Naked Exclusion with Private Offers*

Jeanine Miklós-Thal† Greg Shaffer‡

October 2015

Abstract

We consider a seller’s ability to deter potential entrants by offering exclusive contracts to its downstream buyers. Rasmusen, Ramseyer, and Wiley (1991) showed that this can be a profitable strategy if there is a coordination failure on the part of the buyers. Segal and Whinston (2000) showed that the seller need not rely on a coordination failure if it can make discriminatory “divide-and-conquer” offers. These papers and the literature that followed has assumed that all offers are public. We show that if buyers cannot observe each other’s offers and have passive or wary out-of-equilibrium beliefs, the divide-and-conquer exclusion strategy fails. Equilibria in which the incumbent obtains exclusion for free due to a lack of coordination between buyers, on the other hand, exist for all out-of-equilibrium beliefs.

JEL Classification: L13, L41, L42.

Keywords: exclusive dealing, divide and conquer, coordination failure.

*We thank audiences at the University of Bergen, the University of Toronto, the 2014 International Industrial Organization Conference, and the 2015 European Association for Research in Industrial Economics Conference for helpful comments.

†Simon Business School, University of Rochester; e-mail: jeanine.miklos-thal@simon.rochester.edu.

‡Simon Business School, University of Rochester; e-mail: shaffer@simon.rochester.edu.
1 Introduction

Vertical contracts that prohibit a seller’s customers from dealing with rival sellers have long been controversial in antitrust economics. A key issue is the extent to which such contracts can lead to inefficient and anti-competitive exclusion. At the very least, one must ask why buyers would agree to sign contracts that are written for the purpose of excluding rival sellers. Presumably the buyers would have to be compensated for doing so, and if this compensation would be more than the seller could gain from excluding its rivals, the exclusion would not be profitable.

The Chicago-school view, which has been enormously influential in the courts, is that contracts that are written with the intent to exclude (“exclusive contracts”) will not generally be profitable for an incumbent seller. Typically exposited in a setting with an incumbent seller and one buyer, the argument is that the loss to the buyer from buying at the monopoly price (rather than at the competitive price when there is entry) will generally exceed the incumbent’s gain by the deadweight loss of monopoly pricing. Hence, the incumbent would incur a loss if it had to fully compensate the buyer.

In response, Rasmusen et al. (1991) and Segal and Whinston (2000) have shown that exclusive contracts can indeed be profitable for the incumbent when there are externalities across buyers. In their models, the entrant has scale economies that make it unprofitable to enter unless the entrant can sell to enough buyers. When a buyer signs an exclusive contract, it imposes a negative externality on all other buyers by reducing the profitability of entry. Rasmusen et al. showed that the incumbent may be able to exploit these externalities by taking advantage of a lack of coordination on the part of the buyers. If each buyer believes that enough other buyers will be signing the incumbent’s contract, so that each believes that the entrant will be excluded regardless of its own decision, then each may be willing to sign the incumbent’s contract for little or no inducement. Segal and Whinston showed that with discriminatory offers, the incumbent may be able to exclude the entrant profitably even in the absence of a coordination failure. They showed that the incumbent can adopt a “divide-and-conquer” strategy in which it offers to fully

---

1 Aghion and Bolton (1987) show that an incumbent may be able to extract rents from a more efficient entrant by means of partially exclusive contracts. In a two-buyer version of their model, they also allow for externalities across buyers. However, in contrast to Rasmusen et al (1991) and Segal and Whinston (2000), they assume that each offer can be made conditional on the other buyer’s accept or reject decision.
compensate some buyers — as many as it needs to induce exclusion — in order to earn monopoly profits on the remaining buyers without paying them anything.\textsuperscript{2} Although the buyers who receive compensation are obtained at a loss, the incumbent may nevertheless find exclusion to be profitable if there are enough other buyers to whom it has offered no compensation.

These papers and the literature that followed has assumed that all offers are \textit{publicly observable} (i.e., that each buyer can observe the offers made to every other buyer prior to deciding whether to accept its own offer). In this paper, in contrast, we assume that the incumbent’s offers are \textit{privately observable}. With privately-observable offers, buyers cannot observe each other’s offers prior to deciding whether to accept their own offer.

