distribution with PDF

\[
f(t) = \lambda \exp(-\lambda t)
\]

where \( t \geq 0 \) and the mean and variance are both \( 1/\lambda \). The exponential CDF is

\[
F(t) = 1 - \exp(-\lambda t)
\]

If the Poisson describes the number of occurrences per unit of time, then the exponential describes the duration between occurrences. That is, the Poisson rate describes how many events should occur, on average, per unit of time. If \( \lambda t \) events occur in \( t \) units of time, then the probability that no events occur in an interval of \( t \) is

\[
P(0; \mu = \lambda t) = \frac{\exp(-\lambda t) (\lambda t)^0}{0!} = \exp(-\lambda t)
\]

Therefore, the probability that an event has not occurred after \( t \) time has passed is \( 1 - \exp(-\lambda t) = F(t) \).

Finally, let’s match the exponential distribution to the intensity matrix of a Markov jump process. Following Chapter 3 of Hoel, Port, and Stone, a jump process is a sequence \( X(t) \) that describes the state of a system at time \( t \) in the following way:

\[
X(t) = \begin{cases} 
  x_0, & 0 \leq t < \tau_1 \\
  x_1, & \tau_1 \leq t < \tau_2 \\
  x_2, & \tau_2 \leq t < \tau_3 \\
  \vdots 
\end{cases}
\]

A pure jump process is one that is non-explosive, that is, one for which \( \lim_{n \to \infty} \tau_n = \infty \). The jump times and associated states are random. If the process reaches an absorbing state, it remains there forever, whereas if the process reaches a non-absorbing state \( k \), it remains there for some length of time \( t \), which is distributed according to \( F_k(t) \). After \( t \) elapses, the
process jumps from state $k$ to state $l$ with probability $Q_{kl}$ (define $Q_{kk} = 0$). Moreover, the time and state events are independent. Hence, if we consider a pure jump process beginning in state $k$ at time 0, then

$$\mathbb{P}[\tau_1 \leq t, X(\tau_1) = l \mid x_0 = k] = F_k(t)Q_{kl}$$

Referencing Chapter 5 of *Introduction to Probability Theory*, Hoel, Port, and Stone points out that a pure jump process is Markovian if and only if $F_x(t)$ is exponential for every non-absorbing state $x$. Let $\mathbb{P}_x$ denote the probability of an event conditional on the current state being $x$. Then the Markov property means

$$\mathbb{P}_x [\tau_1 > t + s, \mid \tau_1 > s] = \mathbb{P}_x [\tau_1 > t]$$

Further, let $\mathbb{P}_{xy}(t)$ denote the probability that a process beginning in state $x$ is in state $y$ at time $t$, and let $\mathbb{P}_{xy}(0) = \delta_{xy}$, where

$$\delta_{xy} = \begin{cases} 1, & y = x \\ 0, & y \neq x \end{cases}$$

Then the Markov property also implies

$$\mathbb{P}_{xy}(t + s) = \sum_z \mathbb{P}_{xz}(t)\mathbb{P}_{zy}(s)$$

To relate all of this to the intensity matrix I use in the paper, let $q_x$ be the parameter that defines the exponential distribution $F_x$. Since I always have to be reminded, that means

$$F_x(t) = 1 - \exp(-q_xt)$$

and

$$\mathbb{P}_x [\tau_1 \geq t] = 1 - F_x(t) = \exp(-q_xt)$$

We can describe $\mathbb{P}_{xy}(t)$ as the integral over all possible jumps from time from 0 to $t$ that take the process from state $x$ to state $z$, each weighted by the probability of thereafter transitioning from state $z$ to state $y$. This yields the Chapman-Kolmogorov equation.

$$\mathbb{P}_{xy}(t) = \delta_{xy} (1 - F_x(t)) + \int_0^t f_x(s) \left( \sum_{z \neq x} Q_{xz} \mathbb{P}_{zy}(t - s) \right) ds$$

$$= \delta_{xy} \exp(-q_xt) + \int_0^t q_x \exp(-q_xts) \left( \sum_{z \neq x} Q_{xz} \mathbb{P}_{zy}(t - s) \right) ds$$

46
To get the infinitesimal parameters of the intensity matrix, first replace \( s \) with \( t - s \) to get \(^\text{37}\) 

\[
\mathbb{P}_{xy}(t) = \delta_{xy} \exp(-q_x t) + \int_0^t q_x \exp(-q_x t + q_x s) \left( \sum_{z \neq x} Q_{xz} \mathbb{P}_{zy}(s) \right) ds 
\]

and then differentiate with respect to \( t \) to get

\[
P'_{xy}(t) = -q_x \delta_{xy} \exp(-q_x t) - q_x^2 \exp(-q_x t) \int_0^t \exp(q_x s) \left( \sum_{z \neq x} Q_{xz} \mathbb{P}_{zy}(s) \right) ds + q_x \exp(-q_x t) \left[ \exp(q_x t) \left( \sum_{z \neq x} Q_{xz} \mathbb{P}_{zy}(t) \right) \right] \]

which at \( t = 0 \) reduces to

\[
P'_{xy}(0) = -q_x \delta_{xy} + q_x Q_{xy} \]

