Dynamic Airline Pricing and Seat Availability

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Abstract

Airfares are determined by both intertemporal price discrimination and dynamic adjustment to stochastic demand given limited capacity. In this paper I estimate a model of dynamic airline pricing taking both forces into account. I use an original data set of daily fares and seat availabilities at the flight level. With model estimates, I disentangle key interactions between the arrival pattern of consumer types and remaining capacity under stochastic demand. I find dynamic adjustment to stochastic demand is particularly important as a means to secure seats for high-valuing consumers who arrive close to the departure date. It leads to substantial revenue gains compared to pricing policies which depend on date of purchase but not remaining capacity. In aggregate consumers benefit, despite facing higher fares on average, as a result of more efficient capacity allocation. Finally, I show that failing to account for stochastic demand leads to a systematic bias in estimating demand elasticities.

JEL Classifications: L11, L12, L93

Keywords: dynamic airline pricing, intertemporal price discrimination, capacity constraints, estimation of dynamic models

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1 Introduction

Airlines tend to charge high prices to passengers who search for tickets close to the date of travel. The conventional view is that these are business travelers, and airlines capture their high willingness to pay through intertemporal price discrimination. Airlines also adjust prices on a day-to-day basis as capacity is limited and the future demand for any given flight is uncertain. While fares generally increase as the departure date approaches, prices can actually fall from one day to the next, after a sequence of low demand realizations.

This paper examines pricing in the airline industry taking into account both forces – intertemporal price discrimination and dynamic adjustment to stochastic demand. I use a new flight-level data set to estimate a structural model of dynamic airline pricing where firms face a stochastic arrival of consumers. The mix of consumer types – business and leisure travelers – changes over time, and in the estimated model, late-arriving consumers are significantly more price inelastic than consumers who arrive early on. With model estimates, I simulate the revenue losses associated with a pricing system which allows for intertemporal price discrimination, but not dynamic adjustment. I find these losses to be substantial, suggesting that the addition of dynamic adjustment creates an important complementarity in the pricing channels. My results provide credence as to why airlines have pioneered such complex pricing systems: having prices respond to stochastic demand allows firms to first secure seats for the high-valuing consumers who arrive close to the departure date, and then charge these consumers very high prices.

Existing research demonstrates the importance of intertemporal price discrimination in the airline industry. The view is that business consumers learn of last-minute meetings and are willing to pay a premium in order to reserve a seat, while leisure consumers are more price sensitive and book tickets well in advance. Consistent with the idea of market segmentation, Puller, Sengupta, and Wiggins (2012) find that ticket characteristics, such as advance purchase discount (APD) requirements, explain much of the variation in fares over time. Recently, Lazarev (2013) estimated a model of intertemporal price discrimination and he found a substantial role for this force.

The literature also shows that dynamic adjustment plays an important role in airline pricing. Escobari (2012) and Alderighi, Nicolini, and Piga (2012) find evidence that airlines face stochastic demand and prices respond to remaining capacity. In particular, Escobari (2012) estimates the pricing functions of airlines. He notes that fares decline in the absence of sales while having reduced capacity in any given period results in increased fares.1 These results support the theoretical

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1Puller, Sengupta, and Wiggins (2012) find limited support that seat scarcity explains the variation of fares for major US routes. However, Alderighi, Nicolini, and Piga (2012) find evidence that both characteristics and capacity matter. They find the role of capacity is pronounced in less competitive markets.
predictions of Gallego and Van Ryzin (1994) and a large branch of research in operations management that have studied optimal pricing under uncertain demand, limited capacity, and limited time to sell. This work has been used to inform airline revenue management systems. Such systems allow airlines to respond to stochastic demand by increasing fares when a sellout is likely and fall otherwise, as to not leave as many seats unfilled.

While previous work emphasizes the importance of intertemporal price discrimination and stochastic demand pricing separately, this paper examines both forces together and highlights how they interact. I establish two key points about the interaction. First, intertemporal price discrimination and dynamic adjustment to stochastic demand are complements in the airline industry. This follows because inelastic consumers tend to arrive last. In order to be in a position to price discriminate and set high prices to these late-arriving consumers, the firm will want to allow fares to adjust to realizations in demand. Second, in order to estimate how demand elasticity changes over time, which is needed to calculate the welfare effects of intertemporal price discrimination, it is necessary to take stochastic demand into account. The reason is that by ignoring stochastic demand, the opportunity cost of selling a seat is the same regardless of the date of purchase. But with stochastic demand, the opportunity cost changes over time. This matters because inferences regarding demand elasticity come from the firm’s first-order condition in choice of price, which relates prices and marginal costs to demand elasticities. If marginal costs are incorrect, then the estimated change in demand elasticity is also incorrect.

In order to investigate dynamic airline pricing, a detailed data set of ticket purchases is required. However, the standard airline data sets used in economic studies (Goolsbee and Syverson (2008); Gerardi and Shapiro (2009); Berry and Jia (2010) for example) are either at the monthly or quarterly level. Recently, papers have been using new data to get to the flight level. McAfee and Te Velde (2006) and Lazarev (2013) create data sets containing high frequency fares. Other papers have obtained high frequency data on prices and a measure of quantities. Puller, Sengupta, and Wiggins (2012) use a unique transaction data set from a single computer reservation system. Escobari (2012) and Clark and Vincent (2012) collect fare and flight availability data, where the available number of seats is derived from publicly available seat maps. I create a similar data set with a key improvement. I use a new data source that allows me to see the same flight availability data that travel agents see. Specifically, the seat maps I collect allow me to distinguish between blocked and occupied seats. Without accounting for blocked seats, I find seat maps overstate load factors (seats

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2 There is a large literature on stochastic demand pricing (revenue management, yield management, or dynamic pricing). An overview of the dynamic pricing literature can be found in Elmaghraby and Keskinocak (2003) and Talluri and Van Ryzin (2005). Seat inventory control has also been studied; see Dana (1999).
occupied / capacity) by 10%. In addition, I provide evidence that seat maps are a useful proxy of bookings.

The sample contains the time path of fares and seat availabilities for over 1,300 flights in US monopoly markets. The structure of the data allows me to capture over one hundred departures of a single flight number, where each flight is tracked for sixty days. Descriptive analysis of the data reveals a strong role of remaining capacity in explaining the variation of daily fares. By investigating the pricing decisions of routes with two flights a day, I find that if one flight option is 30% more full, the flight is roughly 35% more expensive compared to the other option. Also, consistent with the ideas of stochastic demand pricing, 35% of fares change daily and 10% of the itineraries in the sample result in a fire-sale.

While the empirical evidence above is informative, it cannot be used to disentangle the interactions between intertemporal price discrimination and stochastic demand pricing. I proceed by estimating a structural model. The model includes three key ingredients: (i) firms have finite capacity and finite time to sell, (ii) firms face a stochastic arrival of consumers, and (iii) the mix of consumers, corresponding to business and leisure travelers, is allowed to change over time. The firm solves a stochastic dynamic programming problem. For the demand system, I assume a stochastic process brings consumers to the market. The consumers that arrive know when they want to travel and solve a static problem, choosing to either buy a ticket on an available flight or exit the market permanently. The demand model differs from earlier theoretical work, including Gale and Holmes (1993), where consumers do not know if they wish to fly and waiting provides more information. In my model the only reason to wait is to bet on price, and since prices tend to increase, I show only a small transaction cost is needed to persuade consumers to decide whether to travel in the current period. In addition, I provide empirical evidence that suggests this is a reasonable assumption.\footnote{Li, Granados, and Netessine (2013) studies dynamic consumer behavior in airline markets. Depending on the specification, they find between 5\% and 20\% of consumers are dynamic.}

To recover the parameters of the model, my identification strategy relies on accounting for the firm’s pricing choice. By solving the firm’s dynamic problem, I recover the shadow price on the capacity constraint which provides a valuable source of identification concerning demand elasticities. Given the panel structure of the data, there is variation in sales given remaining capacity and time to sell, as well as variation in sales across time for a given capacity. This helps to separately identify the arrival rate versus the mix of consumers across time.

Using the model estimates, I compare the allocation of scarce capacity across time under dynamic pricing with several counterfactual pricing systems. I first shut down the use of dynamic pricing so that the monopolist can only charge a uniform price. I find that uniform pricing results
in a significant reallocation of capacity across consumers and time, but the gains in consumer sur-
plus are mitigated as a result of inefficient capacity allocation. Further, uniform pricing results
is a significant decline (6.6%) in revenues, more than offsetting the increase in consumer welfare
(1.4%). As airlines operate under razor thin margins, the decrease in revenues would likely result
in market exit in the long run.\footnote{According to an IATA (2013) industry report, the average fare paid per passenger in 2012 was $181.91, with an average cost per passenger of $225.70. After accounting for auxiliary and cargo revenue, they estimate the net profit per passenger to be $2.56.} Using dynamic and uniform pricing as benchmarks I then allow
the firm to use dynamic pricing, but restrict the frequency of price updates. I find that even minor
restrictions on the frequency of price adjustments results in significant revenue reductions.

I then single out the use of intertemporal price discrimination alone by considering a pricing
system which depends on date of purchase, but not remaining capacity. By comparing uniform
pricing to this intermediate case, and this intermediate case with dynamic pricing, I quantify the
relative importance of intertemporal price discrimination and adjusting to stochastic demand. I find
that roughly half the revenue gains of using dynamic pricing over uniform pricing comes from the
intertemporal price discrimination channel, with the remaining half coming from dynamic adjust-
ment. Dynamic pricing substantially increases revenues (3.5%) over the use of intertemporal price
discrimination alone as firms are able to allocate more seats to late-arriving business consumers,
who are then charged high prices. Additionally, I find that overall, both business and leisure welfare
is higher under dynamic pricing compared to intertemporal price discrimination alone. Although
business consumers are charged higher fares under dynamic pricing, they also benefit from having
more seats available. Leisure consumers benefit from lower fares as dynamic adjustment reduces
the firm’s incentive of holding back capacity in early periods.

Intertemporal price discrimination and dynamic adjustment to stochastic demand are com-
plements in the airline industry because high-valuing consumers arrive late. To highlight this
complementarity, I perform two analyses. First, I reverse the arrival process of consumers so that
the high-valuing consumers arrive first. In this environment, I find that intertemporal price dis-
crimination accounts for a much larger percentage (25% more) of the value of dynamic pricing.
This follows because there is no need for the firm to save seats until close to the departure date.\footnote{This analysis assumes consumers do not wait to purchase. \textit{Stokey} (1979) shows an environment in which a monopolist of durable goods that commits to pricing would not use intertemporal price discrimination as consumers with high valuations would strategically wait to purchase. \textit{Conlisk, Gerstner, and Sobel} (1984) and \textit{Board} (2008) consider durable goods models with time dependent demand.} Second, I hold the mix of business and leisure consumers constant over time. This analysis reveals
the revenue gains associated with dynamic adjustment are half the gains attained under the esti-
mated arrival process where high-valuing consumers show up late. This result is consistent with
the theory of Gallego and Van Ryzin (1994), and emphasizes that stochastic demand pricing is especially valuable in the airline industry because of the particular pattern of consumer arrival.

Finally, I show how estimation approaches that do not take into account stochastic demand will systematically produce biased estimates of the degree to which demand becomes more inelastic as the departure date approaches.

1.1 Related Literature

This paper adds to the growing empirical work on intertemporal price discrimination and dynamic adjustment to stochastic demand. Intertemporal price discrimination can be found in many markets, including video games, Broadway theater, and concerts (Nair (2007), Leslie (2004), and Courty and Paglierio (2012), respectively). A closely related paper to mine is Lazarev (2013), who estimates the welfare effects of intertemporal price discrimination in US monopoly airline markets. His model includes dynamic consumers, but abstracts away from aggregate demand uncertainty. There is a large literature in economics and operations research on stochastic demand pricing. Talluri and Van Ryzin (2004) and Vulcano, van Ryzin, and Chaar (2010) provide insights into the estimation of (discretized) continuous time demand models with myopic consumers. Like Vulcano, van Ryzin, and Chaar (2010), I estimate a discrete choice model with Poisson arrival; however, I also allow for two consumer types. I use information on the pricing decision of the firm to aid in the identification of the parameters. Importantly, McAfee and Te Velde (2006) note that stochastic demand models – models which do not incorporate changes in willingness to pay over time – do not match the positive trend in airfares as the departure date approaches. By investigating both forces simultaneously, my model is able to capture both the positive trend in fares as well as the day-to-day variation in fares, including price declines. To the best of my knowledge, this is the first paper to quantify the complementarities between intertemporal price discrimination and dynamic adjustment to stochastic demand through a structural model.