The rationale for studying the case of privately-observable offers is that when different contracts are offered to different buyers (as is the case when the incumbent makes discriminatory divide-and-conquer offers), negotiations typically occur on a bilateral basis, and these negotiations may not be publicly observable.\textsuperscript{3} It is therefore important to consider whether offering exclusive contracts can still be profitable for the incumbent in these settings (the Chicago-school question), and if so, how the leading responses fare.

Segal and Whinston analyze the case of privately-observable offers in the working paper version of their paper (Segal and Whinston, 1996). They show that with private offers, a divide-and-conquer strategy can be used to induce exclusion in equilibrium when the buyers’ out-of-equilibrium beliefs are such that after receiving an unexpected offer, regardless of what that offer is, each buyer believes that every other buyer has received a “zero offer” from the incumbent (i.e., an offer with no compensation that is expected to be rejected). These out-of-equilibrium beliefs are not appealing, however, because they are tantamount to assuming that after receiving an unexpected offer, each buyer believes that the incumbent has given up trying to deter the entrant. This is troublesome because,

\textsuperscript{2} Divide-and-conquer strategies have also been explored in other applications of contracting with externalities, including corporate takeovers, bankruptcy proceedings, labor relations, litigation and plea bargaining, and vote buying (see, for example, Segal 1999; Posner et al. 2010; and the references therein).

\textsuperscript{3} Although the incumbent might prefer to make its offers public, this may not be feasible. What makes public announcements problematic is that the incumbent may have an incentive to misrepresent to any one buyer how many other buyers have received exclusive contracts, or how many other buyers are expected to sign, in order to induce exclusion cheaply. Also, as we will show, unless it can commit to its offers, the incumbent may have an incentive to engage in multi-lateral deviations from its public announcements.
as is well known from the literature on unobservable contracts in other vertical settings (e.g., Hart and Tirole, 1990; O’Brien and Shaffer, 1992; and McAfee and Schwartz, 1994), characterizing equilibrium outcomes when a seller’s offers are privately observable can be problematic unless sensible restrictions are placed on buyers’ out-of-equilibrium beliefs.  

The main contribution of this paper is to show that when a seller’s offers are privately observable, out-of-equilibrium beliefs are critical in determining the success of a divide-and-conquer strategy, and that unless strong and unrealistic assumptions are made about these beliefs, the strategy will fail. We will show that this is the case, for example, when buyers have “passive” or “wary” beliefs, as commonly assumed in the vertical-contracting literature. In contrast, the existence of equilibria that rely on a coordination failure on the part of the buyers to obtain exclusion do not depend on out-of-equilibrium beliefs.

Under “passive beliefs” (Hart and Tirole, 1990; McAfee and Schwartz, 1994), a buyer who receives an unexpected offer continues to believe that the other buyers have received their equilibrium offers. We find that when buyers have passive beliefs, exclusion cannot be supported in equilibrium via a divide-and-conquer strategy because the incumbent’s offers in the candidate equilibrium are not immune to multilateral deviations. In particular, the incumbent can profitably deviate by offering a small inducement to a buyer who was expecting none, while offering no inducement to a buyer who was expecting to be compensated in full, keeping all other offers the same. Although the latter will reject its offer, the former will accept it, allowing the incumbent to obtain exclusion at a lower cost. Passive beliefs are often justified by assuming that buyers interpret unexpected offers as mistakes or trembles. They are less appealing, however, when buyers interpret unexpected offers as deliberate deviations by the incumbent. In these cases, it may be more natural to assume that buyers have “wary beliefs” (proposed by McAfee and Schwartz, 1994; and elaborated on by Rey and Vergé, 2004; see also Avenel, 2012). Under wary beliefs, a buyer who receives an unexpected offer does not automatically assume that other buyers have received their equilibrium offers, or that the incumbent has given up trying to deter the entrant, but instead believes that the incumbent will act optimally with the

---

4 Without any restrictions on the buyers’ beliefs, the latitude in possible beliefs off-the-equilibrium path typically allows one to support a wide range of perfect-Bayesian equilibria.