Define \( q_{xy} \equiv P'_{xy}(0) \), and we can write the elements of the intensity matrix in a familiar way

\[
q_{xy} = \begin{cases} 
- q_x, & y = x \\
q_x Q_{xy}, & y \neq x 
\end{cases}
\]

and since \( \sum_y Q_{xy} = 1 \), we know that \( \sum_y q_{xy} = q_x = -q_{xx} \). Recall that each non-diagonal element of the intensity matrix is the instantaneous probability (hazard rate) of transitioning from one state to another:

\[
q_{kl} = \lim_{h \to 0} \frac{\mathbb{P}[X_{t+h} = l | X_t = k]}{h}
\]

and the sum of these probabilities represents the rate at which the process leaves state \( k \). Therefore, the CDF of the duration spent in state \( x \) is given by

\[
F_x(t) = 1 - \exp(-t \sum_y q_{xy})
\]

### 8.2 CCP Representation

To apply the two-step procedure requires us to find a way to represent the analytical CCPs as explicit functions of the structural parameters and the empirical CCPs. Consider again

\(^\text{37}\) Use Liebniz rule:

\[
\frac{\partial}{\partial t} \left( \int_{a(t)}^{b(t)} f(t, s) ds \right) = f(t, b(t)) b'(t) - f(t, a(t)) a'(t) + \int_{a(t)}^{b(t)} f_t(t, s) ds
\]

In this case, \( a(t) = 0 \) and \( b(t) = t \)
the ex ante value function for firm $i$ in state $k$:

$$V_{i,k} = \frac{1}{\rho + N\lambda_a + N\lambda_b + \gamma} \left( u_{i,k} + \gamma V_{i,demand,k} + \lambda_a E[V_{i,l(i,k,a)}] + \lambda_b E[V_{i,l(i,k,b)}] + \sum_{i' \neq i} \lambda_a E[V_{i',l(i',k,a)}] + \sum_{i' \neq i} \lambda_b E[V_{i',l(i',k,b)}] \right)$$

and apply what we know of the expectation terms

$$E[V_{i,l(i,k,r)}] = E \max_{j \in X_{i,k,r}} \{ V_{i,l(i,j,k)} + \psi_{j,k} + \epsilon_{ij} \}$$

$$E[V_{i,l(i',k,r)}] = \sum_{j \in X_{i',k,r}} \sigma_{i',j,k} V_{i,l(i',j,k)}$$

to arrive at

$$V_{i,k} = \frac{1}{\rho + N\lambda_a + N\lambda_b + \gamma} \left( u_{i,k} + \gamma V_{i,demand,k} + \sum_{r \in \{a,b\}} \lambda_r \left( E \max_{j \in X_{i,k,r}} \{ V_{i,l(i,j,k)} + \psi_{j,k} + \epsilon_{ij} \} + \sum_{i' \neq i} \sum_{r' \in \{a,b\}} \sigma_{i',j,k} V_{i',l(i',j,k)} \right) \right) \quad (11)$$

### 8.2.1 Incumbents

Applying Proposition 2 of ABBE (2013), we can write the Emax term as follows:

$$E \max_{j \in X_{i,k,r}} \{ V_{i,l(i,j,k)} + \psi_{j,k} + \epsilon_{ij} \} = \gamma_{eul} + \psi_{j,k} + V_{i,l(i,j,k)} - \log(\sigma_{j,k})$$

for any choice $j' \in X_{i,k,r}$. For incumbents facing a capacity adjustment opportunity, we can choose $j' = exit$ and normalize the continuation value of exit to 0,\(^{38}\) giving us

$$E \max_{j \in X_{i,k,a}} \{ V_{i,l(i,j,k)} + \psi_{j,k} + \epsilon_{ij} \} = \gamma_{eul} + \psi_{exit,k} - \log(\sigma_{exit,k}) \quad (12)$$

The same proposition allows us to compare the value functions of different states, as reflected in Proposition 1 of ABBE (2013) below:

$$\gamma_{eul} + \psi_{j,k} + V_{i,l(i,j,k)} - \log(\sigma_{j,k}) = \gamma_{eul} + \psi_{j',k} + V_{i,l(i,j',k)} - \log(\sigma_{j',k})$$

$$V_{i,l(i,j,k)} = V_{i,l(i,j',k)} + \psi_{j',k} - \psi_{j,k} + \log(\sigma_{j,k}) - \log(\sigma_{j',k})$$

where $j$ and $j'$ are elements of the same choice set $X_{i,k,r}$. To compare value functions across choice sets, suppose that player $i$ in state $k$ will always have a continuation choice, $j^*$, that does not change the state. In other words, players can always choose to do nothing, regardless of the type of move opportunity that arises. Combining the continuation choice with the

