Welfare-Enhancing Collusion and Trade*

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Abstract

That collusion among sellers is detrimental to buyers is a central tenet in economics. In the context of trade, we provide an oligopoly model, using only standard ingredients, in which collusion is beneficial for society and can be beneficial for consumers. A differentiated-product duopoly operates in two geographically-separated markets. Each market is home to a single firm, but can import from the foreign firm. Since shipping across markets is costly, every firm has a cost advantage in its home market. Consumers treat the two goods as horizontally-differentiated substitutes and their preferences are identical in both markets. Under oligopolistic competition, each firm has a smaller market share and margin in the away market. The asymmetric margin leads to a market distortion, with more consumers than is socially optimal purchasing the imported variety. Collusion between the two firms (which in this framework is equivalent to a merger to monopoly) partially mitigates this distortion by reducing cross-hauling, raising not only total welfare but also, for sufficiently high trade costs, consumer surplus. “Anti-dumping” regulation induces the socially-optimal allocation.

JEL Codes: F12, L41, D43.

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1 Introduction

Colluding firms typically coordinate on several dimensions other than fixing price, such as assigning geographic regions or customers to each other (Kaplow and Shapiro 2007). Motta (2004) and Harrington (2006) document that an element common to the market sharing scheme of a number of real world cartels is the adoption of a “home-market principle”, by which each cartel member is given preference in supplying its home market—a region, say, where its production facilities are located—at the expense of supplying other regional markets. An implication of a scheme such as this is that while “(i)n a competitive market, one would expect a rise in a firm’s price, ceteris paribus, to result in more imports..., an allocation scheme based on the home-market principle would result in the combination of a higher price and fewer imports” (Harrington 2006, p.36, original emphasis). The incentive to reduce the volume of cross-hauling may be particularly acute in “spatial” industries, where transport (or more generally trade) costs are high relative to product value.

By raising price, collusion among sellers is generally considered by economists to be detrimental to buyers. However, casual interviews with executives in a certain spatial industry that stands accused of explicit collusion has revealed to us their purported belief that, by not wasting resources on cross-hauling, an arrangement based on the home-market principle can enhance aggregate welfare if not consumer welfare. (The executives admitted, of course, to the private benefit.) In this paper, we set out to investigate this possibility result. Using only standard ingredients, we provide a simple oligopoly model in which, relative to (imperfect) competition, collusion is beneficial for society and collusion may be beneficial for consumers.

Our model is a straightforward modification of existing work at the intersection of the trade and industrial organization literatures. There are two geographically separated markets, 1 and 2, each market being home to one firm: firm $A$ is located in market 1 and firm $B$ is located in market 2. To supply consumers in the other market, a firm incurs an additional linear trade cost per unit shipped. Within each market, consumers vary in their preferences over a differentiated good, each market being modeled as a uniform mass of consumers distributed over a unidimensional space of product characteristics, i.e. we embed Hotelling’s unit interval in each market. Each of the two firms produces a single different variety: firm $A$’s offering lies at the left endpoint of the unit interval and firm $B$’s offering lies at the right endpoint. A consumer’s disutility from consuming a variety other than her ideal variety is linear in the distance along this interval. We parameterize the model

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1 Multi-dimensional cartel agreements were a feature of some early cartels (e.g. see Deltas, Serfes and Sicotte 1999).
such that each consumer purchases one of the two goods, thus abstracting away from aggregate
demand effects (which are already well understood) and focusing on strategic market-share effects
(see discussion below).

We derive equilibrium outcomes under two alternative market-based behavioral benchmarks:
(i) a Nash equilibrium in prices—the “(imperfectly) competitive regime”, and (ii) the joint-profit
maximizing cartel—the “(fully) collusive regime”\(^2\). In our framework, given to the presence of
symmetries across both markets, the fully collusive regime is equivalent to a merger to monopoly.
Though we refer throughout to a profit maximizing cartel rather than to a merging firm, our findings
apply fully to a merger between the two firms. Relative to the collusive regime, the competitive
regime is characterized by more trade across geographic markets, as each competitive firm vies to
sell to consumers whose tastes are closer to their product offering than that of the rival firm. But the
perfect cartel still chooses to cross-haul a positive quantity. Intuitively, complete market division
would entail a loss of the surplus that the cartel can extract as, for some consumers, the willingness
to pay for the imported variety is much higher than their willingness to pay for the home variety.\(^3\)

We obtain the result that, in our setting, social welfare under collusion exceeds social welfare
under competition. While the good each consumer chooses in the trade-prone competitive regime
is at least as close to her ideal variety when compared to the collusive regime, this welfare effect in
favor of the competitive regime is dominated by a lower cross-hauling cost in the collusive regime.
This is an example of an industry where better meeting consumers’ tastes for variety through
trade competition may not be socially desirable. From the social viewpoint, competition leads
to “excessive trade” and collusion serves as a mechanism to address this failure. The even more
interesting possibility result is that collusion can increase consumer surplus relative to competition,
with some of the welfare gain generated through collusion being captured by consumers. That is,
competitive trade does bring more variety to a market, but this may come at the expense of a higher
price (for the home good). We then show that the correction mechanism is only partial: while lower
than in the competitive outcome, trade is still too high in the collusive outcome when compared to
society’s first-best outcome.\(^4\)

\(^2\)The perfect cartel’s agreement is, in general, sustainable when firms are sufficiently patient.
\(^3\)In contrast, when products are homogeneous, one might expect optimal cartel agreements to favor full geographic
segmentation. Pinto (1986), for example, models an international homogeneous-good cartel which does not trade in
equilibrium. Also note that in our basic model we assume no home bias, i.e. at equal prices one half of consumers
prefer the imported variety over the home variety. As we later show, our results are strengthened on introducing home
bias. For example, home bias reduces cross-hauling, but by less under competition than it does under collusion.
\(^4\)In a completely different empirically oriented set-up, Fershtman and Pakes (2000) show that collusion can increase
It is instructive to highlight the intuition for the welfare comparisons. Observe that from a welfare point of view, prices cancel out because they are a transfer. Only market allocations matter, and what is important for these are relative prices (or relative cost-adjusted prices). Since markets and all equilibria are symmetric, we can consider welfare effects in a single market. In each market, the “home” firm has a cost advantage over the “away” firm because the home firm does not incur the shipping cost. Consider first the Nash equilibrium. In this equilibrium, the high-cost (away) firm chooses a lower margin than the low-cost (home) firm, because the high-cost firm has a smaller market share and thus more aggressive pricing results in smaller revenue losses from the sales to inframarginal consumers. Therefore, the price difference between the two firms is substantially lower than the cost difference (the trade cost incurred by the importing firm), i.e. there is price discrimination against buyers of the home good, or “dumping” of the imported good. As a result, the number of consumers who purchase from the importing firm is too high relative to the social optimum (a social planner would set the price difference between the two firms equal to the trade cost). Welfare would be higher if relative prices were to change so that some of the consumers purchasing from the high-cost supplier were to switch to the low-cost supplier. The cartel partially eliminates this distortion in an effort to appropriate some of this gain in total surplus. The distortion is not fully eliminated because some price discrimination against each firm’s home-market buyers—who in equilibrium constitute the majority of its buyers—increases the cartel’s surplus. As a result, there still is welfare-reducing cross-hauling by the cartel. A social planner would raise the price of the imported good relative to the home good, no longer price discriminating against buyers of the home variety, and thus would reduce cross-hauling even more than the cartel.

Two qualifications should be made at this point. First, we do not suggest that the mechanism at work (i.e. the price aggressiveness of a high-cost or low-quality firm in imperfect competition) is novel. On the contrary, our modeling ingredients are, as noted, standard, and the result has a

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5 More broadly, a cartel has the potential to increase total welfare when oligopolistic competition between asymmetric firms leads to welfare-reducing distortions in the allocation of consumers to firms, and can potentially do so without relying on side-payments between members. However, this is by no means guaranteed even when aggregate demand effects are absent; in fact, many reasonable models produce the conventional result whereby cartels lower welfare. For example, if firms operate in a single market and after collusion they set prices so as to increase their profits by the same percentage, then the high cost firm would sell more (and the low cost firm less) and welfare would be reduced. The same result would occur if firms split the collusive surplus through Nash bargaining using 50/50 shares. We leave the analysis of the welfare effects of single-market asymmetric cartels, on which there already exists some literature (e.g., Harrington 1991, Athey and Bagwell 2001), for subsequent research. We note, however, that in our model, though there are asymmetries between the firms in each market, the firms (and the cartel agreement) are symmetric over both markets.

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Both the number and the quality of product offerings. This increase can more than compensate for higher prices, and lead to an overall increase in consumer welfare.
“theory of second best” (Lipsey and Lancaster 1957) flavor to it—that if one “Paretian optimum condition” cannot be fulfilled (e.g. marginal cost pricing), then it might not surprise a theorist that welfare can actually increase by departing “further” from the set of Paretian conditions (i.e. switching from Bertrand-Nash to joint-profit maximizing). However, it is our understanding that the applied point of the paper—that a workhorse model confirms that spatial cartels can enhance welfare by restricting trade—has not been made.\textsuperscript{6} Second, as in the standard Hotelling framework, our model assumes that there is no deadweight loss (DWL) from monopoly (i.e. that each consumer’s surplus, gross of the price she pays and the disutility from consuming a variety other than her ideal, is sufficiently high). By abstracting away from aggregate demand effects, our intent is to focus on the possibility result, as argued not least by certain firms themselves—again, that in the presence of costly cross-hauling welfare can rise as the mode of competition shifts from non-cooperative to cooperative. It is in those industries where volume effects are sufficiently small that our result may hold more relevance. Near-zero aggregate volume effects do not seem to be an unreasonable assumption in spatial industries such as cement and sugar (that are among the empirical examples stated below). In mature economies, such an assumption does not seem unreasonable for many household appliances, e.g., refrigerators, ovens, dishwashers, etc. One should realize, however, that any DWL from monopoly should be balanced against the strategic effects we highlight. For industries in which aggregate demand is sufficiently elastic, our welfare conclusions will be overturned.\textsuperscript{7} Yet, to be clear, our conclusions should be continuous as one introduces volume effects.

Recent empirical work suggests that the home-market principle is not a mere theoretical curiosity but an important feature of spatial (international or subnational) cartels. Harrington (2006) describes several global cartels including choline chloride, lysine and methionine, arguing that in the latter case “the home-market principle was, in fact, the instigating factor for cartel formation” (p.35-6). Röller and Steen (2006) examine an official Norwegian cement cartel, documenting the

\textsuperscript{6}We also note that the argument that higher welfare can be attained through some form of coordination (e.g. in Foros et al. 2002, on internalizing investment spillovers by colluding in an investment stage prior to a competitive product market stage) or increased concentration (e.g. in Banal-Estanol 2007, on sharing information between merging parties) has been made previously. Joe Farrell has also pointed out to us that a line of “defense” similar to the one we described is often heard in merger cases, where merger proponents argue that any unilateral effects will be outweighed by cost efficiencies. Whinston (2006, p.18) speculates that in certain settings cartels may benefit society, though—like us—he does not advocate a rule of reason approach to the prosecution of cartels (one should consider administrative costs in designing the optimal policy). Finally, our result is reminiscent of, though conceptually distinct from, suggestions that in some industries competitive markets can lead to “destructive competition” and that cartels can help stabilize them—for more discussion see Deltas, Sicotte and Tomczak (2008) and references therein.