The rest of the paper proceeds as follows. Section 2 describes the data collected for this study. Section 3 presents the model. Section 4 discusses the econometric specification and identification of the model parameters. Section 5 presents the results of estimation. Section 6 presents the counterfactuals. The conclusion follows.

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Lambrecht et al. (2012) provide an overview of empirical work on price discrimination. Interestingly, Jones (10/22/2012) notes that some theaters are now using the same pricing techniques of airlines. An overview of the stochastic demand pricing (or also called: dynamic pricing or revenue management depending on the context) literature can be found in Elmaghraby and Keskinocak (2003) and Talluri and Van Ryzin (2005). Sweeting (2012) analyzes ticket resale markets. Pashigian and Bowen (1991) and Soysal and Krishnamurthi (2012) study clearance sales and seasonal goods, respectively. Zhao and Zheng (2000) and Su (2007) discuss extensions to dynamic pricing models, including consumer dynamics.
2 Data

I create an original data set of high frequency airfares and seat availabilities with data collected from two popular online travel services. The first web service used is a travel metasearch engine. I use the web service to obtain daily fares at the itinerary level.\textsuperscript{8} I obtain all one way and round trip itinerary fares where the length of stay is less than eight days. The fares recorded correspond to the cheapest ticket available for purchase. The second web service returns flight availabilities by allowing users to query real-time seat maps as well as look up detailed fare information. I compare the time series of seat maps to derive seat availabilities and thus, recover bookings across time. The data set contains fare and flight availability data for ten markets collected over a six month period in 2012. In total, the sample contains 1,328 flight departures and more than 80,000 one-way fare/seat map observations.

In the following subsections, I highlight features of the data. I first discuss route selection. I then confront the issue that seat maps may not accurately reflect true flight loads. I perform two analyses that suggest the measurement error in seat maps is likely to be small. I then provide summary statistics for the sample. The last subsection documents preliminary evidence in the data. I document that: (i) prices fluctuate across time, but systematic fare increases are common, (ii) remaining capacity is important in explaining the variation in fares, (iii) there is no evidence that consumers anticipate systematic fare hikes.

2.1 Route Selection

I select markets to study using historical DB1B tables. These publicly available tables contain a 10% of domestic ticket purchases and are at the quarterly level. I define a market in the DB1B as an origin, destination, quarter. I single out markets where:

(i) there is only one carrier operating;\textsuperscript{9}

(ii) there is no nearby alternative airport;

(iii) at least 95% of flight traffic is not connecting to other cities;

(iv) total quarterly traffic is greater than 3,000 passengers;

(v) total quarterly traffic is less than 30,000 passengers;

\textsuperscript{8}I define an itinerary to be a routing, airline, flight number(s), and departure date(s) combination.
\textsuperscript{9}70.12\% of routes in the US are monopoly.
(vi) there is high nonstop traffic.

One important issue with using seat maps is figuring out which itinerary and hence which fare, to attribute to each seat map change. Since airlines offer extensive networks, the disappearance of a single seat could be associated with one of several thousand possible itineraries. This is an important consideration since the pricing of feeder routes tends to be different than main routes. Criteria (iii) addresses this by selecting routes where most traffic is not connecting to other cities. Criteria (iv) corresponds to routes with less than 75% load factor (seats occupied / capacity) of a daily 50-seat aircraft. This criteria removes routes with irregular service. Criteria (v) removes most routes with greater than three flights a day.\textsuperscript{10} I then look for routes with (vi) high nonstop traffic. This criteria is important for establishing the relevant outside option in the demand model. In the data, (vi) is negatively correlated with distance ($\rho = -.5$). Cities with very high nonstop traffic percentages tend to be short flights. Given such short distances, many consumers may choose to instead drive. At the same time, markets with large distances typically have lower nonstop traffic percentages, meaning more consumers purchase tickets such as one stop.

I select five city pairs, or ten directional routings, given the selection criteria above. All directional routings either originate or end in Boston, MA. The other cities are: Portland, OR, San Diego, CA, Austin, TX, Kansas City, MO, and Jacksonville, FL. The selected markets have close to 100% direct traffic, meaning very few passengers connect to other cities. The percent is nonstop traffic ranges between 40% and 70%. Three of the five city pairs are operated by JetBlue. The other airlines in the sample are Delta Air Lines and Alaska Airlines.\textsuperscript{11} The selected markets are all low frequency as at most two flights are operated in either direction daily. Most days see a single flight frequency with double daily service on select routes during peak weeks of the summer or on weekends.\textsuperscript{12}

Two other features of the data are worth being noted. First, both JetBlue and Alaska price itineraries at the segment level. Consumers wishing to purchase round-trip tickets on these carriers in fact purchase two one-way tickets. As a consequence, round-trip fares in these markets are exactly equal to the sum of the corresponding one-way fares. Since fares must be attributed to each seat map change, this feature of the data makes it easier to justify the fare involved. Second, JetBlue does not oversell flights, while most other air carriers do.\textsuperscript{13}

\textsuperscript{10}I implement this criteria to keep the data collection process manageable.
\textsuperscript{11}At the time of data collection, flights between Kansas City and Boston were operated by regional carriers on behalf of Delta Air Lines. Since Delta Air Lines determines the fares for this market, I collectively call these regional carriers Delta.
\textsuperscript{12}This is not exceptional. The average number of frequencies across routes in the US is 1.95 flights per day. Over 60% of US routes see a single fight a day.
\textsuperscript{13}On the legal section of the JetBlue website, under Passenger Service Plan: “JetBlue does not overbook flights. However some situations, such as flight cancellations and reaccommodation, might create a similar situation.”
2.2 Inference and Accuracy of Seat Maps

A seat map is a graphical representation of occupied and unoccupied seats for a given flight at a select point in time before the departure date. Many airlines that have assigned seating present seat maps to consumers during the booking process. When a consumer books a ticket and selects a seat, the seat map changes – an unoccupied seat becomes occupied. The next consumer wishing to purchase a ticket on the flight is offered an updated seat map and has a choice amongst the remaining unoccupied seats. By differencing seat maps across time – in this case daily – inferences can be made about bookings.

**Figure 1:** An example seat map. The white blocks are unoccupied seats, the blue blocks are occupied, the blocks with the X’s are blocked seats, and the blocks with a ”P” are premium, unoccupied seats.
Figure 1 presents a sample seat map. The seat map indicates occupied seats in solid blue. The unshaded blocks correspond to unoccupied seats. Seats with a “P” are available seats, but classified as premium. These seats are toward the front of the aircraft or seats located in exit rows. Finally, the seat map indicates seats currently blocked by the airline with “X”s. Seats that are blocked are usually not disclosed on airline websites; however, I am able to capture this data through the web service used. Seats may be blocked due to crew rest, weight and balance, because a seat is broken, or because the airline reserves handicap accessible seats until the day of departure. In addition, seat blocking may be used to encourage consumers to purchase tickets or upgrade as they give the impression the cabin is closer to capacity. However, the data suggests that airlines predominantly block seats in exit rows and at the front and/or back of the cabin until closer to the departure date or when bookings demand additional seats. The decision is dynamic as over 70% of the flights in the sample experience changes in the number of blocked seats.\footnote{I do not model the decision to block/unblock seats; however, I do take this information into account when determining bookings. Knowing which seats are blocked is important because it allows me to distinguish between consumers canceling or purchasing tickets and airlines adjusting the supply of seats. For example, if it were not possible to distinguish between blocked and occupied seats, if an airline unblocks six seats, I would erroneously conclude six passengers canceled tickets.} For every seat map collected, I aggregate the number of occupied, unoccupied, and blocked seats. I compare the aggregate counts across days to determine net bookings by day before departure.

Unfortunately, seat maps may not be accurate representations of true flight loads. This is especially problematic if consumers do not select seats at the time of booking. This measurement error would systematically understate sales early on, but then overstate last minute sales when consumers without existing seat assignments are assigned seats. From a modeling perspective, this measurement error would lead to an overstatement of the arrival of business consumers. Ideally, the severity of measurement error of my data can be assessed by matching changes in seat maps with bookings, however this is impossible with the publicly available data. In order to gauge the magnitude of the measurement error in using seat maps, I perform two different data analyses, which are only briefly discussed here, but are detailed in the Appendix.

First, I match monthly enplanements using my seat maps with actual monthly enplanements reported in the T100 Segment tables. By comparing these aggregate measures, I find my seat maps understate true enplanements by 0.81% of load factor at the monthly level. To investigate the measurement error at the observation, or flight-day level, I create a separate data set by collecting information from an airline that provides seat maps as well as reported flight loads. With this data, I find seat maps understate reported load factor by 2.3%, with a range of 0% to 4% by day before departure. These two analyses suggest the measurement error associated with the seat maps in
the sample is likely to be small. I proceed by using the capacity of the plane minus the number of occupied and blocked seats as the number of seats available in the proceeding analysis.\textsuperscript{15}

### 2.3 Summary Statistics

Summary statistics for the data sample appear in Table 1. The average oneway ticket in my sample is $282 whereas the average roundtrip fare is $528. The discrepancy in one-way and round-trip fares can be attributed to the flights operated by Delta, since Delta does not price at the segment level but both Alaska and JetBlue do.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>5th pctile</th>
<th>95th pctile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oneway fare ($)</td>
<td>282.23</td>
<td>118.03</td>
<td>129.80</td>
<td>498.80</td>
</tr>
<tr>
<td>Roundtrip fare ($)</td>
<td>528.09</td>
<td>202.30</td>
<td>279.60</td>
<td>917.60</td>
</tr>
<tr>
<td>Load factor</td>
<td>0.85</td>
<td>0.08</td>
<td>0.69</td>
<td>0.98</td>
</tr>
<tr>
<td>Daily booking rate</td>
<td>0.78</td>
<td>1.61</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>Daily fare change ($)</td>
<td>5.77</td>
<td>52.79</td>
<td>-60.00</td>
<td>87.00</td>
</tr>
<tr>
<td>Daily fare change rate</td>
<td>0.35</td>
<td>0.10</td>
<td>0.17</td>
<td>0.52</td>
</tr>
<tr>
<td>Unique fares (per itin.)</td>
<td>12.76</td>
<td>3.50</td>
<td>7</td>
<td>19</td>
</tr>
</tbody>
</table>

\( N_{\text{oneway}} = 80,550 \)

Reported load factor is the number of occupied seats divided by capacity the day flights leave, and is reported between 0 and 1. In my sample, the average load factor is 85%, ranging from 77% to 89% by market. The booking rate corresponds to the mean difference in occupied seats across consecutive days. I find the average booking rate to be 0.78 seats per day, per flight. At the 5th percentile, zero seats per flight are booked a day, and at the 95th percentile, four seats per flight are booked a day. Airline markets are associated with low daily demand as 56.8% of the seat maps in the sample do not change across consecutive days. The fare change rate is an indicator variable equal to one if the itinerary fare changes across consecutive days. I find the daily rate of fare changes to be 35%, so that the itineraries in my sample typically change price 21 times in 60 days.

Due to institutional details concerning airline pricing practices, only a discrete number of fares are seen in the data.\textsuperscript{16} The last row indicates the number of unique fares per itinerary. On average,\textsuperscript{16}

\textsuperscript{15}By treating blocked seats as occupied, I find the monthly load factors for my routes exceed 95%, which is inconsistent with the reported carrier statistics found in the T100. Treating blocked seats as unoccupied results in a 0.81% difference in load factor with reported carrier data.

\textsuperscript{16}Airline pricing practices are discussed in the Appendix.
each itinerary reaches 12.7 unique fares, and given that the average itinerary sees 21 fare changes on average, this implies fares fluctuate up and down usually several times within 60 days.

2.4 Preliminary Evidence from the Data

With a description of the main features of the data complete, I now move into documenting preliminary evidence from the data. This analysis provides additional details concerning the data, but it is also meant to motivate features of the model. First, I document the pricing patterns in the data. This descriptive analysis shows airlines commonly implement systematic fare hikes, but fares frequently change daily. Second, I show that remaining capacity is important in explaining the variation in observed fares. Finally, I investigate if there is evidence that consumers anticipate systematic fare hikes.