\(^{38}\)Note that the adjustment cost of exit is state-specific, so while leaving the industry is worth zero going forward, the context in which a carrier exits (e.g. liquidation, merger, etc.) is allowed to matter.
second expression above gives us the following two equalities:

\[
V_{i,l(i,j_a^*,k)} = V_{i,l(i,j_a,k)} + \psi_{ja,k} - \psi_{ja^*,k} + \log(\sigma_{ja^*,k}) - \log(\sigma_{ja,k})
\]

\[
V_{i,l(i,j_b^*,k)} = V_{i,l(i,j_b,k)} + \psi_{jb,k} - \psi_{ja^*,k} + \log(\sigma_{ja^*,k}) - \log(\sigma_{jb,k})
\]

where the \( j \) choices have subscripts to indicate their relevant choice sets. Recognizing that \( V_{i,l(i,j_a^*,k)} = V_{i,l(i,j_b^*,k)} \), we can write

\[
V_{i,l(i,j_a,k)} + \psi_{ja,k} - \psi_{ja^*,k} + \log(\sigma_{ja^*,k}) - \log(\sigma_{ja,k}) = V_{i,l(i,j_b,k)} + \psi_{jb,k} - \psi_{ja^*,k} + \log(\sigma_{ja^*,k}) - \log(\sigma_{jb,k})
\]

which, assuming there is no instantaneous cost to choosing the status quo, simplifies to

\[
V_{i,l(i,j_b,k)} = V_{i,l(i,j_a,k)} + \psi_{ja,k} + \log(\sigma_{ja^*,k}) - \log(\sigma_{ja,k}) - \psi_{jb,k} - \log(\sigma_{ja^*,k}) + \log(\sigma_{jb,k})
\]

If we set \( j_a = \text{exit} \) and apply the the normalization of equation (20), we get

\[
V_{i,l(i,j_b,k)} = \psi_{exit,k} + \log(\sigma_{ja^*,k}) - \log(\sigma_{exit,k}) - \psi_{jb,k} - \log(\sigma_{ja^*,k}) + \log(\sigma_{jb,k})
\]

For an incumbent facing a bankruptcy adjustment opportunity, we can substitute this expression into Proposition 1 and cancel terms to arrive at

\[
\mathbb{E} \max_{j \in X_{i,k,b}} \{ V_{i,l(i,j,k)} + \psi_{jk} + \epsilon_j \} = \gamma_{eul} + \psi_{exit,k} + \log(\sigma_{ja^*,k}) - \log(\sigma_{exit,k}) - \log(\sigma_{ja^*,k})(13)
\]

Next consider the value to player \( i \) of an opponent’s choice, which moves the state from \( k \) to \( k' \equiv l(i', j, k) \). Note that the value to player \( i \) of being in state \( k' \) does not depend on how player \( i \) arrived in that state. Therefore, if we again let \( j^* \) represent a continuation choice for player \( i \), such that \( l(i, j^*, k') = k' \), then we have \( V_{i,l(i,j',k)} = V_{i,k'} = V_{i,l(i,j^*,k')} \). Proposition 3 of ABBE (2013) applies this equivalence, allowing us to re-write Proposition 1 as follows:

\[
V_{i,l(i',j,k)} = V_{i,l(i,j^*,k')} = V_{i,l(i,j',k')} + \psi_{j^*,k'} - \psi_{j^*,k'} + \log(\sigma_{j^*,k'}) - \log(\sigma_{j^*,k'})
\]

where \( j' \) is any of player \( i \)'s choices in state \( k' \), and we can again set \( \psi_{j^*,k'} = 0 \). In addition, whenever player \( i \)'s choice set in state \( k' \) includes both exit and a continuation choice \( j^* \) (i.e. for a capacity adjustment decision), we can substitute \( V_{i,l(i,exit,k')} = 0 \) for \( V_{i,l(i,j',k')} \) to get an even tidier result:

\[
V_{i,l(i',j,k)} = \psi_{exit,l(i',j,k)} + \log(\sigma_{j^*,l(i',j,k)}) - \log(\sigma_{exit,l(i',j,k)}) (14)
\]
where I have applied \( k' \equiv l(i', j, k) \) to make it clear that we still have an opponent’s move in view. The same expression applies to moves by nature. Expressions (20)-(22) will be valid for all states in which player \( i \) is an incumbent. Substituting them into (19) expresses each value function in terms of CCPs and parameters.