\textsuperscript{7}A more general specification would be to embed a downward-sloping demand at each point of the Hotelling interval. We believe this would add notational clutter without providing intuition beyond what is already given here.
role of a common sales office whose “primary task was to organize sales in a better way, to prevent cross-transportation and unprofitable competition” (p.324). Strand (2002) looks at the European Union’s sugar sector, in which national quotas under the Common Market Organization (CMO) appear to help firms allocate markets geographically; he cites a large sugar buyer saying that “(i)t is in every sugar supplier’s best interest to stay out of each other’s markets” (p.14). Salvo (2010) studies the division of regional markets by firms in the Brazilian cement industry. By quantifying the welfare effect of a real world cartel, under reasonable assumptions regarding the counterfactual competitive regime, future research may yet empirically validate the possibility result we obtain theoretically.\footnote{Moreover, in a world of imperfect information, Motta (2004, p.141) suggests that such “(market allocation schemes) have the advantage of allowing for prices to adjust to new demand and cost conditions without trigger-
ing price wars... (a)s long as each firm does not serve segments of demand (explicitly or tacitly) allocated to rivals... probably explain(ing) why such collusive schemes are often used.” While relevant, the analysis of a reduced occurrence of costly cartel-disciplining price wars à la Green and Porter (1984), alluded to by Motta, is left for future research.}

To the best of our knowledge, this paper also provides a first model where a perfect cartel (i.e. the monopoly outcome) cross-hauls too much product between geographic markets, rather than fully dividing them. Unlike models such as that in Bond and Syropoulos (2008), we do not require constrained cartels—i.e. deviation incentives—for cross-hauling to obtain. In addition to the aforementioned empirical work on spatial cartels, our paper can be related to other strains in the literature. Recent work (Lommerud and Sørgard 2001, Schröder 2007, Bond and Syropoulos 2008) examines the stability of multimarket collusion in the wake of trade liberalization (modeled as a reduction in the trade cost).\footnote{The concern in this strain of the literature is that “cartels are bad” and so if trade liberalization helps to stabilize a cartel, then we should worry about trade liberalization. By contrast, our point is that in certain industries “cartels can be good.” Moreover, unlike this literature, the comparative statics with respect to trade costs do not qualitatively impact the results of our paper (though they do, of course, affect their magnitudes).}\footnote{Regarding the Norwegian cartel’s relation with the broader European continent, Röller and Steen (2006) argue that “competition is a multimarket game where credible threats to enter each other’s markets prevent firms from entering other countries” (p.324). Lommerud and Sørgard (2001) cite a scheme, uncovered by the European Commission in 1994, to limit intra-EU trade of cement (as well as the prosecution in the 1970s of Japanese and European synthetic fiber producers for agreeing to restrain exports to each other’s markets). On how the ability to sell in multiple markets may facilitate collusion, see Bernheim and Whinston (1990).} Similarly, Davidson (1984), Rotemberg and Saloner (1989) and Fung (1992) study single-market cartel stability in light of unilateral trade policy (e.g. tariffs). By contrast, as noted in footnote 2, we abstract away from incentive constraints, implicitly assuming that firms are sufficiently patient. An old literature on spatial (“basing-point”) pricing and “quasi-cooperation” dates back at least to Smithies (1942). Needham (1964) argues that in the absence of side-payments cross-hauling may actually arise under collusion in order to stabilize a cartel. More recently, Thisse and Vives (1988) investigate the profitability of different pricing schemes in...
spatial markets, of which the most relevant to our paper are “mill pricing” (i.e. full pass-through of transport costs) and “uniform pricing” (i.e. the same price paid by consumers across markets, so there is ample price discrimination in favor of distant buyers). They find that while mill pricing can yield higher aggregate profit to the duopoly, uniform pricing obtains in the (imperfectly) competitive equilibrium. Our result is reminiscent of Thisse and Vives (1988) in that relative to collusion and to the first-best social outcome, competition generates more price discrimination in favor of the imported good (the distant buyer). Also, an implication of our framework is that the first-best market allocation can be attained by mandating mill pricing (i.e. ruling out price discrimination).

In the trade literature, Brander and Krugman (1983) show that exogenously moving from autarky (there is, by definition, no cross-hauling) to trade competition in a homogeneous-good Cournot oligopoly where aggregate demand slopes downward—and with free entry—is welfare-enhancing: “(t)he pro-competitive effect of having more firms and a larger overall market dominates the loss due to transport costs in this second best imperfectly competitive world” (p.314). With entry barriers, however, two-way trade in the identical good when the trade cost is high enough is “wasteful”, lowering total (though not consumer) welfare relative to autarky. Indeed, in an extension section of the paper that studies autarky, we obtain a similar result that autarky welfare dominates market-based “trade regimes” (not only competition but also collusion) except when the trade cost is low.\textsuperscript{11} Friberg and Ganslandt (2008) extend Brander and Krugman’s (1983) welfare analysis of autarky (the no-entry case) to a linear-demand differentiated-goods Bertrand oligopoly. When market structure is sufficiently concentrated, they find that trade competition is welfare-enhancing relative to autarky.\textsuperscript{12}

2 The basic model

We consider a geographically segmented industry where goods are horizontally differentiated. To capture the geographic component, we model two local markets, 1 and 2. Shipping product from one market to another—cross-hauling—incurs a unit trade cost \( t > 0 \) (and zero fixed cost). To capture the taste component, we model each local market as a continuum of consumers distributed uniformly over a unidimensional space of product characteristics, defined by the interval \([0, 1]\). The

\textsuperscript{11} Other studies where trade lowers welfare, albeit in very different contexts, include Newbery and Stiglitz (1984) and Eden (2007).

\textsuperscript{12} Neither Brander and Krugman (1983) nor Friberg and Ganslandt (2008) (nor, similarly, Clarke and Collier 2003) are concerned with the welfare effect of collusion, the central motivation of our paper.
disutility from consuming a variety other than one’s ideal variety is linear in the distance along this Hotelling interval, with slope $\theta > 0$. There are two firms, $A$ and $B$, each firm producing one variety. In geographic space, firm $A$’s plant is located in market 1 while firm $B$’s plant is located in market 2. In product space, firm $A$’s product is located at the left endpoint of the unit interval while firm $B$’s product is located at the right endpoint. (We assume barriers to entry are high enough in both geographic and product spaces so that neither firm will build a second plant or introduce a second product.\textsuperscript{13}) The two firms have the same constant marginal cost of production $c \geq 0$.\textsuperscript{14}

Consumers make discrete choices, purchasing one unit or none. Let $x \in [0, 1]$—the consumer’s type—denote the distance from the left endpoint of the unit interval. A firm’s price can vary across the two markets though not across consumers within a market. Consider either one of the local markets, and denote the vector of prices by $p = (p_A, p_B)$; for simplicity, we momentarily omit market subscripts. Denoting the reservation price for one’s ideal product (relative to the outside good) by $V$, consumer type $x$’s (ordinary) demand for good $A$ is

$$q_A(p; x) = \begin{cases} 
1 & \text{if } U_A(p_A; x) := V - \theta x - p_A \geq V - \theta (1 - x) - p_B \text{ and } U_A(p_A; x) \geq 0 \\
0 & \text{otherwise}
\end{cases}$$

while her demand for good $B$ is

$$q_B(p; x) = \begin{cases} 
1 & \text{if } U_B(p_B; x) := V - \theta (1 - x) - p_B > V - \theta x - p_A \text{ and } U_B(p_B; x) \geq 0 \\
0 & \text{otherwise}
\end{cases}$$

(Notice that by specifying a common $V$ across markets, we do not assume any home bias; subsequently, in Section 3.5, we introduce $V_{1A} = V_{2B} > V_{1B} = V_{2A}$.) The location of the “marginal consumer” $\tilde{x}$, defined as the consumer who is indifferent between goods $A$ and $B$, follows from

\textsuperscript{13}We ignore the possibility that a firm’s location in product space is endogenous. Note that collusive firms would choose different locations than competing duopolists.

\textsuperscript{14}The model can readily be extended to more than two local (buyer) markets $m = 1, 2, \ldots, M$, with trade costs $t_{mA} \in (0, t)$ and $t_{mB} \in (0, t)$ from seller locations $A$ and $B$ respectively, provided that overall symmetry between firms $A$ and $B$ is preserved. One could also specify a (sufficiently) low fixed cost of cross-hauling (rather than zero, as we do for simplicity). Finally, in view of the symmetry and full coverage in our model (which implies that each firm’s total output is the same across all equilibria; see below), results would not be qualitatively affected if production costs were increasing in output (one can actually set $c = 0$ wlog).
solving \( U_A(p_A; \tilde{x}) = U_B(p_B; \tilde{x}) \), i.e.

\[
\tilde{x}(p) = \frac{\theta - p_A + p_B}{2\theta}
\]

We next consider equilibrium outcomes under alternative competitive regimes, namely price competition and full collusion. Our focus is the case where, in these equilibrium outcomes, both firms sell in both markets and all consumers purchase an inside good. Thus, and unless otherwise noted, we restrict the space of parameters as follows (see below for verification):

**Assumption A1** (“cross-hauling under collusion”) \( t < 2\theta \): Restricting the cost of cross-hauling between markets to be sufficiently low (relative to the degree of product differentiation) implies that cross-hauling occurs even in the fully collusive regime.\(^\text{15}\)

**Assumption A2** (“full market coverage under competition”) \( 2(V - c) > t + 3\theta \): Restricting the reservation price for one’s ideal product to be sufficiently high (relative to the degree of product differentiation and the cost primitives) implies that even in the competitive regime the consumer located at \( \tilde{x} \), who is indifferent between the inside goods \( A \) and \( B \), prefers these to the outside good.\(^\text{16}\)

Quantity shares for firms \( A \) and \( B \) are then given by \( s_A(p) = \tilde{x}(p) \) and \( s_B(p) = 1 - \tilde{x}(p) \) respectively.

### 2.1 Price competition

Since marginal cost is flat in output, the problem is separable (and analogous) across the two local markets. We consider market 1. In a competitive equilibrium in prices, prices solve the system

\[
\begin{align*}
\max_{p_A} (p_A - c) s_A(p) \\
\max_{p_B} (p_B - c - t) s_B(p)
\end{align*}
\]

\(^\text{15}\)In the competitive regime, cross-hauling occurs if \( t < 3\theta \). We subsequently discuss the case \( 2\theta \leq t < 3\theta \), where cross-hauling occurs in the competitive regime but not in the collusive one.