2.4.1 Pricing Patterns

Figure 2 shows the frequency and magnitude of fare changes across time. The left panel indicates the fraction of itineraries that experience fare hikes versus fare discounts by day before departure, and the right panel indicates the magnitude of these fare changes (i.e. a plot of first differences, conditional on the direction of the fare change). For example, in the left plot, 40 days prior to departure, roughly 20% of fares increase and 20% of fares decrease. The remaining 60% of fares are held constant. Moving to the right panel, the magnitude of fare hikes and declines 40 days out is roughly $50.

The left panel confirms fares change throughout time, including fare declines. The fraction of fares that decline over time is roughly U-shaped, increasing through roughly three weeks prior to departure, peaking at 20%, and then declining to roughly 10% the day before departure. Note that well before the departure date, the number of fare hikes and declines is roughly split even. The fraction of itineraries that experience fare hikes increases over time. There are four noticeable jumps in the line indicating fare hikes. These jumps correspond to crossing 3, 7, 14 and 21 days prior to departure, or when advance purchase discounts placed on many tickets expire. The use of advance purchase discounts (APDs) is consistent with the story of intertemporal price discrimination, since fares increase unconditional on remaining capacity. Surprisingly though, the use of advance purchase discounts is not universal. Only 35% of itineraries experience fare hikes at 21 days and less than 60% increase at 14 days. Just under 70% of itineraries see an increase in fare when crossing the 7-day APD requirement.

\footnote{Advance purchase discounts are sometimes placed at 4, 10, and 30 days prior to departure, but this is not the case for the data I collect.}
Figure 2: Fraction and magnitude of fare changes by day before departure

The right panel shows the magnitude of fare changes towards the departure date, conditional on the direction of the change (increase/decrease). There are two other findings worth mentioning. First is that the magnitude of fare hikes and declines are similar – at around $50 but increasing in time. Second, the magnitude of fare increases when crossing APD days is similar to the magnitude of fare increases on other days.

**Figure 2** shows just how dynamic airline pricing is. Fares are constantly increasing and decreasing. Consistent with the theory of stochastic demand pricing, fire-sales do occur – roughly 10% of the itineraries in these sample decrease over $175 within 24 hours of the departure date. On the other hand, fares systematically increase at routine intervals, which is consistent with airlines segmenting the market between business and leisure customers.

Moving to statistics in levels, **Figure 13** in the Appendix plots the mean fare and mean load factor (seats occupied/capacity) by day before departure. The plot confirms the overall trend in prices is positive, with fares increasing from roughly $225 to over $375 in sixty days. The noticeable jumps in the fare time series occur when crossing the APD fences noted in **Figure 2**. At sixty days prior to departure, roughly 45% of seats are already occupied; consequently, I observe about half the bookings on any given flight. The booking curve for flights in the sample is smooth across time, leveling off roughly three days prior to departure at 85%, which closely matches monthly
enplanement totals found in the T100 Segment data. The fact that fares tend to double but consumers still purchase tickets is suggestive evidence that consumers of different types purchase tickets towards the date of travel.

### 2.4.2 The Role of Remaining Capacity

A key source of the variation in fares can be attributed to the scarcity of seats. I provide descriptive evidence of this two ways. First, I compare fares and load factor when two flights are offered a day. I calculate the difference in fare ($\Delta_{\text{fare}}$) and difference in load factor ($\Delta_{\text{LF}}$) across the two flight options by day before departure. When calculating the difference, I assign the first flight of the pair to be the flight with the greater number of seats occupied. This implies $\Delta_{\text{LF}} > 0$. By comparing fares for the two flights by day before departure, I control for systematic fare changes associated with intertemporal price discrimination.

**Figure 3:** The role of capacity in explaining fare variation

![Nonparametric regression of difference in fare by difference in load factor from comparing fares and load factors for one-way itineraries where two flights are available. $n = 20,062$.](attachment:image.png)

Figure 3 shows nonparametric fitted values as well as the 95% confidence interval of these calculations in percentage terms. The plot suggests that when the two options have the same number of seats occupied, the average difference in fare is close to zero. If one option is 20% more full, the flight which is more full is also 25%, or roughly $60, more expensive. The line remains upward sloping throughout observed differences in load factor, where at the extreme, a flight that is 30% more full is also 35% more expensive.\(^\text{18}\)

\(^{18}\)Applying similar methodology to round trip itineraries with two options – a single outbound flight and the choice of two return flights, or two outbound flights and a single return flight – yields similar results.
I perform a similar analysis using the entire sample by calculating the mean difference in fare and load factor at the flight number, day before departure level. Figure 4 shows nonparametric fitted values of this procedure. The line is again upward sloping across differences in load factor, where flights with lower load factor compared to the average are also less expensive. Likewise, flights that have a higher occupancy compared to the average are also more expensive.

Figure 4: Lowess of mean differences in fare and load factor.

2.4.3 Do consumers dynamically substitute across booking days?

The booking curve of flights plotted in Figure 13 is smooth, even though fares tend to increase by nearly $50 when crossing the advance purchase discount thresholds. This result is surprising since bunching in sales should be seen before the discounts expire if consumers anticipate systematic fare hikes. This is not the case as the only noticeable jump in load factor appears right before flights leave. I test for discontinuities in the booking curve using regressions of the following form:

\[
LF = \overrightarrow{APD} + m(t) + u + \varepsilon,
\]

where \(\overrightarrow{APD}\) are dummy variables corresponding to the day before advance purchase discounts expire, \(m(t)\) is a flexible function in time, and \(u\) are other fixed effects. Regression results appear in Table 6. Across all specifications, I find that none of the advance purchase discount dummies are significant; moreover, the 21 and 3 day advance purchase discount dummies are negative, which is inconsistent with bunching.
The fact that there is no evidence of bunching suggests that consumers either do not anticipate the fences or are possibly restricted in some other way from being able to purchase before the advance purchase discounts expire. Alternatively, it could be the case that consumers substitute to a different departure date, however this does not explain why the booking rate is similar after the discounts expire. I use this feature of the data to motivate a demand system where consumers do not dynamically substitute across booking days. Further, I show after estimating the model that since prices tend to increase across time, this provides little incentive for consumers to wait to purchase tickets.

3 A Model of Dynamic Airline Pricing

In this section, I write down a structural model of dynamic airline pricing where firms face a stochastic arrival of consumers, and the mix of consumer types – corresponding to leisure and business consumers – is allowed to change over time. To make the analysis tractable, I incorporate the following simplifications in the model. First, the model studies the pricing decisions of airlines operating in monopoly markets. In this way, the paper can focus on intertemporal price discrimination and stochastic demand, and avoid the complexities of modeling oligopolistic competition. As noted earlier, a large fraction of airline routes are monopoly. Second, I assume that when consumers first learn about their interest in travel, all travel plan uncertainty is immediately resolved. Consumers pay a fixed cost to come back and check on fares. Since fares tend to increase over time, the combined effect of these assumptions reduce the consumer problem to a static choice of either purchasing a ticket the day the consumer’s travel plans are realized, or not buying at all. Third, while in the actual airline business two consumers buying on the same day might face different fares (e.g. from different fare categories, or buying at different times of the day, or from different web sites), in the model a single fare is offered to consumers each day. Fourth, I model consumers as purchasing one-way tickets. A consumer interested in a round-trip can be thought of as two consumers interested in one-way tickets. As noted earlier, in the collected data, round-trip fares are very close to the corresponding one-way fares. Fifth, I assume firms utilize a finite set of fares. Firms take the set as given, and choose a single fare to offer for each flight daily. This assumption captures the fact that only a discrete number of fares is seen in the data. Finally, I assume firms do not oversell flights since most of the routes studied are operated by an airline that does not oversell flights. If demand exceeds remaining capacity, tickets are rationed.
3.1 Consumer Problem

A market is defined as an origin, destination, search date, departure date. At time $t$, $\tilde{M}_t \in \mathbb{N}$ consumers arrive interested in traveling between the two cities.¹⁹ For each of these newly-arrived consumers, all uncertainty about travel preferences is resolved at this point. This assumption differs from Lazarev (2013) and earlier theoretical work, including Gale and Holmes (1993), where consumer uncertainty exists and this uncertainty can be resolved by delaying purchase until closer to the departure date. In my model, when the date $t$ consumers arrive, they choose to either purchase a ticket on an available flight, exit the market, or pay a cost $\overline{\phi}$ to search again the following day. Throughout the rest of this section and when estimating the model, I assume $\overline{\phi}$ is sufficiently large so that waiting is never optimal. In Section 5.3, I calculate a bound on $\overline{\phi}$ for this assumption to hold, and show that it is relatively small – since fares tend to increase, there is little incentive to wait.

Preferences of consumers follow the earlier two-type consumer approach to study airline markets (Berry, Carnall, and Spiller (1996) and Berry and Jia (2010)). Consumer $i$ is either a business traveler or a leisure traveler. With probability $\gamma_i^t$, consumer $i$ is a business traveler. Consumer $i$ receives indirect utility from product characteristics $X_{jt} \in \mathbb{R}^K$ and price $p_{jt}$. Let $\beta_i, \alpha_i$ denote the taste parameters over $X_{jt}$ and $p_{jt}$, respectively. Let 0 denote the outside option. Each consumer $i$ receives idiosyncratic shocks $\varepsilon_{ijt}$ for each of the flights offered. Let $\varepsilon_{i0t}$ be the taste shock for the outside option. Following the discrete choice literature, consumer $i$ chooses flight $j$ iff

$$U_{ijt}(X, p, \beta, \alpha, \varepsilon) \geq U_{ij't}(X, p, \beta, \alpha, \varepsilon), \forall j' \in J \cup \{0\}.$$  

I assume utility is linear in product characteristics and of the form

$$U_{ijt} = X_{jt} \beta_i - \alpha_i p_{jt} + \varepsilon_{ijt}.$$  

Let $\varepsilon_i = (\varepsilon_{0i}, \varepsilon_{1i}, ..., \varepsilon_{Ji})$ be the idiosyncratic preference shocks for products in the choice set for consumer $i$. Define $y_t = (\alpha_i, \beta_i, \varepsilon_i)_{i=1, ..., \tilde{M}_t}$ to be the vector of preferences for the consumers that enter the market. The demand for flight $j$ at $t$ is a mapping given fares ($p_t$) and consumer preferences ($y_t$), defined as

$$Q_{jt}(p, y) := \sum_{i=0}^{\tilde{M}_t} 1\left[U_{ijt}(X, p, \beta, \alpha, \varepsilon) \geq U_{ij't}(X, p, \beta, \alpha, \varepsilon), \forall j' \in J \cup \{0\}\right] \rightarrow \{0, ..., \tilde{M}_t\},$$

¹⁹More broadly, since $\tilde{M}_t$ is market specific, $\tilde{M}_{t,d}$ consumers look to travel on $d$, $\tilde{M}_{t,d'}$ consumers look to travel on date $d'$, etc.
where \( 1(\cdot) \) denotes the indicator function.\(^{20}\)

Let \( s_t \in \mathbb{N}^J \) denote the remaining capacity for the \( J \) flights at time \( t \), and let \( s_{jt} \) define the capacity for a particular flight. Demand is integer-valued; however, it may be the case that more consumers want to travel than there are seats remaining, i.e. \( Q_{jt}(p,y) > s_{jt} \). Since firms are not allowed to oversell, in these instances, I assume remaining capacity is rationed by random selection. Specifically, I assume the market first allows consumers to enter and choose the product that maximizes utility. Consumers have no knowledge of remaining capacity so the probability of not getting a seat does not affect purchasing decisions. After consumers choose the flights that maximize their utility, the capacity constraints are checked. If demand exceeds capacity for a particular flight \( j \), consumers that selected flight \( j \) are randomly shuffled. The first \( s_{jt} \) are selected and the rest receive their outside option.

Recall that a market is departure date-specific. Although the model assumes consumers arrive and purchase a single one-way ticket, the model does allow for round-trip ticket purchases in the following way. At time \( t \), two consumers arrive. One consumer is interested in leaving on date \( d \), and another consumer is interested in returning on date \( d' \). The consumers receive idiosyncratic preference shocks for each of the available flights, and choose which tickets to purchase. Since the round-trip fares in the sample are very close to the sum of the corresponding one-ways, there is little measurement error in this approach.