\[
V_{i,k}(\rho + N\lambda_a + N\lambda_b + \gamma) = u_{i,k} + \gamma \left[ \psi_{exit,l(demand,k)} - \log(\sigma_{exit,l(demand,k)}) + \log(\sigma_{j,\ast,l(demand,k)}) \right] \\
+ \lambda_a \left[ \gamma_{eul} + \psi_{exit,k} - \log(\sigma_{exit,k}) \right] \\
+ \lambda_b \left[ \gamma_{eul} + \psi_{exit,k} - \log(\sigma_{exit,k}) + \log(\sigma_{j,\ast,k}) - \log(\sigma_{j,\ast,k}) \right] \\
+ \sum_{r \in \{a,b\}} \lambda_r \left( \sum_{j' \neq i \in X_{i',k,r}} \sigma_{i',j,k} \left[ \psi_{exit,l(i',j,k)} - \log(\sigma_{exit,l(i',j,k)}) + \log(\sigma_{j,\ast,l(i',j,k)}) \right] \right)
\]

### 8.2.2 Potential Entrants

For potential entrants, \( exit \) is not an option, so we must apply another substitution in order to eliminate value functions on the right-hand side. As before, apply Proposition 2 to get

\[
\mathbb{E} \max_{j \in X_{i,k,a}} \{ V_{i,l(i,j,k)} + \psi_{jk} + \epsilon_{ij} \} = \gamma_{eul} + \psi_{1,k} + V_{i,l(i,j',k)} - \log(\sigma_{j',k})
\]

but now let \( j' = 1 \), the choice to enter with the lowest possible capacity, and define \( k^* \equiv l(i, 1, k) \). Then, since player \( i \) is an incumbent in state \( k^* \), simply apply equation (22) to \( V_{i,l(i,j',k)} \) to get an expression for the future value of player \( i \)'s move:

\[
\mathbb{E} \max_{j \in X_{i,k,a}} \{ V_{i,l(i,j,k)} + \psi_{jk} + \epsilon_{ij} \} = \gamma_{eul} + \psi_{1,k} + \left[ \psi_{exit,k^*} + \log(\sigma_{j,\ast,k^*}) - \log(\sigma_{exit,k^*}) \right] - \log(\sigma_{1,k})
\]

(15)

To represent the future values of opponents’ moves, apply Proposition 1 as before:

\[
V_{i,l(i',j,k)} = V_{i,l(i,j',k')} = V_{i,l(i,j',k')} + \psi_{j',k'} - \psi_{j,\ast,k'} + \log(\sigma_{j,\ast,k'}) - \log(\sigma_{j',k'})
\]

Choose \( j' = 1 \) and apply equation (22) again to get

\[
V_{i,l(i',j,k)} = \left[ \psi_{exit,l(i,1,k')} + \log(\sigma_{j,\ast,l(i,1,k')}) - \log(\sigma_{exit,l(i,1,k')}) \right] + \psi_{1,k} + \log(\sigma_{j,\ast,k'}) - \log(\sigma_{1,k'})
\]

(16)

Substitute (23) and (24) into (19) to get expressions for each value function

\[
V_{i,k}(\rho + N\lambda_a + (N - 1)\lambda_b + \gamma) = u_{i,k} + \gamma \left[ \psi_{exit,l(demand,k)} + \log(\sigma_{j,\ast,l(demand,k)}) - \log(\sigma_{exit,l(demand,k)}) \right] + \psi_{1,k} + \log(\sigma_{j,\ast,k'}) - \log(\sigma_{1,k'})
\]

\[
+ \lambda_a \left[ \gamma_{eul} + \psi_{1,k} + \left[ \psi_{exit,k} + \log(\sigma_{j,\ast,k}) - \log(\sigma_{exit,k}) \right] - \log(\sigma_{1,k}) \right]
\]

\[
+ \lambda_b \left[ \gamma_{eul} + \psi_{1,k} + \left[ \psi_{exit,k} + \log(\sigma_{j,\ast,k}) - \log(\sigma_{exit,k}) \right] - \log(\sigma_{1,k}) \right]
\]

\[
+ \sum_{r \in \{a,b\}} \lambda_r \left( \sum_{j' \neq i \in X_{i',k,r}} \sigma_{i',j,k} \left[ \psi_{exit,l(i',j,k)} + \log(\sigma_{j,\ast,l(i',j,k)}) - \log(\sigma_{exit,l(i',j,k)}) \right] + \psi_{1,k} + \log(\sigma_{j,\ast,k'}) - \log(\sigma_{1,k'})
\]
where \( k'' \equiv l(demand, k), \ k' \equiv l(i', j, k) \), and I have prohibited potential entrants from filing for bankruptcy.