\(^\text{16}\)The restriction is equivalent to \( (U_B(p_B^C; \tilde{x}^C) = U_A(p_A^C; \tilde{x}^C) > 0 \) (where \( C \) denotes the competitive outcome). The market will remain covered for a region of parameters beyond this inequality. For these parameter values, the nature of competition will no longer be duopolistic but rather the kinked equilibrium of Salop (1979).
with first-order conditions (FOCs)

\[
\frac{(\theta - p_A + p_B)}{(2\theta)} - \frac{(p_A - c)}{(2\theta)} = 0
\]

\[
\frac{(\theta + p_A - p_B)}{(2\theta)} - \frac{(p_B - c - t)}{(2\theta)} = 0
\]

yielding prices

\[
p_{1A}^C = c + \frac{1}{3} t + \theta, \quad p_{1B}^C = c + \frac{2}{3} t + \theta
\]

(2)

and profits

\[
\Pi_{1A}^C = \frac{1}{18\theta} (3\theta + t)^2, \quad \Pi_{1B}^C = \frac{1}{18\theta} (3\theta - t)^2
\]

(3)

(now adding market subscripts, and where the superscript \( C \) denotes the competitive equilibrium).

In equilibrium, the location of the marginal consumer, and thus the quantity share of home firm \( A \), is given by

\[
x_1^C = \frac{1}{2} + \frac{1}{6} \frac{t}{\theta}
\]

Notice that \( A1 \) implies that \( x_1^C < 1 \) and cross-hauling occurs. (It is clear that we have an interior solution for \( x_1^C \) as long as \( t < 3\theta \).) Equilibrium outcomes in market 2 are obtained from interchanging the market-firm subscripts (and \( x_2^C = 1 - x_1^C \)). It is easy to show that a firm’s total profit \( \Pi_{1A}^C + \Pi_{2A}^C \) is increasing in both the trade cost \( t \) and (given \( A1 \)) the degree of product differentiation \( \theta \); intuitively, increasing \( t \) or \( \theta \) relaxes price competition.\(^{17}\)

\[^{17}\text{Proof of this statement follows from noting that } \Pi_{1A}^C + \Pi_{2A}^C = \left( t^2 + 9\theta^2 \right) / (9\theta) \text{ increases in } t \text{ and } \frac{\partial (\Pi_{1A}^C + \Pi_{2A}^C)}{\partial \theta} = 1 - (t/(3\theta))^2 \geq A1 0.\]

2.2 Full collusion

It is well known that when the number of (symmetric) firms is fixed (as in the current setting), then any degree of cooperation, including the joint profit maximum, can be supported as a subgame perfect equilibrium outcome of an infinitely repeated game with perfect monitoring if the discount factor is sufficiently high (e.g. see Fudenberg and Maskin 1986). So, in what follows, we derive the fully collusive outcome assuming that the discount factor is sufficiently close to one so that firms’
incentive compatibility constraints do not bind.\textsuperscript{18} We note that collusion in our set-up is equivalent to a merger to monopoly, though for clarity of exposition we refer to joint profit maximization as arising from collusion.

Clearly, in a fully collusive—or joint-profit maximizing (denoted by the superscript $JM$)—outcome, prices set by the firms leave the marginal consumer in each market, given by (1), with zero surplus. To see this, notice that were the marginal consumer to have positive surplus, joint profits could increase by slightly raising prices (and recall that we have assumed that $V$ is large enough that it is profitable to serve all consumers\textsuperscript{19}). Thus, fully-collusive prices satisfy $U_A(p_A; \hat{x}(p)) = 0$ (a condition that, from the definition of $\hat{x}$, is equivalent to $U_B(p_B; \hat{x}(p)) = 0$), which from (1) can be rewritten as $2V - \theta - p_A - p_B = 0$. Using this locus of prices, the perfect cartel’s problem

$$\max_{p_A, p_B} (p_A - c) s_A(p) + (p_B - c - t) s_B(p) \quad \text{subject to} \quad U_A(p_A; \hat{x}(p)) \geq 0$$

$$\Leftrightarrow \max_{p_A, p_B} (p_A - c) \frac{\theta - p_A + p_B}{2\theta} + (p_B - c - t) \left(1 - \frac{\theta - p_A + p_B}{2\theta}\right) \quad \text{s.t.} \quad U_A(p_A; \hat{x}(p)) \geq 0$$

collapses to the univariate problem

$$\max_{p_A} (p_A - c) \left(\frac{V - p_A}{\theta} + (2V - \theta - p_A - c - t) \left(1 - \frac{V - p_A}{\theta}\right)\right)$$

with FOC

$$\frac{V - p_A}{\theta} - \frac{p_A - c}{\theta} - \frac{\theta - V + p_A}{\theta} + \frac{2V - \theta - p_A - c - t}{\theta} = 0$$

yielding prices

$$p_{1A}^{JM} = V - \frac{1}{4} t - \frac{1}{2} \theta, \quad p_{1B}^{JM} = 2V - \theta - p_{1A}^{JM} = V + \frac{1}{4} t - \frac{1}{2} \theta$$

(4)

and profits

$$\Pi_{1A}^{JM} = \frac{1}{16\theta} (2\theta + t) \left(4V - 4c - t - 2\theta\right), \quad \Pi_{1B}^{JM} = \frac{1}{16\theta} (2\theta - t) \left(4V - 4c - 3t - 2\theta\right)$$

(5)

\textsuperscript{18}An appendix examining the perfect cartel’s incentive constraint, when firms adopt grim trigger strategies that account for the multimarket nature of their contact, is available from the authors.

\textsuperscript{19}We note that at the fully collusive prices derived below, at which the marginal consumer has zero surplus, profit earned on the imported good is positive (though lower than profit on the home good). From A1 and A2, the margin $p_{1B}^{JM} - c - t = V - c - \frac{3}{2}t - \frac{1}{2} \theta = \frac{1}{7} \left(2V - 2c - \frac{3}{2}t - \theta\right) > \frac{1}{8} V - c - \frac{1}{2} \left(t + 2\theta\right) > V - c - \frac{1}{2} \left(t + 3\theta\right) > 0$. 

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The location of the marginal consumer is given by

\[ x_{JM}^{1} = \frac{1}{2} + \frac{1}{4} t \]

where \( x_{JM}^{1} > \bar{x}_{1}^{C} \) such that the quantity share of the home good in the collusive outcome is increased compared to that in the competitive outcome. (Again, interchange the market-firm subscripts for market 2 outcomes, and \( x_{JM}^{2} = 1 - x_{1}^{JM} \).) It is clear from A1 that, though increased relative to competition, the home firm’s share under collusion is less than 1 (we subsequently consider corner solutions\(^{20}\)). The result is captured in the following proposition.

**Proposition 1** The joint-profit maximizing outcome involves less cross-hauling than the competitive equilibrium outcome. Despite the home good and the imported good being equally close to the average consumer’s ideal variety, the cartel trades less product across geographic markets, or “swaps geographic markets” relative to the competitive regime.

Contrary to the competitive outcome, a firm’s total profit \( \Pi_{1A}^{JM} + \Pi_{2A}^{JM} \) decreases in both the trade cost \( t \) (given A1) and the degree of product differentiation \( \theta \). Intuitively, the competitive mechanism is now absent, and a higher \( t \) raises the cost of bringing a variety to market, while a higher \( \theta \) raises the disutility from not consuming one’s ideal variety.\(^{21,22}\)

## 3 Welfare effects

### 3.1 Welfare across the competitive and collusive regimes

We compare social welfare—the sum of consumer surplus and producer surplus—across the competitive and fully collusive regimes. Given our focus on the case where the market is fully covered

\(^{20}\) Outside A1, for \( t \geq 20 \), the perfect cartel does not cross-haul: \( \tilde{x}_{1}^{JM} = 1 \). From \( U_{A}(p_{A}; \tilde{x}_{1}^{JM}) = 0 \), the fully collusive price is \( p_{A}^{JM} = V - \theta \) (and \( p_{A}^{JM} = V \)), and profit is \( V - \theta - c = \frac{1}{2} (2V - 2c - 2\theta) > V - c - \frac{1}{2} (t + 3\theta) > \lambda^{2} \).

\(^{21}\) Proof of these comparative statics follow from noting that \( \partial (\Pi_{1A}^{JM} + \Pi_{2A}^{JM}) / \partial t = (t - 2\theta) / (4\theta) < \lambda^{1} \) and \( \partial (\Pi_{1A}^{JM} + \Pi_{2A}^{JM}) / \partial \theta = - (t^{2} + 4\theta t) / (8\theta^{2}) < 0 \).

\(^{22}\) Since \( \partial (\Pi_{1A}^{JM} + \Pi_{2A}^{JM}) / \partial t < 0 \) and (as noted earlier) \( \partial (\Pi_{1A}^{C} + \Pi_{2A}^{C}) / \partial t > 0 \), the (positive) private gains from collusion decrease in the trade cost \( t \): \( \partial (\Pi_{1A}^{JM} + \Pi_{2A}^{JM} - \Pi_{1A}^{C} - \Pi_{2A}^{C}) / \partial t = \partial (\frac{1}{2} (2V - 2c - t - 3\theta + \frac{1}{4\theta} t^{2})) / \partial t = \frac{1}{4\theta} (t - 18\theta) < \lambda^{1} \). On first thought this may seem counter-intuitive, since the cartel cross-hauls less \( (1 - \tilde{x}_{1}^{JM} < 1 - \bar{x}_{1}^{C}) \). Recall, however, that raising \( t \) softens price competition in the competitive regime.
(i.e. in both regimes all consumers purchase one unit of an inside good, with “gross utility” \( V \)), we can restrict our comparison of welfare across the two equilibrium outcomes to (i) the different total cost of cross-hauling product between geographic markets, borne by firms, and (ii) the different total disutility (“travel cost”) from consuming a variety other than one’s ideal, borne by consumers.

Clearly, the total cost of cross-hauling (into market 1, with market 2 being analogous) under price competition, \( t (1 - \tilde{x}_1^C) \), exceeds the cost from cross-hauling under full collusion, \( t (1 - \tilde{x}_1^{JM}) \), as there is more trade in the competitive regime.