### 3.2 Monopoly Pricing Problem

A monopolist sells tickets for \( J \) flights over a finite horizon. Period \( T \) corresponds to the first period of sale, and \( t = 0 \) is the time at which the flights depart. Each flight has an initial capacity constraint of \( s_{jT} \) seats, which is exogenous to the model. I assume the cost of operation is sunk, so the only cost facing the firm is the opportunity cost of selling seats.\(^{21}\)

The firm maximizes expected discounted revenues. The firm knows on average how many business and leisure travelers will search for tickets over time, but is unsure exactly how many consumers will arrive in any given period. Since fares are posted before consumers arrive, the firm forms expectation over present and future revenues. The decision rule depends on the number of seats remaining and the number of periods left to sell.\(^{22}\)

---

\(^{20}\)Here, and for remainder of the paper, I suppress the dependence of \( Q \) on \( X \) to emphasize the role of price. \( X \) may contain a flight characteristics such as whether a particular flight is a morning or afternoon flight. In addition, \( X \) may contain an indicator for a particular departure date which would allow the firm to use peak-load pricing.

\(^{21}\)I assume the marginal cost per passenger is zero, which is reasonable as almost all flight costs are not influenced by the number of seats occupied.

\(^{22}\)The decision may also depend on other observed (at least to the firm) states \( Z_t \). Note that \( X \subset Z \) since all product characteristics that enter the consumer problem affect purchasing behavior, and consequently, affect expected revenues. Like \( X \), I suppress the notation of \( Z_t \) to highlight the importance of remaining capacity \( s_t \) in the firm problem.
Since excess demand is rationed, by charging prices $p_t$ and receiving consumers $y_t$, the firm can sell at most $\min(Q_t(p, y), s_t)$ seats, where $\min(\cdot)$ is element-wise. This implies the law of motion for remaining capacity can be written as

$$s_{t-1} = s_t - \min\left(Q_t(p, y), s_t\right).$$

Define the incremental revenue for the firm to be

$$R_t(p, y, s) = \min\left(Q_t(p, y), s_t\right) \cdot p_t.$$

The firm is restricted to choosing prices $p_t \in P(s_t)$, where the dependence on seats is noted since only flights with positive remaining capacity are priced.

To write the firm’s problem as a dynamic program, define the value function, $V_t(s)$ to be the discounted expected revenue left to go with remaining capacities $s_t$ and $t$ periods to sell. The restrictions on capacity form two boundary conditions on the value function. The first is that with zero seats remaining, the firm cannot capture additional revenue, which is $V_{jt}(0) = 0$. Second, unsold seats the day the flights leave must be scrapped with zero value implying $V_{j0}(s) = 0$.

The firm’s problem can be written recursively as

$$V_t(s) = \max_{p_t \in P(s_t)} \mathbb{E}_y\left[R_t(p, y, s) + \rho V_{t-1}(s' | y, s)\right]$$

$$s.t.$$

$$\begin{align*}
    s_{t-1} &= s_t - \min\left(Q_t(p, y), s_t\right) \\
    V_{j0}(s) &= 0 \\
    V_{j1}(0) &= 0 \\
    s_t \text{ given,}
\end{align*}$$

where $\rho \in [0, 1]$ is the discount factor (which I set equal to 1).

The value function of the firm illustrates the important interactions between intertemporal price discrimination and dynamic adjustment to stochastic demand. If business consumers are less price sensitive and the proportion of business consumers increases as the departure date approaches, the firm can extract more revenue by increasing fares over time. However, since the arrival of consumers is uncertain, it is possible that a flight may sell out early. This creates an incentive for the firm to save seats until close to the departure date. Looking at the value function, if capacity becomes scarce early on, the firm can increase fares to reduce current period expected sales and revenues, but increase the probability that seats will remain the following day. For example, if the firm sets
If \( p_t = \infty \), then current period expected revenues would be zero, but the probability that \( s_t = s_{t-1} \) would be one. Hence, the firm would enter the subsequent period under \( V_{t-1}(s_t) \) which may be the optimal pricing strategy if the firm expects high-valuing consumers to arrive closer to the departure date. Alternatively, it may be the case that the firm receives a sequence of low demand realizations. In order to not leave as many seats unfilled, the firm may opt to lower prices.

### 3.3 Offering a single flight

Most markets in my sample have one flight a day. I concentrate my analysis to these flights as it greatly simplifies the demand system, as shown in Section 4. When a single flight is offered daily, the monopolist does not need to worry about substitution across flights. Recall that the firm does not know exactly how many consumers nor how many consumers of each type will arrive before the pricing decision is made. This is why the firm takes expectation over \( y_t \). With just a single flight offering, the expected sales for the flight is

\[
Q^e_t(s, p) = \int_{y_t} \min \left( Q_t(p, y), s_t \right) dF(y_t).
\]

By charging a price \( p_t \), the firm has a probability distribution over remaining capacity tomorrow. The value function for the firm can be rewritten as

\[
V_t(s) = \max_{p_t \in P(s_t)} \left[ p_t Q^e_t(s, p) + \rho E[V_{t-1}(s')|s] \right]
\]

\[
= \max_{p_t \in P(s_t)} \left[ p_t Q^e_t(s, p) + \rho \sum_{j=0}^{s_t} \Pr(s_{t-1} = j | p_t, s_t) V_{t-1}(j) \right],
\]

such that \( s_{t-1} = s_t - \min \left( Q_t(p, y), s_t \right) \) and the two boundary conditions hold.

The optimal pricing policy of the firm, \( p(t, s; \theta) \), depends solely on the consumer demand parameters \( \theta \), including taste preferences, the mix of consumer types, and the arrival process. In the next section, I discuss estimation of these parameters.

### 4 Econometric Specification & Estimation

I first parameterize the demand model and derive analytic expressions for purchase probabilities. I model the firm’s pricing decision as a dynamic discrete choice model. To estimate the structural parameters of the model, I solve the dynamic program of the firm.
4.1 Consumer Demand

First I derive the purchase probabilities for the consumer demand system. Recall that the preferences of consumers that arrive to the market are

\[ y_t = (\alpha_i, \beta_i, \epsilon_i)_{i \in 1, \ldots, \tilde{M}_t}. \]

The number of consumers, as well as the relative proportion of each type, that will arrive is not observed by the firm before pricing (or by the econometrician). In order to proceed, I integrate over the distribution of \( y_t \).

**Assumption 1:** Consumer idiosyncratic preferences are distributed Type-1 Extreme Value (T1EV).

I assume the outside option yields a normalized utility \( u_{i0t} = \epsilon_{i0t} \). The distributional assumption on the idiosyncratic preferences leads to the frequently used conditional logit demand system. Since there is only a single product in the choice set,

\[ \varsigma^i_t := \Pr(i \text{ wants to purchase } j | \text{type} = i) = \frac{1}{1 + \exp(-X_{jt}\beta_i + \alpha_i p_{jt})}. \]

The discrete choice literature typically does not model capacity constraints. Given my environment, it is possible that consumers wish to purchase a product, but are unable to due to a violation of the capacity constraint. As noted in the previous section, if demand exceeds available capacity, demand is rationed, and consumers who are not selected for travel receive their outside option. This is why the purchase probabilities state “i wants to purchase j” instead of “i purchases j”. Let \( \varsigma_{0t} \) denote the share of the outside option.

In practice, appropriate data to enter \( X \) would be variables such as if a flight is operated on a holiday or weekend. Preferences over these characteristics could also vary across consumer types and/or routes. While the model allows for these covariates, I assume consumers only care about price, which is not unreasonable as the routes studied are monopoly with a single flight daily.

Let \( B \) denote the business type and \( L \) to denote the leisure type. The probability of a consumer being type-\( B \) is \( \gamma_t \). Then \( \gamma_t \varsigma_t^B \) defines the probability that a consumer is of the business type and wants to purchase a ticket. Consider the market share of the outside good at time \( t \). Conditional on \( k \) consumers arriving, the probability of zero consumers wishing to travel is

\[ \Pr(Q_t = 0 \mid k \in \mathbb{N}) = \sum_{i=0}^{k} \binom{k}{i} \left( (1 - \gamma_t) \varsigma_{0t} \right)^i \left( \gamma_t \varsigma_{0t}^B \right)^{k-i}. \]

---

23I proceed this way because individual decisions are not observed. Using the EM-algorithm would be another way to address this issue. See: Talluri and Van Ryzin (2004) and Vulcano, van Ryzin, and Chaar (2010).

24When I solve the firm problem, adding additional characteristics makes the problem very computationally demanding. I do allow all parameters to be route-specific.
For example, let $k = 2$. Then conditional on two consumers arriving, both consumers want to purchase the outside option. Either both consumers are leisure travelers, both consumers are business travelers, or one of each type arrive. The first two situations correspond to $i = 2$ and $i = 0$, respectively since $((1 - \gamma)\varsigma_{L_{0}})^2$ is the probability of two leisure consumers arriving and wanting to choose the outside option, and $(\gamma\varsigma_{B_{0}})^2$ is the probability two business consumers want to choose the outside option. Lastly, it could be the case that one business and one leisure consumer arrives. There are two possibilities, the first consumer is the business consumer, or vice versa. Hence, $2[(1 - \gamma)\varsigma_{L_{0}}][\gamma\varsigma_{B_{0}}]$ enters the probability.

Next, integrating over the arrival process of consumers yields

$$\Pr\left(Q_{t} = 0 \right) = \sum_{k=0}^{\infty} \Pr(Q_{t} = 0, \tilde{M}_{t} = k) = \sum_{k=0}^{\infty} \Pr(Q_{t} = 0|\tilde{M}_{t} = k)\Pr(\tilde{M}_{t} = k).$$

**Assumption 2:** Consumers arrive according to a Poisson process, $\tilde{M}_{t} \sim \text{Poisson}(\mu_{t}).$\(^{25}\)

The probability mass function for the Poisson distribution is $(\mu_{t})^k e^{-\mu_{t}}/k!$, $k \in \mathbb{N}$. With this assumption, $\Pr\left(Q_{t} = 0 \right)$ has an analytic form and can be written as

$$\Pr\left(Q_{t} = 0 \right) = \sum_{k=0}^{\infty} \frac{\mu_{t}^k e^{-\mu_{t}}}{k!} \sum_{i=0}^{k} \binom{k}{i} \left((1 - \gamma)\varsigma_{L_{0}}\right)^i \left(\gamma\varsigma_{B_{0}}\right)^{k-i}.$$

Implicitly, this depends on both price and capacity remaining. With zero seats remaining, price is infinite so $\varsigma_{L_{0}} = \varsigma_{B_{0}} = 1$, which implies $\Pr\left(Q_{t} = 0 \right) = 1$, as expected. Next, consider the probability of selling a positive number of seats, but not selling out, conditional on $k$. This probability can be written as

$$\Pr\left(Q_{t} = q \mid k \geq q, q < s \right) = \binom{k}{q} \sum_{\ell=0}^{q} \binom{q}{\ell} \left(\gamma\varsigma_{B_{0}}\right)^{\ell} \left((1 - \gamma)\varsigma_{L_{0}}\right)^{q-\ell} \times \left[\sum_{i=0}^{k-q} \binom{k-q}{i} \left((1 - \gamma)\varsigma_{L_{0}}\right)^{i} \left(\gamma\varsigma_{B_{0}}\right)^{k-q-i}\right].$$

In the formula above, the terms following the first sum correspond to all the combinations of having $q$ consumers wanting to purchase a ticket. The terms following the second summation (second line) correspond to all the combinations of the remaining $k - q$ consumers wanting to purchase the outside option. Finally, $kCq$ at the beginning of the equation sorts all the possible combinations of (want to buy, do not want to buy) amongst the $q$ of $k$ consumers that wish to purchase tickets. Of

---

\(^{25}\)Given this assumption, the demand model closely follows Talluri and Van Ryzin (2004) and Vulcano, van Ryzin, and Chaar (2010), except this model has two consumer types.
course, selling $q$ seats requires at least $k = q$ consumers to enter so $\Pr(\text{sell } q \mid k < q, q < C, p) = 0$. Integrating over the Poisson arrival process results in an analytic expression for $\Pr(\text{sell } q \mid q < C, p)$, which is also a Poisson-Binomial mixture.