### 8.3 Duopoly Solution

Following Acemoglu and Akcigit (2012), solving for the value functions is easy. Since costs are linear, any non-zero investment level must satisfy

\[
V_{n+1} - V_n = \lambda \text{ for } n \in \{-2, -1, 0, 1\}
\]

(17)

while any non-zero disinvestment level must satisfy

\[
W_n - W_{n+1} = \theta \text{ for } n \in \{-1, 0, 1, 2\}
\]

(18)

per the first-order conditions for each optimization problem. Combining (17) and (5) gives

\[
V_{-2} = \frac{\pi_{-2} + \lambda(1 + \gamma_2)d_2 + \psi W_{-2}}{r + \psi}
\]

Similarly, combining (18) and (6) yields

\[
W_2 = \frac{\pi'_{2} + \theta(1 + \phi_2)b_2 + \psi'V_2 - R_2b_2}{r + \psi'}
\]

According to (17) and (18), we know that \( V_2 = V_{-2} + 4\lambda \) and \( W_{-2} = W_2 + 4\theta \). These conditions give us a solvable system of two equations:

\[
V_2 = \frac{\pi_{-2} + \lambda(1 + \gamma_2)d_2}{r + \psi} + 4\lambda + \frac{\psi W_{-2}}{r + \psi}
\]

(19)

\[
W_{-2} = \frac{\pi'_{2} + \theta(1 + \phi_2)b_2 - R_2b_2}{r + \psi'} + 4\theta + \frac{\psi'V_2}{r + \psi'}
\]

(20)

It turns out we don’t even need to solve the system, though. We can use everything we have so far to get expressions for optimal investment in each state. First, combine (17) with (1)-(4) and (18) with (7)-(10) to get expressions in terms of value functions, assuming investment and disinvestment are always always positive.
Next, rewrite (19) and (20) as follows

\[ \begin{align*}
x_{*-2}^* &= \frac{\pi - R_2d - \lambda(1 + \gamma_2)d_2 + \psi W_2 - (r + \psi)V_2}{\lambda} \\
x_{*-1}^* &= \frac{\pi_1 - R_1d_1 + \lambda d_{-1} - \lambda(1 + \gamma_1)d_1 + \psi W_1 - (r + \psi)V_1}{\lambda} \\
x_0^* &= \frac{\pi_0 - R_0d_0 + \psi W_0 - (r + \psi)V_0}{\lambda} \\
x_1^* &= \frac{\pi_{-1} - R_{-1}d_{-1} + \lambda(1 + \gamma_1)d_1 - \lambda d_{-1} + \psi W_{-1} - (r + \psi)V_{-1}}{\lambda} \\
x_2^* &= 0 \\
\end{align*} \]

and recall that

\[ \begin{align*}
y_{*-2}^* &= 0 \\
y_{*-1}^* &= \frac{\pi'_1 - R_1b_1 + \theta(1 + \phi_1)b_1 - \theta b_{-1} + \psi'V_1 - (r + \psi')W_1}{\theta} \\
y_0^* &= \frac{\pi'_0 - R_0b_0 + \psi'V_0 - (r + \psi')W_0}{\theta} \\
y_1^* &= \frac{\pi'_{-1} - R_{-1}b_{-1} + \theta b_{-1} - \theta(1 + \phi_1)b_1 + \psi'V_{-1} - (r + \psi')W_{-1}}{\theta} \\
y_2^* &= \frac{\pi'_{-2} - \theta(1 + \phi_2)b_2 + \psi'V_{-2} - (r + \psi')W_{-2}}{\theta} \\
\end{align*} \]

Next, rewrite (19) and (20) as follows

\[ \begin{align*}
\psi W_{-2} - (r + \psi)V_2 &= - (\pi_{-2} + \lambda(1 + \gamma_2)d_2 + (r + \psi)4\lambda) \\
\psi'V_{-2} - (r + \psi')W_{-2} &= - (\pi'_{-2} + \theta(1 + \phi_2)b_2 - R_2b_2 + (r + \psi')4\theta) \\
\end{align*} \]

and recall that

\[ \begin{align*}
W_{-2} &= W_{-1} + \theta = W_0 + 2\theta = W_1 + 3\theta = W_2 + 4\theta \\
V_{-2} &= V_{-1} + \lambda = V_0 + 2\lambda = V_{-1} + 3\lambda = V_{-2} + 4\lambda \\
\end{align*} \]

Then we need not even solve explicitly for either value function. We can simply substitute the expressions above into (21)-(30). Starting from the top, let’s sub in for investment
intensities:

\[ x_{-2}^* = \frac{\pi_2 - R_2 d_2 - \lambda (1 + \gamma_2) d_2 + \psi W_2 - (r + \psi)V_2}{\lambda} \]
\[ = \frac{\pi_2 - R_2 d_2 - \lambda (1 + \gamma_2) d_2 + \psi (W_{-2} - 4\delta) - (r + \psi)V_2}{\lambda} \]
\[ = \frac{\pi_2 - R_2 d_2 - \lambda (1 + \gamma_2) d_2 - (\pi_{-2} + \lambda (1 + \gamma_2) d_2 + (r + \psi)4\lambda) - \psi 4\delta}{\lambda} \]
\[ = \frac{\pi_2 - \pi_{-2} - R_2 d_2 - 4\delta \psi}{\lambda} - (4(r + \psi) + 2(1 + \gamma_2) d_2) \]