The total consumer taste disutility under price competition is
\[
\int_0^{\tilde{x}_1^C} \theta x dx + \int_{\tilde{x}_1^C}^1 \theta (1 - x) dx = \frac{1}{2} \theta \left( 2 \left( \tilde{x}_1^C \right)^2 - 2 \tilde{x}_1^C + 1 \right),
\]
while that under full collusion is obtained similarly. It should also be clear that the former is lower than the latter, as the marginal consumer in the competitive regime lies closer to the midpoint of the Hotelling interval than the marginal consumer in the collusive regime \((\frac{1}{2} < \tilde{x}_1^C < \tilde{x}_1^{JM})\); mechanically, the quadratic expression in brackets defines a convex parabola with minimum at \( x = \frac{1}{2} \). Relative to the collusive regime, the good each consumer chooses in the competitive regime is (weakly) closer to her ideal variety.

We now combine the total cost of cross-hauling and the total consumer taste disutility in each of the two regimes to obtain the following result:

**Proposition 2** Social welfare under full collusion exceeds social welfare under price competition, since (i) the (collusive) effect of “swapping”—i.e. reducing cross-hauling between—geographic markets, dominates (ii) the (competitive) effect of consumers choosing products that are closer to their ideal variety. Further, the welfare gains under collusion relative to competition are increasing in the unit trade cost \( t \) and decreasing in the degree of product differentiation \( \theta \).

**Proof** From the above analysis, for \( t > 0 \), the (per market) increase in social welfare in the collusive outcome relative to the competitive one is
\[
W^{JM} - W^C = t \left( 1 - \tilde{x}_1^C \right) + \frac{1}{2} \theta \left( 2 \left( \tilde{x}_1^C \right)^2 - 2 \tilde{x}_1^C + 1 \right) - t \left( 1 - \tilde{x}_1^{JM} \right) - \frac{1}{2} \theta \left( 2 \left( \tilde{x}_1^{JM} \right)^2 - 2 \tilde{x}_1^{JM} + 1 \right)
= \frac{7}{144} \frac{t^2}{\theta} > 0
\]
This is increasing in \( t \) and decreasing in \( \theta \).
It is immediate from (2) and (4) above (and recalling the symmetry) that a firm price discriminates against its home-market consumers (or, equivalently, in favor of its geographically-distant consumers) in both competitive and collusive regimes. But it is under competition that price discrimination is more pronounced, in that $p_{2A}^C - p_{1A}^C = p_{1B}^C - p_{1A}^C = \frac{1}{3}t < p_{2A}^{JM} - p_{1A}^{JM} = p_{1B}^{JM} - p_{1A}^{JM} = \frac{2}{3}t < t$.\footnote{Under competition, there is more “dumping” (in the Brander and Krugman 1983 sense) or, borrowing other terms from the trade and spatial literatures, there is a greater degree of “pricing to market” or “freight absorption” (again in the sense that the imported good’s price upcharge falls short of the trade cost).}

From the social point of view, the oligopolistically competitive equilibrium is characterized by “excessive consumption of the imported variety”, or “excessive trade”. Collusion serves as a mechanism to correct this failure, but only partially, as we show in the subsequent subsection.

It is also worth pointing out that the welfare gain from collusion occurs even for a low, but positive, unit trade cost. As the trade cost approaches zero, social welfare converges under the two alternative regimes, i.e. as $t \to 0$, we have that $\bar{x}_{1}^{JM} \to \bar{x}_{1}^{C} \to \frac{1}{2}$. The setup then approaches the standard Hotelling model, in which collusion does not result in welfare losses, leading only to a transfer from consumers to producers.

Quite striking is the further finding that, relative to competition, collusion can be good even for consumers, a result we state in the following proposition.

**Proposition 3** There exist parameter values for which consumer surplus under full collusion exceeds consumer surplus under price competition. Specifically, collusion brings gains to consumers relative to competition when the unit trade cost $t$ is sufficiently high (relative to the reservation price for one’s ideal product $V$) that $A2$ is within $5t^2 / (72\theta)$ of binding.

**Proof** We compute $CS^{JM} = \int_{0}^{\bar{x}_{1}^{JM}} (V - \theta x - p_{1A}^{JM}) \, dx + \int_{\bar{x}_{1}^{JM}}^{1} (V - \theta (1 - x) - p_{1B}^{JM}) \, dx$ and, similarly, $CS^{C}$ (we omit expressions for brevity; notice that $CS^{C}$ is equivalent to the area of two trapezia, while $CS^{JM}$ collapses to the area of two triangles as the marginal consumer has zero surplus). Then consider the condition $CS^{JM} - CS^{C} > 0 \iff 5t^2 / (72\theta) - (2V - 2c - t - 3\theta) > 0$.

From A2, $2V - 2c - t - 3\theta$ is positive (and decreasing in $t$). The first term of the inequality is also positive (and increasing in $t$). Clearly, when $t$ is high enough and A2 is not too slack, the condition can hold; more formally, $CS^{JM} > CS^{C}$ iff A2 is within $5t^2 / (72\theta)$ of binding. ■

It may seem surprising that collusion can raise aggregate consumer surplus. To understand why this may happen, consider the case in which the marginal consumer has only a small positive surplus under competition, so that consumers with somewhat higher values of $x$ would obtain negative utility
from the purchase of the home good. The cartel wishes to reduce the share of the imported good and increase that of the home good, while still covering the market (recall the space of parameters we consider). If it attempted to accomplish this solely by increasing the price of the imported good, then some of the consumers who have hitherto chosen the imported good would choose to purchase nothing. The only way to continue covering the market is to simultaneously lower the price of the home good. This raises welfare for most consumers. Therefore, when collusion raises consumer welfare relative to competition, the price on the imported good rises, but the price on the home good declines. In this region of parameters, consumers who under competition were already buying the home good (and even some near-marginal consumers who were buying their preferred imported good but now switch to the even cheaper home good) are made better off through collusion. This gain in consumer welfare dominates both the loss suffered by consumers who carry on buying the now dearer imported good and the loss experienced by some consumers who have been induced to switch to their less-favored home variety.

3.2 First-best social outcomes

The immediate question then is how distortionary are the market-based behavioral regimes derived above? We now compute the set of first-best outcomes, where social welfare is maximal, and compare them to the competitive and collusive outcomes. As we explain, what characterizes a first-best social outcome is the price difference between the home good and the imported good, which is equal to the trade cost. We then provide price levels for two alternative (and extreme) first-best outcomes, where the division of surplus between producers and consumers is reversed: prices set by a “business-friendly” social planner, and prices set by a “consumer-friendly” social planner. To be clear, our planner’s bias between pro-business and pro-consumer does not affect the price of the imported good relative to the home good, which determines the welfare trade-off between meeting consumers’ love of variety and saving on trade costs.

Denote this first-best outcome by the superscript FB and consider market 1 (again market 2 is

\[ p_{1M}^J < p_{1A}^J \iff \frac{t}{6} - (2V - 2x - t - 3\theta) > 0 \text{ and } t/6 > \frac{5t^2}{120} \text{. So when } A2 \text{ is within } 5t^2/(720) \text{ of binding (and thus } CS^{JM} > CS^{C} \text{ as per the proof of Proposition 3), } A2 \text{ is also within } t/6 \text{ of binding and hence } p_{1M}^J < p_{1A}^J. \]

Relative to competition, collusion impacts consumer welfare both by shifting prices (up for the imported variety and—for t high—down for the home variety) and by changing the market allocation (against imports). Since the latter effect hurts consumers and is not accounted for in typical calculations of “customer damages,” we note that whenever collusion turns out to raise consumer welfare, it is also the case that customer damages will be negative (i.e. no “damage” occurs).
analogous). Express the location of consumer $\tilde{x}_1^{FB}$, who is indifferent between the two inside goods, as lying at a distance $d$ to the right of the midpoint of the unit interval of product characteristics, i.e. $\tilde{x}_1^{FB} = \frac{1}{2} + d$. For this marginal consumer to be indifferent to buying the home good or the imported good, the fact that she finds the home good less appealing must be offset by a price difference in its favor. The relative taste disutility of the home good is that of traveling a distance $2d$ (a distance $d$ to the midpoint $\frac{1}{2}$, and then another $d$), costing the marginal consumer $2d\theta$. Now, the social planner equates this relative disutility $2d\theta$ with the cost of cross-hauling $t$, i.e. $2d\theta = t$, from which $d = t/(2\theta)$ and the location of the marginal consumer follows:

$$\tilde{x}_1^{FB} = \begin{cases} \frac{1}{2} + \frac{1}{2}\theta & \text{if } t < \theta \\ 1 & \text{otherwise} \end{cases}$$

Relative to the trade-prone competitive and collusive regimes, the social planner reduces wasteful cross-hauling across borders, opting for less trade—and none at all when $\theta \leq t < 2\theta$—and a greater quantity share for the home good: $1 \geq \tilde{x}_1^{FB} > \tilde{x}_1^{JM} > \tilde{x}_1^C$. (Intuitively, for $t \geq \theta$ the degree of cost asymmetry exceeds the degree of product differentiation and there is a corner solution.) There is no price discrimination against a firm’s home-market buyers, as the price difference is equated to the trade cost $t$, in contrast to the market-based regimes where price discrimination was substantial (i.e. the price difference was as low as $\frac{1}{2}t$ under collusion and $\frac{1}{3}t$ under competition).

We summarize the welfare result in the following proposition.

**Proposition 4** Social welfare under full collusion, though higher than under price competition, is suboptimal. Relative to the fully collusive outcome, a social planner would raise the price of the imported good relative to the price of the home good, further restricting the penetration of imports, i.e. the social planner would further enhance geographic market-swapping at the expense of consumers’ taste for variety.

**Proof** When $0 < t < \theta$, the (per market) increase in social welfare in a first-best outcome relative to full collusion is

$$W^{FB} - W^{JM} = t(1 - \tilde{x}_1^{JM}) + \frac{1}{2}\theta \left(2(\tilde{x}_1^{JM})^2 - 2\tilde{x}_1^{JM} + 1\right) - t(1 - \tilde{x}_1^{FB}) - \frac{1}{2}\theta \left(2(\tilde{x}_1^{FB})^2 - 2\tilde{x}_1^{FB} + 1\right)$$

$$= \frac{1}{16} \frac{t^2}{\theta} > 0$$
This is increasing in $t$ and decreasing in $\theta$. When $\theta \leq t < 2\theta$,

$$W^{FB} - W^{JM} = \frac{1}{16\theta} (3t - 2\theta)(2\theta - t) > 0$$

Notice that $\frac{\partial (W^{FB} - W^{JM})}{\partial t} = (4\theta - 3t) / (8\theta)$ which is positive for low $t$ and negative for high $t$. Also, $\frac{\partial (W^{FB} - W^{JM})}{\partial \theta} = (3t^2 - 4\theta^2) / (16\theta^2)$ which is negative for low $t$ and positive for high $t$. ■

Intuitively, though an improvement over competition, the perfect cartel still imports too much product, as it is eager to cater to consumers’ taste for variety in order to extract maximal surplus. Seen from the socially optimal outcome, the cartel disproportionately raises the price of the home good, which has a large market share, leading to large revenue gains from the sales to (the many) inframarginal consumers.  