Demand is latent in situations where flights are observed to sell out since it is possible some consumers are forced to the outside option. These probabilities can be constructed based off the fact that at least $s_t$ seats are demanded. For example, if $s_t = 2$ and the flight is observed to sell out, that implies at least 2 seats were demanded, which also has an analytic expression of a Poisson-Binomial mixture. Since capacity is assumed to be monotonically decreasing, all purchase probabilities have been defined, which appear in Equation 4.1 - Equation 4.3 below. Collectively call these probabilities $f'(s' \mid s, p, t).$

\[
\Pr(Q_t \geq s \mid s, p) = \sum_{q=s}^{\infty} \sum_{k=q}^{\infty} \frac{\mu_t^k e^{-\mu_t}}{k!} \left( \binom{k}{q} \sum_{\ell=0}^{q} \binom{q}{\ell} \left( (1 - \gamma_t)s_t^B \right)^\ell \left( \gamma_t s_t^L \right)^{q-\ell} \times \right) \sum_{i=0}^{k-q} \binom{k-q}{i} \left( (1 - \gamma_t)s_t^0 \right)^i \left( \gamma_t s_t^B_0 \right)^{k-q-i} \right]
\]

\[
\Pr(Q_t = q \mid 0 < q < s, p) = \sum_{k=q}^{\infty} \frac{\mu_t^k e^{-\mu_t}}{k!} \left( \binom{k}{q} \sum_{\ell=0}^{q} \binom{q}{\ell} \left( (1 - \gamma_t)s_t^B \right)^\ell \left( (1 - \gamma_t)s_t^L \right)^{q-\ell} \times \right) \sum_{i=0}^{k-q} \binom{k-q}{i} \left( (1 - \gamma_t)s_t^0 \right)^i \left( \gamma_t s_t^B_0 \right)^{k-q-i} \right]
\]

\[
\Pr(Q_t = 0 \mid s > 0, p) = \sum_{k=0}^{\infty} \frac{\mu_t^k e^{-\mu_t}}{k!} \sum_{i=0}^{k} \binom{k}{i} \left( (1 - \gamma_t)s_t^L \right)^i \left( \gamma_t s_t^B_0 \right)^{k-i} .
\]

4.2 Solving the dynamic program of the firm

The firm’s pricing decision depends on remaining capacity and time to sell. I assume fares are chosen from a discrete set, which is exogenous to the model. I define this set to be the set of observed fares in the data. This assumption accurately reflects that fares for any given flight tend to fluctuate between relatively few distinct prices. This assumption allows me to write down the firm’s problem a dynamic choice model.

Since the firm’s decision depends on observed states, the model cannot capture why two flights that have the same number of seats remaining and time left to sell are priced differently in the data. To account for this, I assume the firm faces states $(\omega(p_t))_{p_t \in P(s_t)}$, which are choice-specific

\[26] To be clear, $s'$ denotes the number of seats sold in the current period given remaining capacity $s$. 

23
shocks observed by the firm, but not the econometrician, that affect the pricing decision. These
states allow the econometrician to account for the fact that a firm may choose two different prices
under identical states. Following Rust (1987), I make the following assumptions:

**Firm Assumptions:**

(i) The choice shocks are distributed Type-1 Extreme Value (T1EV);

(ii) The per-period payoff function and choice shocks are separable;

(iii) Conditional independence is satisfied.

These assumptions collectively lead to a dynamic discrete choice model, or more specifically, a
dynamic logit model. Due to conditional independence, the transition probabilities correspond to
the probabilities derived in the last section.\(^{27}\) In addition, these functions also can be used to define
the expected per period revenues, since \(\sum_{s'} f(s'|s,p,t) \cdot s' = Q^e(p,s)\).

Let \(V_t(s,\omega)\) be the discounted expected revenue at state \((s_t,\omega_t)\). \(V_t(s,\omega)\) solves

\[
V_t(s,\omega) = \max \left\{ \begin{array}{c}
v_t(p^1, s) + \omega(p^1) \\
v_t(p^2, s) + \omega(p^2) \\
\vdots \\
v_t(p^{|P|}, s) + \omega(p^{|P|})
\end{array} \right\},
\]

where \(\{v_t(p^k, s)\}_{k \in 1..|P(s_t)|}\) are the choice specific value functions and can be written as

\[
v_t(p^k, s) = Q^e_t(s, p^k)p^k_t + \rho \mathbb{E}_{s', \omega'|s, p^k} V(s', \omega') = ER_t(p^k, s) + EV_t(p^k, s).
\]

The T1EV assumption along with conditional independence imply the expected value functions
have a closed form and can be computed as

\[
EV_t(p, s) = \int_{s'} \left[ \sigma \ln \left( \sum_{p' \in P(s')} \exp \left( \frac{ER_t(p', s') + EV_t(p', s')}{\sigma} \right) \right) \right] f_t(s'|s, p) ds',
\]

\(^{27}\)That is, \(f(s', \omega'|s, \omega, p, t) = f(\omega'|s', t) f(s'|s, p, t)\)
where \( \sigma \) is the scale parameter the choice shocks (\( \omega \)).\(^{28}\) Lastly, the assumptions on the firm’s problem imply the conditional choice probabilities also have a closed form and can be computed as

\[
CP_t(p, s) = \frac{\exp(ER(p, s) + EV(p, s)/\sigma)}{\sum_{p' \in P(s)} (\exp(ER(p', s) + EV(p', s))/\sigma)}.
\]

### 4.3 Estimation Approach

In this section, I discuss estimation of the structural parameters. There are two possible approaches. The first strategy is to estimate the demand parameters without accounting for the optimal choice in price. This strategy would rely solely on the purchase probabilities derived in Section 4.1. However, as pointed out in Talluri and Van Ryzin (2004), for a specific price, the quantity of tickets sold in a given period can be higher either because of a higher overall arrival rate, or because consumers become more inelastic for a given arrival rate.\(^{29}\) The second approach is to estimate the model by accounting for the firm’s choice in price. I take this approach and argue this information provides additional identification power of the structural parameters.

To account for the firm’s pricing decision, I incorporate the dynamic discrete choice model of the firm (Section 4.2). With the assumptions in Section 4.2, given a set of flights \( (F) \) each tracked for \( (T) \) periods, the likelihood for the data is

\[
L(data|\theta) = \max_{\theta} \prod_F \prod_T CP_t(p, s) f(s'|s, p, t).
\]

Note that under the first strategy, the likelihood for the data is \( \prod_F \prod_T f(s'|s, p, t) \). By accounting for the firm’s pricing decision, the likelihood is disciplined through the conditional choice probabilities, and that Bellman’s equation is satisfied. One thing to note is that the parameters in the transition probabilities and the parameters in the per-period payoff function, which enter \( CP \), are the same – they are both functions of \( \theta \). For this reason, the transition probabilities cannot be estimated in a first stage (which would be just estimating the consumer demand model), as is

\[^{28}\text{Additionally,}
\]

\[
v_t(p, s) = ER_t(p, s) + \beta EV_t(p, s).
\]

\[
= \sum_{s' \neq 0} f(s'|s, p, t) p_t s' + \sum_{q=0} f(s'|s, p, t) V_{t-1}(s - s')
\]

\[
= \sum_{s' \neq 0} \left( f(s'|s, p, t) \cdot \left[p_t s' + \beta V_{t-1}(s - s') \right] \right).
\]

\[^{29}\text{Additional restrictions could be made, such as assuming the arrival process does not change over time, but this requires knowing the functional form of the mixture of consumer types. Table 7 provides Monte Carlo evidence that the demand parameters can be recovered under these assumptions.}\]
typically done using this framework, and in alternative methods of estimating dynamic models, including Hotz and Miller (1993) and Bajari, Benkard, and Levin (2007).

To estimate the structural parameters of the model, I solve the dynamic program of the firm using mathematical programming with equilibrium constraints (MPEC), shown to be consistent for the likelihood in Su and Judd (2012). The estimation procedure equates to solving the constrained maximization problem

\[
\max_{\theta, EV} \sum_{F} \sum_{T} \ln(CP_t(p, s)) + \ln(f(s'|s, p, t))
\]

such that \( EV = T(EV, \theta) \).

Recall that the arrival rate (\( \mu \)) and mixture of consumer types (\( \gamma \)) is allowed to change over time. I specify both sets of parameters to be continuous functions in time. I assign the Poisson arrival rates (\( \mu \)) to be a 4th-degree polynomial series. For the probability on types, I specify \( \gamma \) as a logistic function,

\[
\gamma_t = \frac{\exp(\gamma_1 + \gamma_2 t)}{1 + \exp(\gamma_1 + \gamma_2 t)^\gamma_2 \geq 0}
\]

This functional form assumption implies \( \gamma_t \in (0, 1), \forall t \) and that the probability of being a business consumer is increasing over time. However, this specification does not require the proportion of business consumers to strictly increase over time as \( \gamma_2 = 0 \) is allowed.

Since the firm knows the demand elasticities of consumers as well as the arrival process, studying the firm’s incentives to set prices given remaining capacity and time to sell relays important information regarding the structural parameters. By solving the firm’s dynamic program, I recover the shadow price of capacity across time and the pricing policy functions of the firm, which through the markup rule, provide a source of identification concerning demand elasticities. Variation in sales for a given number of seats and time left to sell help to identify the arrival process, while random realizations of capacity yield different prices for a given time period, that identify the mixture of consumers and their disutilities to price.

5 Results

In this section, I discuss the parameter estimates and the fit of the model. I then discuss how the estimated pricing policies relate to theory on dynamic pricing. Finally, I return to the assumption that the cost for consumers to search again the next period is sufficiently high that waiting is never optimal. I show only a small transactions cost is needed to make this assumption valid. In the next section, I conduct the counterfactual exercises.
5.1 Model Estimates and Fit

I estimate the model by city pair. I utilize observations for the last 45 days prior to departure, as average prices are relatively constant and bookings are low between 45 and 60 days prior to departure. Here I discuss the results for one city pair, which I use to conduct the counterfactual exercises in the next section. In the Appendix, I compare results across routes.\(^{30}\)

Parameter estimates appear in Table 8. All parameters are significant at the 1% level. The parameter estimates imply business consumers are over three times less price sensitive than leisure consumers, and are willing to pay up to 75% more in order to secure a seat. Figure 14 plots fitted values of the Poisson arrival rate and probability on types across time. Estimates of the arrival rate start around eight persons per flight 45 days prior to departure. The process peaks roughly three weeks prior to departure with a rate just under 12 persons per flight. From there, the potential market decreases in time with the lowest arrival of consumers appearing the day before departure. Here the arrival rate is just under 8 persons per flight.

The model estimates suggest a large shift in the makeup of consumers across time. More than a month prior to departure, the share of business travelers is close to zero. Starting at approximately two weeks prior to departure, corresponding to the 14-day advance purchase discount, the share of business consumers in the market increases dramatically from 20% fourteen days to nearly 80% the day before departure.

Figure 15 plots the median model fares and observed fares by day before departure.\(^{31}\) The plot shows that the model fares are quite similar to observed fares, with differences of usually less than $25 between 45 and 10 days prior to departure. The model accurately picks up the increasing pattern of fares within three weeks of the departure date. The model predicts fares to increase to their highest levels between seven to three days prior to departure, whereas in the data, the highest fares occur over the last five days prior to departure. With model prices, flights with excess capacity result in fire-sales, with the 50% percentile decreasing nearly $75. The reason for this last-minute decline in prices is that there is no value of holding capacity in the last period. Recall in the data fire-sales do occur – roughly 10% of the time, but presumably, fire-sales do not occur more often because otherwise business consumers would learn that delaying purchase results in lower fares.

5.2 Optimal Pricing

Figure 5 plots the optimal pricing policies and expected revenues for the firm as a function of remaining capacity. The left panel plots the optimal price given remaining capacity for three

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\(^{30}\) These tables and the corresponding counterfactual analysis will appear in a forthcoming draft.

\(^{31}\) Mean fares appear in the counterfactuals.
selected periods, corresponding to 15, 30, and 45 days prior to departure. The right plot indicates the expected revenues associated with remaining capacities for these three periods.

**Figure 5: Estimated Policy Functions and Expected Revenues**

The right panel shows that expected revenues are increasing in capacity for a given period, i.e. $V_t(s) \leq V_t(s+1)$. Second, the panel shows that expected revenues are increasing in time to sell for a given capacity, i.e. $V_t(s) \leq V_{t+1}(s)$. These results are consistent with the theory on dynamic pricing found in Gallego and Van Ryzin (1994). Expected revenues flatten out for a given period because the firm cannot capture additional revenue when there is sufficiently high capacity remaining. The plot shows that the probability of a sell out is close to zero if the firm has at least 30 seats remaining with 15 days left to sell. With 30 days remaining, there is excess capacity when at least 55 seats are remaining.