\[ x_{-1}^* = \frac{\pi_1 - R_1 d_1 + \lambda d_{-1} - \lambda (1 + \gamma_1) d_1 + \psi W_1 - (r + \psi)V_1}{\lambda} \]
\[ = \frac{\pi_1 - R_1 d_1 + \lambda d_{-1} - \lambda (1 + \gamma_1) d_1 + \psi (W_{-2} - 3\delta) - (r + \psi)(V_2 - \lambda)}{\lambda} \]
\[ = \frac{\pi_1 - R_1 d_1 + \lambda d_{-1} - \lambda (1 + \gamma_1) d_1 - (\pi_{-2} + \lambda (1 + \gamma_2) d_2 + (r + \psi)4\lambda) - \psi 3\delta + \lambda (r + \psi)}{\lambda} \]
\[ = \frac{\pi_1 - \pi_{-2} - R_1 d_1 - 3\delta \psi}{\lambda} - (3(r + \psi) + (1 + \gamma_2) d_2 + (1 + \gamma_1) d_1 - d_{-1}) \]

\[ x_0^* = \frac{\pi_0 - R_0 d_0 + \psi W_0 - (r + \psi)V_0}{\lambda} \]
\[ = \frac{\pi_0 - R_0 d_0 + \psi (W_{-2} - 2\delta) - (r + \psi)(V_2 - 2\lambda)}{\lambda} \]
\[ = \frac{\pi_0 - R_0 d_0 - (\pi_{-2} + \lambda (1 + \gamma_2) d_2 + (r + \psi)4\lambda) - \psi 2\delta + 2\lambda (r + \psi)}{\lambda} \]
\[ = \frac{\pi_0 - \pi_{-2} - R_0 d_0 - 2\delta \psi}{\lambda} - (2(r + \psi) + (1 + \gamma_2) d_2) \]

\[ x_1^* = \frac{\pi_{-1} - R_{-1} d_{-1} + \lambda (1 + \gamma_1) d_1 + \lambda d_{-1} + \psi W_{-1} - (r + \psi)V_{-1}}{\lambda} \]
\[ = \frac{\pi_{-1} - R_{-1} d_{-1} + \lambda (1 + \gamma_1) d_1 + \lambda d_{-1} + \psi (W_{-2} - \delta) - (r + \psi)(V_2 - 3\lambda)}{\lambda} \]
\[ = \frac{\pi_{-1} - R_{-1} d_{-1} + \lambda (1 + \gamma_1) d_1 - \lambda d_{-1} - (\pi_{-2} + \lambda (1 + \gamma_2) d_2 + (r + \psi)4\lambda) - \psi \delta + 3\lambda (r + \psi)}{\lambda} \]
\[ = \frac{\pi_{-1} - \pi_{-2} - R_{-1} d_{-1} - \delta \psi}{\lambda} - ((r + \psi) + (1 + \gamma_2) d_2 - (1 + \gamma_1) d_1 + d_{-1}) \]
And now, disinvestment intensities:

\[ y_{-1}^* = \frac{\pi_1' - R_1 b_1 + \theta(1 + \phi_1)b_1 - \theta b_{-1} + \psi' V_1 - (r + \psi')W_1}{\theta} \]
\[ = \frac{\pi_1' - R_1 b_1 + \theta(1 + \phi_1)b_1 - \theta b_{-1} + \psi'(V_2 - \lambda) - (r + \psi')(W_{-2} - 2\theta)}{\theta} \]
\[ = \frac{\pi_1' - R_1 b_1 + \theta(1 + \phi_1)b_1 - \theta b_{-1} - (\pi_2' + \theta(1 + \phi_2)b_2 - R_2 b_2 + (r + \psi')4\theta)}{\theta} - \lambda \psi' + (r + \psi')3\theta \]
\[ = \frac{\pi_1' - \pi_2' + R_2 b_2 - R_1 b_1 - \lambda \psi'}{\theta} - ((r + \psi') + (1 + \phi_2)b_2 - (1 + \phi_1)b_1 + b_{-1}) \]

\[ y_0^* = \frac{\pi_0' - R_0 b_0 + \psi' V_0 - (r + \psi')W_0}{\theta} \]
\[ = \frac{\pi_0' - R_0 b_0 + \psi'(V_2 - 2\lambda) - (r + \psi')(W_{-2} - 2\theta)}{\theta} \]
\[ = \frac{\pi_0' - R_0 b_0 - (\pi_2' + \theta(1 + \phi_2)b_2 - R_2 b_2 + (r + \psi')4\theta) - 2\lambda \psi' + (r + \psi')2\theta}{\theta} \]
\[ = \frac{\pi_0' - \pi_2' + R_2 b_2 - R_0 b_0 - 2\lambda \psi'}{\theta} - (2(r + \psi') + (1 + \phi_2)b_2) \]