Now, to illustrate, consider alternative price levels within the set of first-best social outcomes. A pro-business ("pro - b") social planner, wishing to maximize producer surplus conditional on total welfare being optimal, would set prices such that the marginal consumer’s surplus is fully extracted, $U_A(p_A; \tilde{x}_1^{FB}) = 0$, that is

$$p_{1A}^{FB, pro-b} = V - \theta \tilde{x}_1^{FB} = V - \frac{1}{2} \min (t + \theta, 2\theta), \quad p_{1B}^{FB, pro-b} = p_{1A}^{FB, pro-b} + t = V + t - \frac{1}{2} \min (t + \theta, 2\theta)$$

Profits are then given by

$$\begin{align*}
\Pi_{1A}^{FB, pro-b} &= \begin{cases} 
\frac{1}{4\theta} (\theta + t)(2V - 2c - t - \theta) & \text{if } t < \theta \\
V - c - \theta & \text{otherwise}
\end{cases} \\
\Pi_{1B}^{FB, pro-b} &= \begin{cases} 
\frac{1}{4\theta} (\theta - t)(2V - 2c - t - \theta) & \text{if } t < \theta \\
0 & \text{otherwise}
\end{cases}
\end{align*}$$

A firm’s total profit in this business-friendly first-best solution $\Pi_{1A}^{FB, pro-b} + \Pi_{2A}^{FB, pro-b}$ falls short

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26 Allowing side-payments would not change the cartel allocation, since in their absence each firm already enjoys one half the monopoly profit over both markets.
relative to the fully collusive outcome\textsuperscript{27}:

\[
\Pi_{1A}^{FB,pro-b} + \Pi_{2A}^{FB,pro-b} - (\Pi_{1A}^{JM} + \Pi_{2A}^{JM}) = \begin{cases} 
-\frac{1}{8} \frac{t^2}{\theta} < 0 & \text{if } t < \theta \\
-\frac{1}{8\theta} (2\theta - t)^2 < 0 & \text{otherwise}
\end{cases}
\]

One might at first be surprised that, in an environment with no aggregate demand effects, the collusive allocation and the pro-business social planner’s outcome differ. To understand why this is the case, recall that the marginal consumer earns zero surplus in both outcomes. However, on switching from pro-business first-best prices to the cartel’s prices, the typical (i.e. non-marginal) home good consumer is made worse off, while the typical imported good consumer is made better off (and by the same amount). Since there are more consumers who purchase the home good, the collusive regime leads to more surplus extraction than the pro-business social optimum. This additional surplus outweighs the increase in cross-hauling costs. This essentially follows from the envelope theorem: marginally shifting the marginal consumer’s to the left has a small effect on total welfare but a first-order effect on surplus extraction, and thus it is profitable for the cartel. Progressively larger shifts to the left yield incrementally smaller increases in surplus extraction (as the market shares become more equal) but they increase hauling costs linearly; at the cartel outcome, these effects cancel each other out at the margin.

Alternatively, a pro-consumer (“pro – c”) social planner, wishing to maximize consumer surplus conditional on total welfare being optimal, would adopt marginal cost pricing\textsuperscript{28}:

\[
p_{1A}^{FB,pro-c} = c, \quad p_{1B}^{FB,pro-c} = p_{1A}^{FB,pro-c} + t = c + t
\]

Much as we compared firm profits in the preceding business-friendly first-best solution relative to the collusive outcome, we may wish to compare consumer surplus in this consumer-friendly first-best solution \(CS^{FB,pro-c}\) relative to the competitive outcome \(CS^{C}\). Computing \(CS^{FB,pro-c}\) as we did...
for $CS^C$ earlier, we obtain

$$CS^{FB,pro-c} - CS^C = \begin{cases} \frac{1}{3\theta} \left( 2t^2 + 9\theta^2 \right) & \text{if } t < \theta \\ \frac{1}{3\theta} \left( t(18 - t) + 27\theta^2 \right) & \text{otherwise} \end{cases}$$

from which it follows (recall A1) that consumer surplus rises relative to the competitive regime.\(^{29}\)

Figure 1 summarizes quantity shares and (relative) prices in market 1 in each of the three regimes. (The figure is drawn for a combination of parameters satisfying $t < \theta$—so that the social planner would allow imports to penetrate—as well as $CS^{JM} > CS^C$.) In all three regimes, the quantity share of the home good increases in the trade cost $t$ and decreases in the degree of product differentiation $\theta$. In particular, as $t \to 0$, the location of the marginal consumer in all three regimes approaches the midpoint of the unit interval and each consumer, facing common prices across goods, buys the good that more closely resembles her ideal product; as mentioned above, we are now in the standard Hotelling setup.

Moving in the other direction, consider for a moment raising the trade cost beyond the upper bound set by A1, $t < 2\theta$. Figure 2 summarizes the quantity cross-hauled (and welfare), in each regime, on moving beyond the restricted space of parameters, as a function of $t$. As noted, for $t \geq 2\theta$, the fully collusive cartel would now make the same trade-off as would the social planner between meeting consumers’ taste for variety and saving on trade costs, which would be to not cross-haul at all, i.e. in this region, $\tilde{x}_1^{JM} = \tilde{x}_1^{FB} = 1$. Social welfare under full collusion would then be maximal, i.e. $W^{JM}|_{t \geq 2\theta} = W^{FB}$. Specifically, for $2\theta \leq t < 3\theta$, while collusion would eliminate cross-hauling, this would still not be the case under price competition.\(^{30}\) Further raising $t$, for $t \geq 3\theta$, cross-hauling would now cease also in the competitive regime, $\tilde{x}_1^{C} (= \tilde{x}_1^{JM} = \tilde{x}_1^{FB}) = 1$, with competition now also yielding optimal social welfare.\(^{31}\) Notice the “concavity” of the problem: at the low end, as $t \to 0$, all three regimes coincide in terms of the degree of geographic market-swapping—this

\(^{29}\)Conceptually, this consumer-friendly first-best solution should not be confused with the optimization of consumer surplus, as it maximizes the sum of consumer and producer welfare. It is easy to show, however, that in this case both solutions coincide. Intuitively, producer surplus in the consumer-friendly first-best solution is zero and thus the entire surplus is obtained by the consumers. Since total surplus is maximal and the individual rationality constraints must be satisfied, there is no possible allocation that would yield higher surplus to consumers.

\(^{30}\)There would thus still be welfare gains from collusion relative to competition: similar to the earlier welfare calculations, noting that $\tilde{x}_1^{C} < \tilde{x}_1^{JM} = 1$, we have $W^{JM} - W^C = t \left( 1 - \tilde{x}_1^{C} \right) + \frac{1}{2} \theta \left( 2 \left( \tilde{x}_1^{C} \right)^2 - 2 \tilde{x}_1^{C} + 1 \right) - \frac{1}{2} \theta = (5t - 3\theta) (3\theta - t) / (36\theta) > 0$ for $2\theta \leq t < 3\theta$.

\(^{31}\)It is easy to show that A2 binds at $t = 3\theta$.\(^{18}\)
is minimal—and welfare—the consumer taste disutility is minimal, as each consumer acquires the
good that is closest to her ideal. Again, this is standard Hotelling. At the high end, as \( t \) reaches
3\( \theta \), all three regimes again coincide: the degree of market-swapping is now maximal—there is no
cross-hauling—and both competition and collusion are first-best.

The inefficiency of oligopoly outcomes in markets with asymmetric firms has been noted previously in the literature. In our paper, the source of inefficiency is the price discrimination against
buyers whom a firm has an advantage in serving from the standpoint of lower trade costs.\(^\text{32}\)

As one may expect, the inefficiency would disappear were firms able to price discriminate within
markets. For every consumer \( x \), a monopolist would compare \( V - \theta x \) (the price it would charge for
the home good and extract all the consumer’s surplus) against \( V - \theta (1 - x) - t \) (the price on the
imported good that extracts all the consumer’s surplus, minus the trade cost it incurs); equating
the two expressions yields the first-best allocation \( \tilde{x}^{FB}_1 \). As for a competitive duopoly, were firms to
dispute every consumer separately (i.e. there are no inframarginal consumers), the home firm would
win consumers \( x < \tilde{x}^{FB}_1 \) while the importer would win consumers \( x > \tilde{x}^{FB}_1 \); to see this, note that
both firms would be equally placed to win the consumer at \( \tilde{x}^{FB}_1 \) since the relative taste disutility
from consuming the home good, \( 2 \left( \frac{1}{2} \right) \theta \), equals the relative price discount offered by the home
firm, \( t \) (there would be marginal cost pricing at \( \tilde{x}^{FB}_1 \) as the effects of product differentiation and
cost asymmetry offset one another).

3.3 Government interventions: Tax and subsidy policies, and price regulation

We now examine how the first-best can be implemented. We show that a social planner, rather
than setting prices directly (as suggested in Proposition 4), can replicate the socially optimal market
allocation through a system of taxes and subsidies. Say that a government, with oversight respon-
sibility over the two local markets, can (in each market) impose a unit tax (tariff) \( \tau \geq 0 \) on sales of
the imported good and a unit subsidy \( \omega \geq 0 \) on sales of the home good. The tax and subsidy policy
is set prior to the firms setting prices. (Alternatively, in an international context, one can envision
two countries coordinating to reciprocally tax imports and subsidize the domestically-produced va-
riety. Clearly, a government acting unilaterally and taking into consideration only domestic welfare,

\(^{32}\)E.g. In Bester and Petrakis’ (1996) spatial model, price discrimination under imperfect competition reduces both
efficiency and firm profits (for a generalization see Liu and Serfes 2004). Our paper goes beyond this literature by
examining the effects of collusion relative to both competition and first-best. Also, our paper is concerned with a
trade context.
i.e., attaching no value to foreign firms or consumers, would not choose the first best tariff level.)

For a market-based regime with either price competition or full collusion, the following proposition describes the symmetric tax and subsidy policy that yields the welfare-optimal market allocation.

**Proposition 5** An appropriate tax and subsidy policy can be used to induce the market-based regime—either price competition or full collusion—to limit trade across geographic markets to the socially optimal level. In particular, an optimal unit tax on imports and unit subsidy on home good sales pair \( (\tau, \omega) \) satisfies (i) \( \tau + \omega = 2t \) in the competitive regime, and (ii) \( \tau + \omega = t \) in the collusive regime.

**Proof** See the appendix.

The proposition states necessary conditions for the first-best market allocation, \( \bar{x}_{FB}^1 \) \( (> \bar{x}_{JM}^1 \succ \bar{x}_{C}^1) \), to be replicated in both competitive and collusive regimes. The reason why these conditions are not sufficient is that individual rationality constraints, for both firms and consumers, need to be satisfied as well. Consider an example, for each regime, of a policy that attains first-best. In the competitive regime, the social planner could optimally tax the imported good at \( \tau = 2t/3 \) and subsidize the home good at \( \omega = 4t/3 \). In the collusive regime, the social planner could optimally tax the imported good at \( \tau = t \) and not subsidize the home good.