The left panel plots the policy function of the firm, $p(s,t)$. As already mentioned, with 30 seats remaining and 15 days left to sell, the probability of a sell out is close to zero. In return, the firm charges a low price. However, the price charged by the firm with 30 seats remaining is increasing in time to sell; that is, $p(30, -15) < p(30, -30) < p(30, -45)$. This result is driven by the complementarity between intertemporal price discrimination and dynamic adjustment to stochastic demand when business consumers arrive close to the departure date. With additional time to sell for a given capacity, the probability of a sell out increases and in particular, there is an increased probability that a sell out would occur under a low price before business consumers become active.
By charging a higher price early on when capacity is expected to be scarce, the firm can save seats for the high-valuing consumers who arrive late.

5.3 Allowing Consumers to Wait

The demand model assumes that waiting is never optimal. This assumption was motivated by the fact that there are no discontinuities in bookings immediately before advance purchase discounts expire. In this section, I study the incentives of consumers to wait in purchasing tickets.

I change the model in the following way: after consumers arrive, each consumer has the option to either buy a ticket, choose not to travel, or wait one additional day to decide. By choosing to wait, each consumer retains her private valuations (the \( \varepsilon \)'s) for traveling but may be offered a new price tomorrow. Consumers do not have perfect foresight, so they forecast both fares and remaining capacity for the next period. Additionally, each consumer has to pay a transactions cost \( \phi_i \) to wait. This cost reflects the disutility consumers incur when needing to return to the market the next period. The goal of this section is to derive a waiting cost \( \bar{\phi} \) such that if all consumers have a waiting cost at least as high as \( \bar{\phi} \), then no one will wait. I then calculate the waiting cost in the data.\(^\text{32}\)

Dropping the \( i \) subscript, the choice set of a consumer arriving at time \( t \) in a model of waiting is

\[
\max \left\{ \varepsilon_0, \beta - \alpha p_t + \varepsilon_1, EU^{\text{wait}}(p, s) - \phi \right\},
\]

where \( EU^{\text{wait}} \) is the expected value of waiting and can be written as

\[
EU^{\text{wait}}(p, s) = \mathbb{E}_{p'|p, s} \left[ \max \{ \varepsilon_0, \beta - \alpha p_t - 1 + \varepsilon_1 \} \right],
\]

To derive \( \bar{\phi} \), I first investigate the decision to wait for the marginal consumer, or a consumer such that \( \varepsilon_0 = \beta - \alpha p_t + \varepsilon_1 \). This consumer has no incentive to wait if the price tomorrow is at least as high as today. If price drops, the gain from waiting is

\[
u_{t-1} - u_t = (\beta - \alpha p_{t-1} + \varepsilon_1) - (\beta - \alpha p_t + \varepsilon_1) = \alpha (p_t - p_{t-1}),
\]

\(^{32}\)For the proceeding analysis, I assume capacity is infinite. This means \( \bar{\phi} \) is not the lower bound on waiting costs because the probability of not getting a seat creates an additional incentive to not wait, i.e. there is a positive probability of being offered an infinite price. By ignoring capacity, the consumer does not forecast the arrival process, or make decisions due to possible rationing.
which implies the expected gains from waiting are \( \Pr(p_{t-1} < p_t) \mathbb{E}[\alpha(p_t - p_{t-1}) | p_{t-1} < p_t] \). Hence, an indifferent consumer will not wait if \( \phi_i > \overline{\phi} := \Pr(p_{t-1} < p_t) \mathbb{E}[\alpha(p_t - p_{t-1}) | p_{t-1} < p_t] \).

**Proposition 1:** With \( \overline{\phi} := \Pr(p_{t-1} < p_t) \mathbb{E}[\alpha(p_t - p_{t-1})] \), then all consumers will choose not to wait.

**Proof:** See Appendix.

In monetary terms, \( \tilde{\phi} = \overline{\phi} / \alpha = \Pr(p_{t-1} < p_t) \mathbb{E}[\alpha(p_t - p_{t-1})] \) defines the the transaction cost a consumer would have to face in order to never wait. These statistics can be calculated in the data. Figure 6 plots \( \tilde{\phi} \) across time using the model estimates. I find that the average transactions cost required to make consumers not wait to be $5.49. For many days, the transactions required to make consumers not wait is close to zero. Notably, \( \tilde{\phi} \) is higher close to the departure date as there are greater incentives to wait under model prices. In particular, business consumers who arrive the day before departure must incur a high waiting cost to persuade them not to wait. By waiting, they can capture fire-sale prices.

**Figure 6:** Plot of mean transactions costs that would induce consumers not to wait

![Average transactions cost across time](image)

The calculated waiting costs suggest that the assumption that consumers do not dynamically substitute is very reasonable for almost all days prior to departure. The result is driven by the upward price trend. Since prices tend to increase, there is little incentive to wait a day to purchase once consumers first learn about their interest in travel. Further, while the required transaction cost is high close to the departure date, this analysis analyzed the indifferent consumer. Consumers who prefer to either purchase or not purchase would require a lower cost to not wait.
6 Analysis of the Estimated Model

In this section, I use the estimated model to examine three issues. In Section 6.1, I first perform three exercises that reduce the firm’s ability to price discriminate. I investigate uniform pricing, dynamic pricing but restrict the number of price changes, and a pricing system which depends on date of purchase, but not remaining capacity. Section 6.2 uses the estimated model to highlight the complementarity between intertemporal price discrimination and stochastic demand pricing. Finally, in Section 6.3, I show how estimation approaches that do not take into account stochastic demand will systematically produce biased estimates of the degree to which demand becomes more inelastic as the departure date approaches.

For each exercise, I use the empirical distribution of remaining capacity 45 days prior to departure as the initial capacity condition.\textsuperscript{33} I maintain the assumption that prices are chosen from the price set observed in the data. Relaxing this assumption so that prices are continuous yields qualitatively similar findings. For each counterfactual, I calculate the following benchmarks:

- Fare: overall mean fare for flights that have not sold out;
- LF: mean load factor the day flights leave;
- Sell outs: percent of flights that sell out after the last pricing period;
- Rev: mean revenue across flights;
- $CS_L$: mean leisure consumer surplus;
- $CS_B$: mean business consumer surplus;
- SW: mean daily welfare across flights (less sunk costs).

6.1 The Welfare Effects of Flexible Pricing

In the model, the firm can set prices flexibly over time, to respond to changes in the consumer composition, and in response to random realizations of demand. At the opposite extreme is a pricing system that sets a uniform price over the entire time period. This subsection compares these extreme cases: dynamic pricing to uniform pricing. I also examine an intermediate case, where prices depend upon time to departure, but not on random fluctuations in demand. By comparing uniform to the intermediate case, and the intermediate to the full, the following analysis separates out the gains from intertemporal price discrimination from the gains to adjusting to stochastic demand.

\textsuperscript{33}I simulate 100,000 flights using this distribution. This provides an initial capacity condition between 6 and 83 seats. This controls for the fact that I model pricing for the last 45 days prior to the departure date. With just six seats remaining, the probability of selling out exceeds 85% under dynamic pricing versus less than 5% for a flight with 83 seats.
Uniform Pricing

I start by removing the firm’s ability to adjust prices as the departure date approaches. The firm maximizes expected revenues subject to the constraint that it must charge a uniform price across time. The price is solely dependent on the initial capacity condition.

Under uniform pricing, a high fare ensures the firm can successfully secure seats for business consumers, but doing so prices leisure consumers out of the market. At the same time, going after only the business market results in unused capacity which can be filled by lowering the fare. The optimal pricing strategy is one that balances saving seats for the high-valuing customers who arrive close to the departure date along and filling seats that would otherwise be scrapped.

<table>
<thead>
<tr>
<th>Policy / Mean</th>
<th>Fare</th>
<th>LF$^f$</th>
<th>Sell outs</th>
<th>Rev$^f$</th>
<th>CS$^{i_L}$</th>
<th>CS$^{i_B}$</th>
<th>SW$^t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic</td>
<td>$208.70</td>
<td>95.69%</td>
<td>28.65%</td>
<td>$9,956.51</td>
<td>$52.59</td>
<td>$171.97</td>
<td>$955.85</td>
</tr>
<tr>
<td>Uniform</td>
<td>$197.62</td>
<td>91.87%</td>
<td>40.46%</td>
<td>$9,299.37</td>
<td>$51.46</td>
<td>$184.50</td>
<td>$951.35</td>
</tr>
<tr>
<td>Difference (%)</td>
<td>-5.30%</td>
<td>-3.82%</td>
<td>11.81%</td>
<td>-6.60%</td>
<td>-2.15%</td>
<td>7.29%</td>
<td>-</td>
</tr>
</tbody>
</table>

Results for the counterfactual appear in Table 2. The left panel plots mean fare, weighted by initial capacity, across time for flights that have not yet sold out. The plot shows that the uniform price is higher relative to dynamic prices well in advance of the departure date. An optimal uniform fare of nearly $200 means only leisure consumers with high private valuations actually purchase.
The fare is sufficiently high that the firm can save (some) seats for the high-valuing consumers who arrive close to the departure date.

The fact that fares are relatively higher under uniform pricing early on, but relatively lower closer to the departure date results in a significant reallocation of capacity across time. This is shown in the right panel, which plots the booking curve, or mean cumulative seats sold, towards the departure date. The uniform pricing booking curve is bowed out as fewer consumers purchase under the relatively higher fare early on. Relatively lower fares close to the departure date results in a higher booking rate compared to dynamic pricing through the day of departure. However, even with this increase in the booking rate, the overall load factor for flights under uniform pricing is lower than dynamic pricing. I find that 17% fewer seats are purchased by leisure consumers under uniform pricing. At the same time, nearly 17% more seats are booked by business consumers under uniform pricing. Since more leisure consumers purchase tickets than business consumers, the net change in bookings under uniform pricing is negative.

There is also a large reallocation within consumer type. Figure 7 plots the fraction of consumer types that purchase conditional on entering the market (i.e. number of consumers that buy / number of consumers that enter). The left panel shows that under dynamic pricing, the purchase rate amongst leisure consumers is close to 11% well before the departure date. As prices increase under dynamic pricing, very few leisure consumers purchase. On the other hand, under uniform pricing, fares are high early on resulting in a 33% decline in purchases by leisure consumers. The take up rate is constant, but also declines towards the departure date. This is because under uniform pricing, many flights sell out in advance. For business consumers, the lower fares offered by uniform pricing results in higher purchasing rates across all periods. Again, the uniform pricing rate declines towards the departure date since many flights sell out in advance.

The change in allocation under uniform pricing is mitigated because of two forces. First, as discussed above, uniform pricing implies leisure consumers with high private valuations purchase throughout time, whereas under dynamic pricing essentially no leisure consumers purchase when fares go over $250. While leisure consumers are made worse off due to high prices early on, it also means some successfully purchase closer to the departure date and overall, leisure consumer surplus decreases over 2% under uniform pricing. Second, while business consumers benefit from lower prices, the increase in business consumer surplus is mitigated because of the increased number of early sell outs. An important consequence of uniform pricing is that the firm cannot control the booking rate of flights that turn out to be popular. Uniform pricing increases sell outs by 12%, which consequently forces more late-arriving business consumers to the outside option. Hence, business consumer surplus increases 7% under uniform pricing.
On average, leisure consumer welfare declines and business consumer welfare increases under uniform pricing. Overall, I find consumer welfare is 1.36% higher under uniform pricing compared to dynamic pricing. The consumer welfare gains in the absence of price discrimination are relatively modest as a result of inefficient capacity allocation. Further, compared to dynamic pricing, revenues fall 6.6% under uniform pricing. As airlines operate razor thin margins, the decline in revenues is significant, and suggests the firm would probability choose to exit the market in the long run. Moreover, total welfare is lower under uniform pricing compared to dynamic pricing.

**The Role of Frequent Price Adjustments**

The previous exercise compared the extremes in pricing capabilities of the firm – either the firm maintains a single price across time, or the firm can update prices daily. Now I allow the firm to use dynamic pricing, with the restriction that prices must be maintained for $k$ days. I conduct four counterfactuals, corresponding to $k = 3, 5, 9, 15$. The idea here is that dynamic pricing is clearly valuable to the firm, but it is not necessarily true that daily price adjustments are needed to obtain the revenues observed under (daily) dynamic pricing.