\[ y_1^* = \frac{\pi_{-1}' - R_{-1} b_{-1} + \theta b_{-1} - \theta(1 + \phi_1)b_1 + \psi' V_{-1} - (r + \psi')W_{-1}}{\theta} \]
\[ = \frac{\pi_{-1}' - R_{-1} b_{-1} + \theta b_{-1} - \theta(1 + \phi_1)b_1 + \psi'(V_2 - 3\lambda) - (r + \psi')(W_{-2} - \theta)}{\theta} \]
\[ = \frac{\pi_{-1}' - R_{-1} b_{-1} + \theta b_{-1} - \theta(1 + \phi_1)b_1 - (\pi_2' + \theta(1 + \phi_2)b_2 - R_2 b_2 + (r + \psi')4\theta) - 3\lambda \psi' + (r + \psi')\theta}{\theta} \]
\[ = \frac{\pi_{-1}' - \pi_2' + R_2 b_2 - R_{-1} b_{-1} - 3\lambda \psi'}{\theta} - (3(r + \psi') + (1 + \phi_2)b_2 + (1 + \phi_1)b_1 - b_{-1}) \]

\[ y_2^* = \frac{\pi_{-2}' - \theta(1 + \phi_2)b_2 + \psi' V_{-2} - (r + \psi')W_{-2}}{\theta} \]
\[ = \frac{\pi_{-2}' - \theta(1 + \phi_2)b_2 + \psi'(V_2 - 4\lambda) - (r + \psi')W_{-2}}{\theta} \]
\[ = \frac{\pi_{-2}' - \theta(1 + \phi_2)b_2 - (\pi_2' + \theta(1 + \phi_2)b_2 - R_2 b_2 + (r + \psi')4\theta) - 4\lambda \psi'}{\theta} \]
\[ = \frac{\pi_{-2}' - \pi_2' + R_2 b_2 - 4\lambda \psi'}{\theta} - (4(r + \psi') + 2(1 + \phi_2)b_2) \]

Summarizing, the set of investment and disinvestment intensities is as follows:
\[ x_{-2}^* = \max \left\{ 0, \frac{\pi_2 - \pi_2^* - R_2 d_2}{\lambda} - 4\theta \psi - (4(r + \psi) + 2(1 + \gamma_2)d_2) \right\} \]

\[ x_{-1}^* = \max \left\{ 0, \frac{\pi_1 - \pi_2 - R_1 d_1}{\lambda} - 3\theta \psi - (3(r + \psi) + (1 + \gamma_2)d_2 + (1 + \gamma_1)d_1 - d_{-1}) \right\} \]

\[ x_0^* = \max \left\{ 0, \frac{\pi_0 - \pi_2 - R_0 d_0}{\lambda} - 2\theta \psi - (2(r + \psi) + (1 + \gamma_2)d_2) \right\} \]

\[ x_1^* = \max \left\{ 0, \frac{\pi_1 - \pi_2 - R_{-1} d_{-1} - \theta \psi}{\lambda} - ((r + \psi) + (1 + \gamma_2)d_2 - (1 + \gamma_1)d_1 + d_{-1}) \right\} \]

\[ x_2^* = 0 \]

\[ y_{-2}^* = 0 \]

\[ y_{-1}^* = \max \left\{ 0, \frac{\pi_1 - \pi_2 + R_2 b_2 - R_1 b_1 - \lambda \psi'}{\theta} - ((r + \psi') + (1 + \phi_2)b_2 - (1 + \phi_1)b_1 + b_{-1}) \right\} \]

\[ y_0^* = \max \left\{ 0, \frac{\pi_0 - \pi_2 + R_2 b_2 - R_0 b_0 - 2\lambda \psi'}{\theta} - (2(r + \psi') + (1 + \phi_2)b_2) \right\} \]

\[ y_1^* = \max \left\{ 0, \frac{\pi_1 - \pi_2 + R_2 b_2 - R_{-1} b_{-1} - 3\lambda \psi'}{\theta} - (3(r + \psi') + (1 + \phi_2)b_2 + (1 + \phi_1)b_1 - b_{-1}) \right\} \]

\[ y_2^* = \max \left\{ 0, \frac{\pi_2 - \pi_2 + R_2 b_2 - 4\lambda \psi'}{\theta} - (4(r + \psi') + 2(1 + \phi_2)b_2) \right\} \]