Intuitively, as can be seen in the appendix, an optimal tax and subsidy would induce competitive or cartelized firms to set prices such that the price of the imported good exceeds the price of the home good exactly by \( t \). That is, price discrimination against firms’ home-market consumers is eliminated, and excessive cross-hauling ceases.

A natural question concerns how government, facing the “highly wasteful” competitive regime, can implement a (suboptimal) tax and subsidy policy to replicate the market allocation observed in the “less wasteful” collusive regime \( \bar{x}_{JM}^1 \) \( (> \bar{x}_{C}^1) \) (i.e. as calculated in Section 2.2, free of tax and subsidy). As we show in the appendix, a necessary (though again insufficient) condition for government to replicate the cartel’s market allocation is that the \( (\tau, \omega) \) pair satisfies \( \tau + \omega = \frac{1}{2}t \). For example, the competitive duopoly would be induced to cross-haul the same amount of product as the cartel would (or, equivalently, price discriminate just as the cartel would, \( p_{2A}^{JM} - p_{1A}^{JM} = \frac{1}{2}t \)) were, say, \( (\tau, \omega) = (\frac{1}{4}t, \frac{1}{4}t) \). In this particular example, as we show, both consumer surplus and firm profits turn out to be higher relative to the tax-and-subsidy-free competitive outcome.
In sum, tax and subsidy policies can be used in any regime to replicate another regime’s market allocation scheme, namely (and intuitively, recalling Figure 1) (i) \( \bar{x}_1^{C} + \omega = 2t \bar{x}_1^{FB} \), (ii) \( \bar{x}_1^{JM} + \omega = t \bar{x}_1^{FB} \) and (iii) \( \bar{x}_1^{C} + \omega = 1/2 t \bar{x}_1^{JM} \), with the latter characterizing an “intermediate” (suboptimal) level of intervention.

Finally, notice that as a potentially simpler alternative to the optimal tax and subsidy policy above, the government can induce the first-best market allocation by mandating mill pricing, i.e. enacting “anti-dumping” regulation. On prohibiting price discrimination for the same product across both markets—and it is likely that this form of price regulation would be more politically palatable than direct command-and-control pricing measures—a socially optimal outcome ensues. Intuitively, recalling that aggregate volume effects are assumed away, inefficiency arises solely from the fact that, absent intervention, a (competitive or cartelized) importer chooses to absorb a portion of the freight costs. Firms would now be required to either fully pass through transport costs to consumers or outsource shipping to a competitive third-party industry (which would, in equilibrium, charge a price equal to the transport cost).

3.4 Can autarky improve welfare over market-based trade regimes?

We have examined the extent of cross-hauling—or trade—under two alternative market-based “trade regimes”—competitive trade and collusive trade—comparing this to the socially-optimal amount of trade (recall Figure 2 for a summary). In contrast, the trade literature typically compares one trade regime (competitive trade) against the absence of trade (autarky). For completeness, we now derive the autarkic outcome and study the welfare effect of autarky relative to the trade equilibria (both competitive and collusive regimes).

Is it possible that certain (competitive or even collusive) markets lead to so much wasteful trade that society is better off in autarky? Instinctively, one would think not. Imagine the two markets initially shut off from each other. Why would a government forestall the entry in each market of a high-cost (importing) firm that better meets the tastes of some consumers and (recalling that a perfect cartel engages in trade) is privately profitable? We show, however, that the government can improve welfare over market-based trade regimes by directly imposing autarky. The reason is that once entry is allowed, the government cannot dictate the scale of entry. Though autarky can only be worse than allowing limited trade, under first-best, it can be better than allowing unrestricted trade. In other words, it is ideal if the government can limit trade, say through a tax and subsidy
policy, but if that is not possible (say because, in an international setting, only the blunt instrument of blocking entry through health/safety regulations is available, rather than the finer instrument of tariffs), then it may make sense to ban trade outright.

In the autarkic regime (denoted by the superscript $\text{AUT}^K$), each market is a monopoly. (Due to the symmetry, we again consider market 1, and thus firm $A$, and omit market-firm subscripts.) Within the space of parameters restricted by $A_1$ and $A_2$, there are two kinds of monopoly outcomes. The first case occurs for $V$ high enough that the autarkic monopolist fully covers the market. As can be verified below, this occurs for $V - c \geq 2\theta$. (In the appendix, we show that a sufficient, though not necessary, condition for $V - c \geq 2\theta$ to hold, given $A_2$, is $\theta \geq \theta_A$.) In this full coverage case, the monopolist sets price such that the consumer at $x = 1$ has zero surplus, i.e. $V - \theta - p^{\text{AUT}^K} = 0$, so $p^{\text{AUT}^K} = V - \theta$ and thus $\Pi^{\text{AUT}^K} = V - c - \theta$. In the second (complementary) case, where $V - c < 2\theta$, full coverage is not optimal for the monopolist. At the monopoly price, the consumer at $\bar{x}^{\text{AUT}^K} < 1$ is indifferent between the inside good and the outside good, i.e. $V - \theta \bar{x}^{\text{AUT}^K} - p^{\text{AUT}^K} = 0$, or $\bar{x}^{\text{AUT}^K} = (V - p^{\text{AUT}^K}) / \theta < 1$.\footnote{We use $\bar{x}$ rather than $\bar{x}$ since all along $\bar{x}$ has denoted the location of the consumer who—when trade is allowed—is indifferent between either inside good $A$ or $B$.}

The monopolist’s problem in this case is then:

$$\max_p \left( p - c \right) \frac{V - p}{\theta}$$

yielding $p^{\text{AUT}^K} = \frac{1}{2} (V + c)$ and $\Pi^{\text{AUT}^K} = (V - c)^2 / (4\theta)$, and where the share of the inside good is $\bar{x}^{\text{AUT}^K} = (V - c) / (2\theta) < 1$. In summary, the autarkic price and per-market profit are given by

$$p^{\text{AUT}^K} = \begin{cases} \frac{1}{2} (V + c) & \text{if } V - c < 2\theta \text{ (incomplete coverage)} \\ V - \theta & \text{otherwise (full coverage)} \end{cases}$$

and

$$\Pi^{\text{AUT}^K} = \begin{cases} \frac{1}{4\theta} (V - c)^2 & \text{if } V - c < 2\theta \text{ (incomplete coverage)} \\ V - \theta - c & \text{otherwise (full coverage)} \end{cases}$$

The following proposition states regions in parameter space for which society would be better off under autarky relative to market-based trade regimes. Put simply, autarky welfare-dominates
market-based trade regimes except when the trade cost $t$ is low; for these low $t$ cases, the welfare shortfall under autarky narrows as $t$ is raised.

**Proposition 6** For a sufficiently high unit trade cost, social welfare under autarky exceeds social welfare under full collusion (and thus exceeds social welfare under price competition). In particular, $W^{AUTK} > W^{JM} (> W^C)$ holds (i) for $\theta \leq t (< 2\theta)$ (here there is full market coverage under autarky); and (ii) for $\frac{2}{3}\theta < t < \theta$ and $V - c \geq 2\theta$ (i.e. whenever there is full market coverage under autarky). Further, (iii) for $\frac{2}{3}\theta < t \leq \frac{2}{3}\theta$ and $V - c \geq 2\theta$, social welfare under autarky exceeds social welfare under price competition but is lower than social welfare under full collusion, i.e. $W^{JM} \geq W^{AUTK} > W^C$. Outside these regions, any welfare shortfall under autarky relative to price competition (and thus relative to full collusion) narrows as the trade cost increases. In particular, (iv) for $t \leq \frac{2}{3}\theta$ and $V - c \geq 2\theta$, increasing $t$ raises both $W^{AUTK} - W^C \leq 0$ and $W^{AUTK} - W^{JM} < 0$ toward zero; and (v) for $V - c < 2\theta$ (market coverage is incomplete under autarky, occurring for $t \leq 2(V - c) - 3\theta < \theta$), increasing $t$ raises both $W^{AUTK} - W^C \leq 0$ and $W^{AUTK} - W^{JM} \leq 0$ toward zero (and possibly beyond).

**Proof** See the appendix. ■

Notice that statement (i) of the proposition is quite intuitive. Recall that in the interval $\theta \leq t (< 2\theta)$ the first-best social outcome involves no cross-hauling— unlike the collusive regime, let alone the competitive regime, where cross-hauling obtains. Since in this region the autarkic monopolist would fully cover the market, autarky is thus first-best. Statements (ii) through (iv) pertain also to regions where there is full market coverage in autarky: conditional on full coverage, autarky is preferred to price competition (if not to full collusion) for $t$ no less than $\frac{2}{3}\theta$. Statement (v) says that in the region where there is incomplete market coverage in autarky (this is a strict subspace of $t < \theta$), it may be that either $(W^{JM}) W^C > W^{AUTK}$, $W^{JM} > W^{AUTK} \geq W^C$ or $W^{AUTK} \geq W^{JM} (> W^C)$; importantly, however, as $t$ increases in this region (holding other parameters fixed) the undesirability of autarky relative to market-based regimes diminishes and may be reversed.\(^{34}\)

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\(^{34}\)A comment on unilateral trade policy is in order. Say that a local market’s government can ban imports but still allow outbound trade (i.e. exports) and that the other local market does not reciprocate. We know that foreign sales are profitable for the home firm. So whenever autarky is preferred over a trade regime, it must be the case that a unilateral import ban welfare-dominates autarky (i.e. the banning of inbound and outbound trade). Further, whenever autarky lowers welfare relative to (reciprocal) trade, it may be that a unilateral import ban still welfare-dominates trade.
3.5 The effects of home bias

Very often, consumers in a market favor locally-sourced products over competing imports. Part of the reason may be national sentiment and concern over local jobs (witness the prominent “Made in the USA” label on goods produced and sold in the American market), or environmental considerations (e.g. the green movement promotes consumption of local produce in an attempt to curb greenhouse emissions generated from transportation). Similarly, local market knowledge may enable home brands to better appeal, on average, to national tastes.