**Figure 8** plots the revenue loss compared to the baseline case of daily price adjustments, for the four counterfactuals. For example, while uniform pricing reduces revenue by 6.6% compared to daily dynamic pricing, the ability to update prices every 15 days increases revenue by over 1%. The difference in revenues between 15 day adjustments and 9 day adjustments is quite large. I find that
Figure 8: Comparing different degrees of dynamic pricing

Revenues are 1.1% lower under 5-day adjustments compared to daily dynamic pricing. Under 3-day adjustments, revenues are 0.5% less than under daily adjustments. There are two ways to interpret these results. The first is that while 9 and 15 day adjustments result in significant revenue declines, adjustments made at 3 and 5 day intervals result in revenues similar to daily adjustments. At the same time, even under 3-day adjustments, revenues are nearly half a percent lower compared to daily adjustments. As reported by the IATA (2013), the margins for airlines are very small (around 1%), suggesting that even the losses associated with 3 and 5 day adjustments would lead to a significant decline in profits in percentage terms.

The Use of Intertemporal Price Discrimination Alone

I next single out the intertemporal price discrimination force by investigating pricing policies which depend on day until departure, but not on the scarcity of seats. Viewed from the dynamic pricing perspective, this counterfactual investigates pricing with the dynamic adjustment force shut down and thus, quantifies the complementarity between the two pricing forces. Specifically, this counterfactual quantifies the additional revenue gains possible by dynamically adjusting fares.

Under intertemporal price discrimination alone the optimal sequence of fares, $p^*$, solves

$$p^* \in \arg\max_{p \in \mathcal{P} \times \mathcal{T}} \mathbb{E}_y \left[ \sum_t \min \left\{ Q_t(p, y), s_{t+1} - \min \left( Q_{t+1}(p, y), s_{t+1} \right) \right\} p_t \big| s_T \right],$$
where \( s_{t+1} - \min \left( Q_{t+1}(p, y), s_{t+1} \right) \equiv s_t \) is the capacity remaining after the previous period’s demand is realized and \( s_T \) is the initial capacity condition. The decision space of the firm’s problem has cardinality \( |\mathcal{P}|^T \). I simplify the problem by adding the restriction that the firm only adjusts fares on the usual advance purchase discount days – 3, 7, 14, and 21 days prior to departure.\(^{34}\)

\[ \text{Table 3: Dynamic vs. Intertemporal Price Discrimination} \]

<table>
<thead>
<tr>
<th>Policy / Mean</th>
<th>Fare</th>
<th>LF</th>
<th>Sell outs</th>
<th>Rev</th>
<th>( CS_L )</th>
<th>( CS_B )</th>
<th>SW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic</td>
<td>$208.70</td>
<td>95.69%</td>
<td>28.65%</td>
<td>$9,956.51</td>
<td>$52.59</td>
<td>$171.97</td>
<td>$955.85</td>
</tr>
<tr>
<td>Discrim. Only</td>
<td>$205.94</td>
<td>90.01%</td>
<td>31.43%</td>
<td>$9,608.13</td>
<td>$52.01</td>
<td>$171.54</td>
<td>$942.29</td>
</tr>
<tr>
<td>Difference (%)</td>
<td>-1.32%</td>
<td>-5.68%</td>
<td>2.78%</td>
<td>-3.49%</td>
<td>-1.01%</td>
<td>-0.25%</td>
<td>-</td>
</tr>
</tbody>
</table>

Results for this counterfactual appear in Table 3. The time path of prices under intertemporal price discrimination alone are monotonically increasing, with a substantial increase in fares when crossing the 14-day advance purchase discount. This corresponds to the increase in proportion of business consumers given the model estimates. The fares offered to leisure consumers well before the departure date are slightly higher under intertemporal price discrimination alone. The reason for this is because without dynamic adjustment, there is an additional incentive for the firm to reserve capacity for business consumers. In order to do this, the firm charges higher prices early on, which decreases the number of seats purchased by leisure consumers compared to dynamic

\[ \text{I have also investigated the pricing decision over } |\mathcal{P}|^T \text{ using simulated annealing. I obtain similar findings compared to this setup.} \]

\(^{34}\)
Importantly, as the pricing becomes more flexible, the fares offered early on are lower. Fares under intertemporal price discrimination alone are substantially lower than uniform pricing well in advance of the date of travel. Under dynamic pricing, fares are even lower. The point where leisure consumers stop purchasing is similar under both intertemporal price discrimination and dynamic pricing, and as a result, overall leisure consumer surplus has the following order: $CS_L^{\text{dynamic}} > CS_L^{\text{discrim.}} > CS_L^{\text{uniform}}$. That is overall, but within consumer type there is also a reallocation, and the ordering is reversed for the leisure consumers that arrive closer to the departure date. Business consumer surplus is essentially unchanged as prices are either higher or lower under dynamic pricing depending on when they arrive. Moreover, without dynamic adjustment, firms have a reduced ability to save seats for late-arriving business consumers, which results in a 2.8% increase in sell outs.

Not only is consumer surplus (marginally) higher under dynamic pricing, revenues are substantially higher. Revenues under intertemporal price discrimination alone are 3.5% lower compared to dynamic pricing. These results demonstrate there is a significant complementarity between the pricing channels. Allowing firms to adjust prices dynamically results in additional revenues from early-arriving leisure consumers. At the same time, dynamic adjust allows airlines to secure seats on flights that are realized to have high demand. When the business consumers arrive, they receive higher fares compared to pricing under intertemporal price discrimination alone. In terms of value, these results suggest 50% of the revenue gains associated with dynamic pricing over uniform pricing comes from dynamic adjustment.

### 6.2 The Complementarity of Intertemporal Price Discrimination and Stochastic Demand Pricing

This subsection shows how intertemporal price discrimination and stochastic demand pricing are complements in the airline industry. It arises because of the particular pattern of consumer arrival. The consumers who arrive last are the ones with the highest willingness to pay, which creates an incentive for firms to save seats until close to the departure date. If seats become scarce early on, the firm’s optimal pricing strategy is to sharply increase fares, which reduces the rate at which seats are sold. With capacity reserved, the firm then price discriminates toward the late-arriving consumers by charging them high fares.

The point about this complementarity can be made with a simple example. Consider a reversal of the arrival process, where high-valuing consumers arrive first. In particular, suppose there are

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35Revenue Management Overview states revenue management systems have increased airline revenues by 3-9%.
three periods and a flight has two seats. In each period, a consumer arrives with a 50/50 probability. In the first period \((t = 1)\), if the consumer arrives, the reservation price is $1000. For the remaining two periods \((t = 2, 3)\) the reservation price is $200. In this environment, the profit maximizing policy is to set \(p_1 = 1000\) and \(p_2 = 200\), and \(p_3 = 200\). There is no need for price to respond to demand since the high-valuing consumer is guaranteed a seat. In this case, there are no gains to having a pricing system that reacts to demand realizations.

Now reverse the order of arrival so that in the first two periods, the reservation price is $200, and in the last period, the reservation price is $1,000. Under dynamic pricing, the firm will charge a price of $200 in the first period. If a seat sells, the firm will set a price \(p_2 > 200\), and finally, will set a price of $1,000 in the last period. If the seat does not sell, the firm will charge \(p_2 = 200\) and \(p_3 = 1000\), which yields an expected profit of $650.\(^{36}\) However, if prices are not allowed to respond to demand, the firm will set up a fare schedule that reserves the seat for the potential business consumer. By charging \((p_1, p_2, p_3) = (\infty, 200, 1000)\), the firm has expected revenues of $600. Hence, the airline would be willing to pay up to $50 to utilize a pricing system that responds to demand realizations.

This simple example demonstrates the importance of dynamic adjustment when high-valuing consumers arrive last. In particular, it is important that airlines price to keep seats available until close to the departure date. To get a sense of the magnitude of this force for the airline industry, I perform the same exercise above using the estimated model. I reverse the arrival process of consumers and compare revenues under dynamic pricing with a pricing system that only depends on the date of purchase. Note that if the arrival process was constant, and the mix of consumer types did not change over time, reversing the order would have no effect on revenues. Thus the magnitude of the difference will depend upon how stochastic demand is, and the extent to which elasticity varies. Both of these are pinned down in the estimation exercise.

**Reversal of the Arrival Process**

Table 4 presents the counterfactual results of dynamic pricing, intertemporal price discrimination alone, and uniform pricing when the arrival process is reversed. Compared to when business consumers arrive late (observed arrival process), a reversal of the arrival process brings: lower fares, higher load factors, more sell outs, and increased revenues (see Table 5 for comparisons). Overall consumer surpluses are also higher under a reversal of the arrival process, with the exception of leisure consumer surplus under uniform pricing, which is lower, but very close to the observed arrival process.

\(^{36}\)If the seat sells, the expected revenues for the remaining period are $500. If the seat does not sell, the expected revenues are \(1/2 \cdot 200 + 1/2 \cdot 1000\). Hence, \(ER = 1/2(200) + 1/2(500) + 1/2(1/2 \cdot 200 + 1/2 \cdot 1000) = 650\)
Table 4: Reversal of the arrival process

<table>
<thead>
<tr>
<th>Policy / Mean</th>
<th>Fare</th>
<th>LF</th>
<th>Sell outs</th>
<th>Rev</th>
<th>CSₐ</th>
<th>CSₜ</th>
<th>SW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamic</td>
<td>$205.61</td>
<td>96.69%</td>
<td>49.67%</td>
<td>$10,086.01</td>
<td>$52.86</td>
<td>$172.60</td>
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<td>Discrim. Only</td>
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</tr>
<tr>
<td>Uniform</td>
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<td>91.96%</td>
<td>40.83%</td>
<td>$9,304.30</td>
<td>$51.34</td>
<td>$190.20</td>
<td>$957.15</td>
</tr>
</tbody>
</table>

With a reversal of the arrival process, firms have no incentive to hold remaining capacity. This is particularly noticeable under intertemporal price discrimination alone. Under the reversed arrival process, leisure consumers who arrive within 21 days of the departure date receive fares that are $10-$20 lower on average. Fares increase slightly under dynamic pricing closer to the departure date for flights with scarce capacity, driving up mean fares. Flights that have not already sold out result in fare-sales the day before departure as the firm tries to fill any remaining open seats. The lower prices offered with a reversed arrival process result in substantially higher sell outs. As shown in Table 5, the ordering of percent of sell outs is reversed under the two arrival processes. The percent of sell outs under dynamic pricing nearly doubles. Under intertemporal price discrimination alone, sell outs are 17% higher.

The most important feature of the reversed arrival process is the role of intertemporal price discrimination. While revenues are $100 greater per flight with dynamic pricing across arrival processes and only $5 per flight more under uniform pricing, the difference in revenues under intertemporal price discrimination alone is $250 across arrival processes. The other relevant measure is the revenue across policies within arrival process. As previously discussed, 50% of the revenue
Table 5: Comparing Arrival Processes

<table>
<thead>
<tr>
<th>Policy</th>
<th>Arrival / Mean</th>
<th>Fare</th>
<th>LF</th>
<th>Sell outs</th>
<th>Rev</th>
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<td>Dynamic</td>
<td>Reversed</td>
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<td>$9,956.51</td>
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<tr>
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<td>Reversed</td>
<td>$204.89</td>
<td>93.72%</td>
<td>48.79%</td>
<td>$9,851.89</td>
</tr>
<tr>
<td></td>
<td>Observed</td>
<td>$205.94</td>
<td>90.01%</td>
<td>31.43%</td>
<td>$9,608.13</td>
</tr>
<tr>
<td>Uniform</td>
<td>Reversed</td>
<td>$196.53</td>
<td>91.96%</td>
<td>40.83%</td>
<td>$9,304.30</td>
</tr>
<tr>
<td></td>
<td>Observed</td>
<td>$197.62</td>
<td>91.87%</td>
<td>40.46%</td>
<td>$9,299.37</td>
</tr>
</tbody>
</table>

gains of using dynamic pricing over uniform pricing can be attributed to intertemporal price discrimination. The remaining can be attributed to dynamic adjustment. However, when the arrival process is reversed, 75% of the revenue gains can be attributed to intertemporal price discrimination. The reason for this is because the role of dynamic adjustment is unique to the arrival process. When business consumers arrive late, dynamic adjustment is used to reserve capacity across time. With business consumers arriving early, firms can capture their willingness to pay solely through the intertemporal price discrimination channel. The role of dynamic adjustment is simply to fill remaining seats after the business market is served.