8.3.1 Duopoly Implications: Steady-State

While investment rates are informative, they do not tell the whole story. The distribution of industry structures in equilibrium may change when \( R_n \) changes. Therefore, we compute the steady-state distribution, \( \mu \), a vector of long-run probabilities. The long-run rate at which the process leaves state \( i \) must equal the sum of the long-run rates at which the process enters state \( i \). The steady-state vector \( \mu \) is a solution to

\[ \mu'Q = 0 \]

\[ \sum_i \mu_i = 1 \]

where \( Q \) is the infinitesimal generator, or the intensity matrix, of the continuous-time Markov process and has elements \( q_{ij} \). The matrix \( Q \) corresponds to the matrix \( P - I \) in discrete-time
Markov processes. The row sums in $Q$ are zero, such that

$$q_{ii} \equiv \sum_{j=1, j \neq i}^{N} -q_{ij}$$

Given our equilibrium (dis)investment intensities, we can construct $Q$ as follows:

$$Q = \begin{bmatrix}
 q_{11} & d_2(1 - \gamma_2) + x_{-2} & d_2\gamma_2 & \psi & 0 & 0 \\
 x_1 + d_{-1} & q_{22} & x_{-1} + (1 - \gamma_1)d_1 & 0 & \psi & 0 \\
 2\gamma_0d_0 & 2(x_0 + (1 - \gamma_0)d_0) & q_{33} & 0 & 0 & \psi \\
 \psi' & 0 & 0 & q_{44} & b_2(1 - \phi_2) + y_2 & b_2\phi_2 \\
 0 & \psi' & 0 & y_{-1} + b_{-1} & q_{55} & y_1 + (1 - \phi_1)b_1 \\
 0 & 0 & \psi' & 2\phi_0b_0 & 2(y_0 + (1 - \phi_0)b_0) & q_{66}
\end{bmatrix}$$

The condition $\mu'Q = 0$ yields the balance equations

$$\mu_i q_i = \sum_{j=1, j \neq i}^{N} \mu_j q_{ji}$$

which we express in long form as

\begin{align*}
 u_2(x_1 + d_{-1}) + u_32\gamma_0d_0 + u_4\psi' &= u_1(d_2 + x_{-2} + \psi) \\
 u_1(d_2(1 - \gamma_2) + x_{-2}) + u_32(x_0 + (1 - \gamma_0)d_0) + u_5\psi' &= u_2(x_1 + d_{-1} + x_{-1} + (1 - \gamma_1)d_1 + \psi) \\
 u_1d_2\gamma_2 + u_2(x_{-1} + (1 - \gamma_1)d_1) + u_6\psi' &= u_3(2x_0 + 2d_0 + \psi) \\
 u_5(y_{-1} + b_{-1}) + u_62\phi_0b_0 + u_1\psi &= u_4(b_2 + y_2 + \psi') \\
 u_4(b_2(1 - \phi_2) + y_2) + u_62(y_0 + (1 - \phi_0)b_0) + u_2\psi &= u_5(y_{-1} + b_{-1} + y_1 + (1 - \phi_1)b_1 + \psi') \\
 u_4b_2\phi_2 + u_5(y_1 + (1 - \phi_1)b_1) + u_3\psi &= u_6(2y_0 + 2b_0 + \psi') \\
 u_1 + u_2 + u_3 + u_4 + u_5 + u_6 &= 1
\end{align*}

The system can be solved for $\mu$ when constraint (37) is substituted in, but the expression is many pages long. Absent a simplified expression, I parameterized the model in MATLAB and verified that changes in $R_n$ have the same effect in steady-state as they do on the intensities for a given level. In particular, steady-state investment in upturns falls with $R$, while steady-state disinvestment in downturns rises with $R$. To illustrate, the figure below presents the steady-state distribution of investment and disinvestment intensities as functions of reorganization cost for a parameterization of the theoretical model.
\begin{align*}
  u_2 (x_1 + d_{-1}) + u_3 2 \gamma_0 d_0 + u_4 \psi' &= u_1 (d_2 + x_{-2} + \psi) \quad \text{(38)} \\
  u_1 (d_2(1 - \gamma_2) + x_{-2}) + u_3 2 (x_0 + (1 - \gamma_0) d_0) + u_5 \psi' &= u_2 (x_1 + d_{-1} + x_{-1} + (1 - \gamma_1) d_1 + \psi) \quad \text{(39)} \\
  u_1 d_2 \gamma_2 + u_4 (x_{-1} + (1 - \gamma_1) d_1) + u_6 \psi' &= u_3 (2x_0 + 2d_0 + \psi) \quad \text{(40)} \\
  u_5 (y_{-1} + b_{-1}) + u_6 2 \phi_0 b_0 + u_1 \psi &= u_4 (b_2 + y_{-2} + \psi') \quad \text{(41)} \\
  u_4 (b_2(1 - \phi_2) + y_2) + u_5 2 (y_0 + (1 - \phi_0) b_0) + u_2 \psi &= u_5 (y_{-1} + b_{-1} + y_{-1} + (1 - \phi_1) b_1 + \psi') \quad \text{(42)} \\
  u_4 b_2 \phi_2 + u_5 (y_1 + (1 - \phi_1) b_1) + u_3 \psi &= u_6 (2y_0 + 2b_0 + \psi') \quad \text{(43)} \\
  u_1 + u_2 + u_3 + u_4 + u_5 + u_6 &= 1 \quad \text{(44)}
\end{align*}
References