We now ask what the effect of “home bias” in consumer preferences would be in each of the three (trade) environments we have considered. It is clear that if consumers favor the local brand, less cross-hauling will occur in all regimes (in a sense, one can think of home bias as a trade cost, though the former operates from the demand side while the other operates from the supply side). Less clear is which regime, if any, will be most affected. To fix ideas, we generalize the earlier setup by specifying, for each consumer, an additional willingness to pay $h$ for the home variety relative to the imported variety, i.e. $V_{1A} = V_{2B} = V + h$ while maintaining $V_{1B} = V_{2A} = V$. The consumer who is indifferent between goods $A$ and $B$ is now located at

$$\bar{x}^{hb}(p) = \frac{\theta - p_A + p_B + h}{2\theta}$$

where superscript $hb$ denotes the presence of home bias. (Restate restrictions A1’ and A2’, which define the space of interest, as $t + h < 2\theta$ and $2(V - c) > t + 3\theta - h$ respectively.) Repeating the derivations of the preceding sections (and again considering market 1), the competitive equilibrium is now characterized by prices

$$p_1^{C,gb} = p_1^C + \frac{1}{3}h, \quad p_1^{C,gb} = p_1^C - \frac{1}{3}h$$

and home-good quantity share

$$\bar{x}_1^{gb} = \bar{x}_1^C + \frac{1}{6}\frac{h}{\theta}$$

where the absence of $hb$ in the superscript denotes the particular case where there is no home bias, $h = 0$, seen earlier. As expected, increasing home bias (from zero) raises the share of the home good. The relative change in shares results, in equilibrium, in a reduction in the relative price of the imported good, $p_1^{C,gb} - p_1^{C,gb} = \frac{1}{3}(t - 2h)$. Similarly, prices and the quantity share of the home
good in the fully collusive regime are now

\[ p_{JM,hb}^{JM} = p_{JM}^{JM} + \frac{3}{4} h, \quad p_{JM,hb}^{JM,lb} = p_{JM,lb}^{JM} + \frac{1}{4} h \]

\[ \tilde{x}_{JM,hb}^{JM} = \tilde{x}_{JM}^{JM} + \frac{h}{4 \theta} \quad (8) \]

Comparing (8) and (7), an increase in the home bias raises the share of the home good by more in the collusive regime than in the competitive regime \((+h/(4\theta) v. +h/(6\theta))\). In other words, the presence of home bias results in more cross-hauling in the non-cooperative equilibrium relative to collusion, reinforcing the result of Proposition 1. It is easy to verify that \(p_{JM,lb}^{JM,lb} - p_{JM,lb}^{JM} = p_{JM,lb}^{JM,lb} - p_{JM,lb}^{JM,lb} = \frac{1}{2} (t - h)\): though home bias again raises price discrimination in favor of buyers of the imported good, it does so by less under collusion than under competition \((-h/2 v. -2h/3)\), reinforcing our finding of collusion as a (partial) correction mechanism. Finally, consider the socially first-best market allocation. As in Section 3.2, write the location of the marginal consumer as \(\tilde{x}_{FB,lb}^{FB} = \frac{1}{2} + d\).

The relative taste disutility of the home good for this consumer is now \(2d\theta - h\), which the social planner equates with the cross-hauling cost \(t\), and thus \(\tilde{x}_{FB,lb}^{FB} = \frac{1}{2} + \frac{1}{2} (t + h) / \theta\), or

\[ \tilde{x}_{FB,lb}^{FB} = \begin{cases} \tilde{x}_{FB}^{FB} + \frac{h}{2 \theta} & \text{if } t + h < \theta \\ 1 & \text{otherwise} \end{cases} \]

Relative to the competitive and collusive outcomes, the effect of home bias—expanding the home-good share, restricting cross-hauling—is most pronounced under first-best \((+h/(2\theta))\), reinforcing our earlier result. To understand why, notice that \(p_{1B}^{FB,lb} - p_{1A}^{FB,lb} = t\) (this is immediate from, e.g., (6)): as in the earlier setup without home bias \((h = 0)\), the planner equalizes a same variety’s price-cost margins across the two markets. In contrast, dumping is maximal—and increasing in \(h\)—in the competitive regime. The effect of \(h\) on the collusive market division is intermediate to the socially optimal and competitive regimes, as was the case for \(h = 0\).
4 Concluding remarks

In the context of trade where aggregate demand effects are small, we have provided a model—employing only standard ingredients—where the following unconventional and clear-cut combination of results obtains: (i) (perfect) collusion reduces, though does not eliminate, trade relative to competition, leading to a cartel allocation consistent with the “home-market principle”; (ii) this collusive reduction in trade (and thus reduction in the heterogeneity of consumption) enhances total welfare; (iii) the welfare gain from collusion occurs even when the trade cost is low (with this welfare gain increasing in the trade cost and decreasing in the degree of product differentiation); (iv) the cartel’s reduction in trade can even enhance consumer welfare relative to the competitive regime; (v) even collusion involves some degree of excessive trade relative to the welfare optimum; and (vi) for a sufficiently high trade cost, even the (extreme) prohibition of trade—i.e. imposing autarky—improves welfare over the (tax-and-subsidy-free) competitive and collusive trade regimes.

These results have direct implications for antitrust policy with regard to coordination in spatial markets, and cross-border trade in particular. We believe they may be useful in assessing international cartels which, as recent evidence suggests (e.g. Levenstein and Suslow 2004 selectively count 42 international cartels that were successfully prosecuted during the 1990s), are an important phenomenon in the contemporary global economy. The results suggest that “anti-dumping” regulation may find a role in optimal trade policy.

Our analysis assumed exogenous trade costs. The result that a cartel chooses a lower level of cross-hauling than a competitive duopoly suggests the possibility that, in the absence of a cartel, competing firms might lobby to (symmetrically) raise trade costs. For example, they might oppose the formation of a trade agreement. We also assumed that perfect collusion (monopoly) can be implemented, thus ignoring the possibility that changes in trade costs may in fact impact the sustainability of the cartel (e.g. Bond and Syropoulos 2008) and that cartelized firms may attempt to influence these costs. This comparative static will likely depend on the nature of punishment strategies employed (Friedman-type trigger strategies or Abreu-type strategies with a stick and carrot structure). These questions, while interesting, are left as future research since they are of derivative interest relative to the issues examined in the present paper.

Beyond the confines of this particular modeling framework, we speculate that ours is but one example where collusion can result in Pareto superior outcomes—i.e. both for firms and for consumers—even in the absence of any direct transfers between the firms. What is particularly
interesting in our framework is that we obtain this win-win possibility result even in the absence of firm asymmetries across all markets. The welfare-reducing distortion that the cartel mitigates arises from the comparative advantage of firms in supplying particular markets. A systematic examination of the conditions under which collusion can yield such Pareto superior outcomes appears to be an interesting research agenda.

References


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A Appendix: Proof of Proposition 5 (and related statements)

We start by considering the collusive regime. With the tax and subsidy, the perfect cartel’s univariate problem of Section 2.2 changes to

$$\max_{p_A} (p_A - c + \omega) \frac{V - p_A}{\theta} + (2V - \theta - p_A - c - t - \tau) \left(1 - \frac{V - p_A}{\theta}\right)$$

(For brevity, we write the proof for an interior solution, $t < \theta$; otherwise the equilibrium outcome is the corner solution of Section 3.2.) This yields prices and profits

$$p_{1A}^M (\tau, \omega) = V - \frac{1}{4} (t + \tau + \omega) - \frac{1}{2} \theta, \quad \Pi_{1A}^M (\tau, \omega) = 2V - \theta - p_{1A}^M (\tau, \omega) = V + \frac{1}{4} (t + \tau + \omega) - \frac{1}{2} \theta$$

$$p_{1B}^M (\tau, \omega) = 2V - \theta - p_{1B}^M (\tau, \omega) = 2V + \frac{1}{4} (t + \tau + \omega) - \frac{1}{2} \theta$$

$$\Pi_{1B}^M (\tau, \omega) = \frac{1}{16 \theta} (2\theta + t + \tau + \omega) (4V - 4c - t - \tau + 3\omega - 2\theta)$$

The marginal consumer, who has zero surplus, is now located at:

$$\bar{x}_{1A}^M (\tau, \omega) = \frac{1}{2} + \frac{1}{4} \frac{t + \tau + \omega}{\theta}$$

For the first-best market allocation to attain, i.e. $\bar{x}_{1A}^M (\tau, \omega) = \bar{x}_{1A}^{FB}$, it is clear from Section 3.2 that $(t + \tau + \omega) / (4\theta) = t / (2\theta)$ and thus a necessary condition is

$$\bar{x}_{1A}^M \rightarrow \bar{x}_{1A}^{FB} : \quad \tau + \omega = t$$

(Alternatively, one can compute, as in Section 3.1, the sum of the total cost of cross-hauling and the total consumer taste disutility, $t \left(1 - \bar{x}_{1A}^M (\tau, \omega)\right) + \int_0^{\bar{x}_{1A}^M (\tau, \omega)} \theta x dx + \int_{\bar{x}_{1A}^M (\tau, \omega)}^1 \theta (1 - x) dx$, and minimize this sum with respect to the policy instruments.) As expected, $p_{1B}^M (\tau, \omega) - p_{1A}^M (\tau, \omega) = \tau + \omega = t$.

To verify the example provided in the text, $(\tau, \omega) = (t, 0)$, notice that by construction of the cartel’s univariate problem, the marginal consumer’s utility is zero, while profits on both the home
good and the imported good are non-negative. (To see this, notice that $\Pi_{1A}^{JM}(\tau = t, \omega = 0) = (\theta + t) (2V - 2c - t - \theta) / (4\theta) > A^2 0$ and $\Pi_{1B}^{JM}(\tau = t, \omega = 0) = (\theta - t) (2V - 2c - 3t - \theta) / (4\theta) > t<0 \theta$ $(\theta - t) (2V - 2c - t - 3\theta) / (4\theta) > A^2 0$.) So individual rationality constraints are satisfied.

Now consider the competitive regime. In the presence of a tax and a subsidy, prices in the competitive equilibrium solve the modified system (cf. Section 2.1)

$$\max_{p_A} (p_A - c + \omega) s_A(p)$$

$$\max_{p_B} (p_B - c - t - \tau) s_B(p)$$

yielding prices and profits (again, for brevity, we write the proof for an interior solution, $t < \theta$, otherwise the corner solution of Section 3.2 applies)

$$p_{1A}^C(\tau, \omega) = c + \frac{1}{3} (t + \tau - 2\omega) + \theta, \quad p_{1B}^C(\tau, \omega) = c + \frac{1}{3} (2t + 2\tau - \omega) + \theta$$

$$\Pi_{1A}^C(\tau, \omega) = \frac{1}{18\theta} (3\theta + t + \tau + \omega)^2, \quad \Pi_{1B}^C(\tau, \omega) = \frac{1}{18\theta} (3\theta - t - \tau - \omega)^2$$

The equilibrium location of the marginal consumer is given by

$$\hat{x}_1^C(\tau, \omega) = s_{1A}(p(\tau, \omega)) = \frac{1}{2} + \frac{1}{6} \frac{t + \tau + \omega}{\theta}$$

Similar to the above, for the first-best market allocation to attain, i.e. $\hat{x}_1^C(\tau, \omega) = \hat{x}_1^{FB}$, it follows that $(t + \tau + \omega) / (6\theta) = t / (2\theta)$ and thus a necessary condition is

$$\hat{x}_1^C \rightarrow \hat{x}_1^{FB} : \quad \tau + \omega = 2t$$

Note, similarly, that $p_{1B}^C(\tau, \omega) - p_{1A}^C(\tau, \omega) = t+\omega=2t.$

Now verify that individual rationality constraints are satisfied in the example provided in the text, $(\tau, \omega) = (2t/3, 4t/3)$. The marginal consumer’s utility is $U_A(p_{1A}^C(2t/3, 4t/3); \hat{x}_1^{FB}) = \frac{1}{2} (2V - 2c - \frac{1}{3} t - 3\theta) > \frac{1}{2} (2V - 2c - t - 3\theta) > A^2 0$. Profits on both the home good and the imported good are clearly positive.