Constant Arrival Process

The last counterfactual demonstrated the importance of dynamic adjustment in complementing intertemporal price discrimination when business consumers arrive late. The counterfactual also established that the role of dynamic adjustment is unique to the arrival process. In this counterfactual, I investigate pricing under a constant arrival process. Demand is still stochastic, but the mix of consumers that arrive over time is held constant. This shuts down the use of the intertemporal price discrimination channel. In this environment, prices adjust across time solely due to the scarcity of seats under stochastic demand.

Figure 9 plots the mean price under dynamic pricing and uniform pricing under constant arrival.\textsuperscript{37} Notably, instead of prices increasing over time, prices simply fluctuate around the levels offered under uniform pricing. Fares increase for flights that are realized to be scarce close to the departure date in a similar fashion to the dynamic pricing policies under the reversed arrival

\textsuperscript{37}Under constant arrival, I take the probability of a consumer being of the business type to be the mean of $\hat{\gamma}$ for all periods
process. Flights are then offered under fire-sale prices as to not leave as many seats unfilled. One of the predictions found in Gallego and Van Ryzin (1994) is that the value of dynamic pricing is lower under constant arrival. My empirical findings support this theory. While revenues increase 6.6% by using dynamic pricing over uniform pricing under observed arrival, the revenue gains under constant arrival are half of that.

6.3 Consequences of Ignoring Stochastic Demand

In this section, I illustrate that in order to conduct welfare analysis in markets where both the intertemporal price discrimination and dynamic adjustment forces operate, it is important to take into account the uncertainty about demand. In particular, by ignoring stochastic demand, an empirical analysis will fail to take into account the opportunity cost of holding back capacity tends to fall over time, which leads to a systematic bias in estimating demand elasticities.

Consider the pricing of a single flight. Suppose demand is stochastic but the empirical approach ignores the uncertainty about demand. In this case, the firm faces a static problem which can be written as

$$\max_p \sum_t Q_t(p_t)p_t \quad \text{s.t.} \quad \sum_t Q_t(p_t) \leq s.$$
Letting \( c(s) \) be the shadow price of capacity, the firm problem can be written as the following unconstrained problem:\[^{38}\]

\[
\max_p \quad \sum_t Q_t(p_t)p_t - c(s) \left( \sum_t Q_t(p_t) - s \right)
\]

\[
\Leftrightarrow \max_p \quad \sum_t Q_t(p_t) \left( p_t - c(s) \right) + c(s)s.
\]

Letting \( c \) be the opportunity cost at the optimum, Lerner’s index reveals

\[
\frac{p_t - c}{p_t} = \frac{1}{e(D)(p_t)},
\]

where \( e(D)(p_t) \in \mathbb{R}_+ \). Rearranging terms to solve for price yields the markup rule, \( p_t = \frac{e(D)(p_t)}{e(D)(p_t) - 1}c \).

Then, taking the ratio of prices over time, in this case with the first period, reveals

\[
\frac{p_t}{p_T} = \frac{e(D)(p_t)}{e(D)(p_T) - 1}c = \frac{e(D)(p_T)}{e(D)(p_T) - 1},
\]

(Ignoring stochastic demand)

In the equation above, the opportunity costs cancel which reveals that information on prices directly relate to elasticity ratios. On the other hand, when accounting for stochastic demand, the opportunity cost changes over time and the terms do not cancel. Instead,

\[
\frac{p_t}{p_T} = \frac{e(D)(p_t)}{e(D)(p_T) - 1}c_t.
\]

(Accounting for stochastic demand)

In particular, the opportunity cost of selling a given seat tends to be lower closer to the departure date \( (c_t \leq c_T) \), because if a seat is not sold in the current period there is less of a chance it will be sold in the future. As a result, an empirical analysis ignoring stochastic demand will result in biased demand elasticities, especially close to the departure date as the opportunity cost tends to fall.

**Figure 10** demonstrates this result, by plotting the relative elasticity ratios when accounting for stochastic demand and when ignoring stochastic demand using the model estimates. Notably, as the opportunity cost falls closer to the departure date, the bias becomes significant.

[^{38}]: Here I assume price is not restricted to the discrete set of fares, otherwise the firm problem is an integer programming problem.
7 Concluding remarks

There are two broad rationales for product prices that change over time: segmentation of consumers who differ in their willingness to pay, and changes in scarcity – or shadow costs – arising from stochastic demand. In this paper, I study the interactions of these forces by investigating the pricing decisions of airlines in US monopoly markets. I create a novel data set of high frequency fares and seat availabilities to estimate a structural model of dynamic airline pricing. In the model, firms face a stochastic arrival of consumers. The mix of consumer types, corresponding to leisure and business travelers, changes over time.

I show dynamic adjustment to stochastic demand complements intertemporal price discrimination in the airline industry. The complementarity arises because price inelastic consumers tend to arrive close to the date of travel. I find there are significant revenue losses associated with a pricing system that depends on the date of purchase, but not on the random realizations of demand. There are two reasons for this. First, dynamic adjustment allows firms to secure seats for business consumers who arrive close to the departure date. These consumers are then charged high prices. Second, dynamic adjustment reduces the incentive to hold back capacity well before the date of travel. Firms offer lower fares early on which results in additional revenue from leisure consumers who would otherwise not purchase.

Compared to pricing policies that depend only on the date of purchase, I find dynamic adjustment increases consumer surplus. Leisure consumers benefit from the lower fares offered early on.
While business consumers face higher prices under dynamic adjustment, the ability of firms to save seats reduces the probability of an early sell out. This allows some business consumers to receive seats on flights that would have been sold out otherwise.

One of the limitations of this study is that I only examine the pricing decisions in monopoly markets. Recent research has suggested the ability of airlines to respond to scarcity may be reduced in more competitive markets. An open question would be to examine the complementarities in the pricing channels in oligopoly markets. In addition to presence of multiple equilibria, this research would have to address what information airlines know about their competitors. Airlines surely keep track of competitor fares, but it is not clear that airlines keep track of their competitors’ seat availabilities.

References


Chris Jones. 10/22/2012. “How theater ticket prices are changing like airline fares.” Chicago Tribune.


8 Appendix

Proof of Proposition 1:

Take a consumer such that \( \varepsilon_0 < \beta - \alpha p_t + \varepsilon_1 \). Then there exists a \( \bar{p} > p \) such that \( \varepsilon_0 = \beta - \alpha \bar{p} + \varepsilon_1 \).

The expected gain for this consumer waiting is

\[
\Pr(p_{t-1} < p_t) \mathbb{E}[\alpha(p_t - p_{t-1}) | p_{t-1} < p_t] + \Pr(p_{t} < p_{t-1} \leq \bar{p}) \mathbb{E}[\alpha(p_t - p_{t-1}) | p_{t} < p_{t-1} \leq \bar{p}] - \phi \leq 0,
\]

which shows that a consumer that prefers to purchase today would not wait under the defined \( \phi \).

Next, consider a consumer such that \( \varepsilon_0 > \beta - \alpha p_t + \varepsilon_1 \). Then there exists a \( \underline{p} < p_t \) such that \( \varepsilon_0 = \beta - \alpha \underline{p} + \varepsilon_1 \). The expected gain from this consumer waiting is

\[
\mathbb{E}[\text{gain}] = \Pr(p_{t-1} < \underline{p}) \mathbb{E}[\beta - \alpha p_{t-1} + \varepsilon_1 - \varepsilon_0 | p_{t-1} < \underline{p}] - \Pr(p_{t-1} < p_t) \mathbb{E}[\alpha(p_t - p_{t-1}) | p_{t-1} < p_t]
\]

\[
\leq \Pr(p_{t-1} < p_t) \left[ \mathbb{E}[\beta - \alpha p_{t-1} + \varepsilon_1 - \varepsilon_0 | p_{t-1} < p_t] - \mathbb{E}[\alpha(p_t - p_{t-1}) | p_{t-1} < p_t] \right]
\]

\[
\leq \Pr(p_{t-1} < p_t) \left[ \mathbb{E}[\beta - \alpha p_{t-1} + \varepsilon_1 - \varepsilon_0 | p_{t-1} < p_t] - \mathbb{E}[\alpha(p_t - p_{t-1}) | p_{t-1} < \underline{p}] \right]
\]

\[
= \Pr(p_{t-1} < p_t) \left[ \mathbb{E}[\beta - \alpha p_{t-1} + \varepsilon_1 - \varepsilon_0 - \alpha(p_t - p_{t-1}) | p_{t-1} < p_t] \right]
\]

\[
= \Pr(p_{t-1} < p_t) \mathbb{E}[\beta + \varepsilon_1 - \varepsilon_0 - \alpha p_t | p_{t-1} < p_t]
\]

\[
= \Pr(p_{t-1} < p_t) \Pr(p_{t-1} < \underline{p}) \left[ \beta - \alpha \underline{p} + \varepsilon_1 - \varepsilon_0 \right] \leq 0.
\]

Hence, a consumer that prefers the outside option today would also not wait. \( \blacksquare \)
A Additional Data and Computational Details

A.1 Accuracy of seat maps

I perform two analyses to address the potential measurement error with using seat maps to proxy bookings. First, I match statistics of my data with reported carrier data found in the T100 US Segment tables. The T100 Segment tables report actual monthly enplanements for an origin, destination, air carrier. I aggregate the number of occupied seats the day flights leave to the monthly level for each route-month and compare with the T100. A scatter plot comparing monthly enplanements can be seen in Figure 11 Most points lie very close to the 45-degree line (zero measurement error at the monthly level) indicating a close match between actual monthly enplanements and the totals calculated from seat maps. The points highlighted in blue correspond to the city pair Kansas City, Boston, operated by Delta Air Lines. Delta switched regional carriers in this market during August 2012. My programming scripts failed to pick up the changes in flight numbers, which resulted in more than half of the month’s flights from being tracked. However, the other means are still close to the 45-degree line. By comparing these aggregate measures, I find my seat maps understate true enplanements by 0.81% of load factor at the monthly level.

Figure 11: Comparison of monthly enplanements of T100 US Segment and sample seat maps

This is a promising result, but comparing enplanements with the T100 does not address the issue of consumers purchasing tickets but not selecting seats at the time of booking. To this end, I create an additional data set from an airline that provides flight loads and seat maps on its website. I collect 18,107 seat maps and reported flight load by randomly selecting routes and
days until departure over a five month period. Unfortunately, the airline does not operate in the markets I study, so this constitutes an out of sample analysis. Further, this data is only for business class cabins with a range of 25 to over 50 seats. Of course, a one seat difference is a much higher percentage of load factor compared to the capacities in my data. This may exaggerate the measurement error percentage. At the same time, it may be that more economy class consumers do not select tickets at the time of booking. With this data set, I find that the seat maps understate actual loads by 2.3% on average. Few seat maps overstate the number of occupied seats, so the direction is the bias is mostly downward.

I use this secondary data set to estimate the measurement error by day before departure. Figure 12 plots a nonparametric fit of the measurement error along with average load factor by day before departure. This plot shows that the measurement error remains small across time, between 0.0% and 4.3% of load factor. At the same time, the measurement error is not flat indicating some days are more accurate than others. The worst measurement error between two and three weeks prior to departure. Seat maps are most accurate either well in advance or close to the departure date.
B Additional Tables & Figures

Table 6: Dynamic Substitution Regressions

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<td>(0.041)</td>
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<td></td>
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<td>(0.062)</td>
<td>(0.062)</td>
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Route clustered standard errors in parentheses

* p < 0.05, ** p < 0.01, *** p < 0.001
**Figure 13:** Mean fare and load factor by day before departure.

**Table 7:** Monte Carlo Experiments

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<th>$\mu$</th>
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<th>$\gamma_{slope}$</th>
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<td>.07</td>
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<td>F=100</td>
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<td>1.004</td>
<td>0.0690</td>
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</table>

*a F is the number of flights. The time horizon is $T = 10$. 
vertical dotted lines indicate advance purchase discount days*
Table 8: Parameter Estimates

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<td>$\gamma_1$</td>
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<td>$\mu_2$</td>
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<td>$\mu_3$</td>
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<td>0.000404</td>
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*Standard errors calculated using parametric bootstrap, n = 10,440.

Figure 14: Fitted values of the arrival process: probability on consumer types and Poisson rates.
Figure 15: Comparison of model prices with observed prices.