Still considering the competitive regime, for the cartel market allocation to attain, i.e. $\hat{x}_1^C(\tau, \omega) =$
\(\tilde{x}_1^{JM}\), a necessary condition is \((t + \tau + \omega) / (6\theta) = t / (4\theta)\) or

\[\tilde{x}_1^C \rightarrow \tilde{x}_1^{JM} : \quad \tau + \omega = \frac{1}{2}t\]

where, as expected, \(p_{1B}^C(\tau, \omega) - p_{1A}^C(\tau, \omega) = \tau + \omega = \frac{1}{2}t\), \(p_{1B}^{JM} - p_{1A}^{JM} = \frac{1}{2}t\).

Verifying the example provided, \((\tau, \omega) = (\frac{1}{4}t, \frac{1}{4}t)\), profits on both home and imported goods are similarly positive, and the marginal consumer’s utility is \(U_A(p_{1A}^C(\frac{1}{4}t, \frac{1}{4}t); \tilde{x}_1^{JM}) = \frac{1}{2}(2V - 2c - t - 3\theta) > A^2\) 0. Thus individual rationality constraints are met. For this example, both consumer surplus and firm profits turn out to be higher relative to the tax-and-subsidy-free competitive outcome. Calculating \(CS^C(\frac{1}{4}t, \frac{1}{4}t) = \int_0^{\tilde{x}_1^{JM}} (V - \theta x - p_{1A}^C(\frac{1}{4}t, \frac{1}{4}t)) dx + \int_{\tilde{x}_1^{JM}}^1 (V - \theta (1 - x) - p_{1B}^C(\frac{1}{4}t, \frac{1}{4}t)) dx\) and subtracting \(CS^C\) (as mentioned in Section 3.1, \(CS\) is the area of two trapezia), it follows that \(CS^C(\frac{1}{4}t, \frac{1}{4}t) - CS^C = \frac{5}{144}t^2 \theta > 0\). Also, each firm’s profit increases by \(\Pi_{1A}^C(\frac{1}{4}t, \frac{1}{4}t) + \Pi_{1B}^C(\frac{1}{4}t, \frac{1}{4}t) - \Pi_{1A}^C - \Pi_{1B}^C = \frac{5}{36}t^2 \theta > 0\).

B Appendix: Proof of Proposition 6

Within the space of parameters defined by A1 and A2, we begin by examining the subspace where full market coverage obtains in autarky (as shown in the text, this occurs if \(V - c \geq 2\theta\)), followed by the subspace where coverage in autarky is incomplete (i.e. \(V - c < 2\theta\)). Consider A1: \((0 < )t \leq 2\theta\). Notice that a sufficient condition for full coverage in autarky is \(t \geq \theta\), since \(V - c > A^2\) \(\frac{1}{2}t \geq 3\theta\) \(\geq t > 2\theta\). For \(t < \theta\), full coverage in autarky may or may not obtain, as \(\frac{1}{2}(t + 3\theta) < t \theta < 2\theta\) and thus \(V - c \geq 2\theta\) (e.g. say that \(t\) is low and A2 almost binds: then \(2(V - c) \simeq 3\theta < 4\theta\), so in autarky there is incomplete coverage). To emphasize, incomplete coverage \((V - c < 2\theta)\) implies that \(t < A^2 2(V - c) + 3\theta < V - c < 2\theta\).

Now compute (per-market) social welfare under autarky. Consider the first case, of full coverage: \(V - c \geq 2\theta\). Consumer surplus is \(\frac{1}{2}t\) (the area of a triangle with height \(V - p^{AUTK} = \theta\) and unit width) and producer surplus is \(\Pi^{AUTK} = V - c - \theta\), the sum of which yields social welfare: \(W^{AUTK} = V - c - \frac{1}{2}t\). Next consider the complementary case, of incomplete coverage: \(V - c < 2\theta\). Consumer surplus is \((V - c)^2 / (8\theta)\) (the area of a triangle with height \(V - p^{AUTK} = \frac{1}{2}(V - c)\) and width \(\tilde{x}^{AUTK} = (V - c) / (2\theta)\)) and producer surplus is \(\Pi^{AUTK} = (V - c)^2 / (4\theta)\), with welfare totaling \(W^{AUTK} = 3(V - c)^2 / (8\theta)\).

Next compute social welfare in each of the two market-based trade regimes. (Recall that in these
regimes, throughout the space of parameters, the market is fully covered and cross-hauling occurs.) Consumer surplus is calculated as explained in Section 3.1 (equivalent to the area of two trapezia, which in the collusive regime collapses to the area of two triangles as the marginal consumer has zero surplus). Producer surplus is given by the sum of a firm’s profit on home sales and its profit on foreign sales, as stated in Sections 2.1 (competitive regime) and 2.2 (collusive regime). For brevity, we simply state the sum of consumer surplus and producer surplus in each regime: $W^C = (36V\theta - 36c\theta - 18t\theta + 5t^2 - 9\theta^2) / (36\theta)$ and $W^{JM} = (16V\theta - 16c\theta - 8t\theta + 3t^2 - 4\theta^2) / (16\theta)$.

We now calculate welfare differences across regimes, first considering the parameter subspace for which there is full coverage under autarky (i.e. $V - c \geq 2\theta$). We compute $16\theta (W^{AUTK} - W^{JM}) = -3t^2 + 8t\theta - 4\theta^2$ which, being concave in $t$ and having roots $t = \frac{2}{3}\theta, 2\theta$, is strictly positive over the interval $\frac{2}{3}\theta < t < 2\theta$: hence (conditional on full coverage, and recalling Proposition 2) we have $W^{AUTK} > W^{JM} > W^C$. This proves statements (i) and (ii). Further, proof of the second part of statement (iv) follows from noting that $-3t^2 + 8t\theta - 4\theta^2$ is negative for $t < \frac{2}{3}\theta$, but increasing in $t$. Similarly, we compute $36\theta (W^{AUTK} - W^C) = -5t^2 + 18t\theta - 9\theta^2$ which is strictly positive over the interval $\frac{2}{3}\theta < t < 3\theta$. So for $\frac{2}{3}\theta < t \leq \frac{2}{3}\theta$ (and full coverage) we have $W^{JM} \geq W^{AUTK} > W^C$, proving statement (iii). Proof of the first part of statement (iv) follows, similarly, from noting that, for $t \leq \frac{2}{3}\theta$, $-5t^2 + 18t\theta - 9\theta^2$ is (weakly) negative and increasing in $t$.

It remains to prove statement (v), pertaining to the parameter subspace for which market coverage in autarky is incomplete (i.e. $V - c < 2\theta$, implying that $t \leq 2(V - c) - 3\theta < \theta$). We compute $16\theta (W^{AUTK} - W^{JM}) = 2(V - c) (3V - 3c - 8\theta) + (-3t^2 + 8t\theta + 4\theta^2)$. Since $V - c > A^2 0$ and, conditional on incomplete coverage, $V - c - 2\theta < 0 \iff 3V - 3c - 6\theta < 0 \iff 3V - 3c - 8\theta < 0$, the first bunch of terms is negative. It is also invariant in $t$. Noting that, over the interval $0 < t \leq 2V - 2c - 3\theta < \theta$, the parabola defined by $-3t^2 + 8t\theta + 4\theta^2$ is positive and increasing in $t$, it follows that $W^{AUTK} - W^{JM} \leq 0$ and that $W^{AUTK} - W^{JM}$ increases in $t$. Similarly, we compute $16\theta (W^{AUTK} - W^C) = 2(V - c) (3V - 3c - 8\theta) + \frac{4}{9} (-5t^2 + 18t\theta + 9\theta^2)$ where the (same $t$-invariant) first bunch of terms is negative and, over the interval $0 < t \leq 2V - 2c - 3\theta < \theta$, the parabola $\frac{4}{9} (-5t^2 + 18t\theta + 9\theta^2)$ is positive (in fact, consistent with Proposition 2, larger than $-3t^2 + 8t\theta + 4\theta^2$) and increasing in $t$. It follows that $W^{AUTK} - W^C \leq 0$ and that $W^{AUTK} - W^C$ increases in $t$. This proves (v). In this subspace of incomplete coverage under autarky, we further show that as $t \to 0^+$, $(W^{AUTK} - W^{JM} <) W^{AUTK} - W^C < 0$. The left inequality follows from Proposition 2. The right inequality follows from noting that as $t \to 0^+$, $8\theta (W^{AUTK} - W^C) \to (V - c) (3V - 3c - 8\theta) + 2\theta^2 < 0$. To see this, notice that $(V - c) (3V - 3c - 8\theta) + 2\theta^2 < 0 \iff - (3V - 3c - 8\theta) (V - c) > 2\theta^2$, and that $V - c < 2\theta \iff - (3V - 3c - 8\theta) > 2\theta > 0$ and $V - c > A^2$.
\( \frac{1}{2} t + \frac{3}{2} \theta > \theta > 0 \). Also in this subspace of incomplete coverage under autarky, we show (by example) that as \( t \to 2V - 2c - 3\theta < \theta \), \( W^{AUTK} - W^C \) remains negative (e.g. \( V = 3, c = 1.2, \theta = 1 \)) or can become positive (e.g. \( V = 3, c = 1.05, \theta = 1 \)). Similarly, as \( t \to 2V - 2c - 3\theta < \theta \), \( W^{AUTK} - W^{JM} (< W^{AUTK} - W^C) \) remains negative or can become positive (see the same respective examples).
Figure 1: Quantity shares and prices for the home good and the foreign good in market 1, for different trade regimes (in the restricted space of parameters): Price competition (top panel), full collusion (middle panel), and the socially first-best outcome (bottom panel). Market 2 is symmetric. Drawn to scale for parameter values $V = 1$, $c = .42$, $t = .25$, $\theta = .3$. (Vertical measures start at $c$; relative to vertical scale, horizontal scale is scaled up by a factor of 2.)
Figure 2: The extent of cross-hauling across the different trade regimes, within and beyond the restricted space of parameters (A1). Market 1’s import share \(1 - \tilde{x}_1\) (left axis) and share of home good \(\tilde{x}_1\) (right axis, inverted scale) against trade cost \(t\).