5 Passive beliefs are the most commonly assumed beliefs in the vertical-contracting literature with privately-observable offers. See, for example, the discussion in Rey and Tirole (2007).
other buyers given the offer it has received. Buyers with wary beliefs are more suspicious about the incumbent’s motives than buyers with passive beliefs in the sense that they expect the incumbent to act opportunistically whenever it is profitable for it to do so.

Although wariness on the part of the buyers has been shown in other settings to limit the ability of the incumbent to act opportunistically (this is one of the key observations in Rey and Vergé, 2004), we find that it has no effect in the setting we consider here. This is so in spite of the fact that upon receiving an unexpected offer wary buyers recognize that multilateral deviations by the incumbent are not only possible but likely. The reason for the failure is that any buyer who receives an unexpected offer and who has wary beliefs will expect the incumbent to have deviated with others in a way that continues to ensure that the entrant will be deterred. Given this, it will be optimal for the buyer to accept its offer, even if the offer fails to fully compensate the buyer for the absence of upstream competition.

After showing that a divide-and-conquer strategy cannot be supported in equilibrium when buyers have passive or wary beliefs, we characterize necessary and sufficient conditions for such an equilibrium to exist (when it exists with publicly-observable offers). We show that for any beliefs that satisfy these conditions, either (i) a buyer believes that the incumbent’s offers to the other buyers are not optimal given the offer it has received (e.g., as is the case with zero-offer beliefs), or (ii) a buyer believes that the incumbent has incorrect expectations about which offers will be accepted off the equilibrium path (e.g., a buyer believes that the incumbent expects it to accept an offer that is not in its equilibrium acceptance set, when in fact it would be optimal to reject any such offer).

In their experimental investigation of the settings in Rasmusen et al. (1991) and Segal and Whinston (2000), Landeo and Spier (2009) consider the case of privately-observable offers as an extension. Like us, they also argue that divide-and-conquer equilibria may fail to exist. However, their non-existence result is not based on buyers having passive or wary beliefs (or any other beliefs), but on the fact that the incumbent’s offer space in their experiments is bounded away from zero. They state (p. 1869) that “If we allowed our incumbent seller to offer zero (or to make no offer at all) to one of the buyers, then [Segal and Whinston’s] divide-and-conquer equilibrium would exist here as well”. As we will show, however, this claim implicitly depends on buyers having certain beliefs (e.g.,
zero-offer beliefs) and does not hold if, for example, buyers have passive or wary beliefs.

The rest of the paper is organized as follows. Section 2 presents the model and discusses the types of exclusion that can arise when the incumbent’s offers are publicly observable. Section 3 solves the model when the incumbent’s offers are not publicly observable. We consider both passive and wary beliefs, and show that in both cases exclusion can only arise from a coordination failure on the part of the buyers. We also consider other cases and characterize necessary and sufficient conditions for an equilibrium with divide-and-conquer offers to exist. Section 4 extends the model to allow for price commitments at the contracting stage. Section 5 concludes.

2 Model

We follow Rasmusen et al (1991) and Segal and Whinston (2000) and assume there is an incumbent seller, a potential entrant, and a set of $N$ buyers. The game has three stages. In the first stage, the incumbent offers each buyer $i$ a contract that contains (i) an exclusivity provision that commits the buyer to purchasing only from the incumbent and (ii) an offer to pay the buyer a lump-sum amount of $x_i \geq 0$. The contracts are offered simultaneously and the $N$ buyers simultaneously decide whether to accept or reject their offers. In the second stage, the potential entrant decides whether to enter or not. In the third stage, the incumbent and the entrant (if it is active) compete by setting prices.

When setting prices in stage three, the incumbent is able to discriminate between those buyers who have signed an exclusive contract (the “signed buyers”) and those buyers who have not (the “free buyers”). In contrast, the potential entrant, if it enters, can only compete for the free buyers. It is assumed that competition for the free buyers results in zero profit for the incumbent from these buyers if the potential entrant enters. It is further assumed that the incumbent earns a monopoly profit of $\pi > 0$ per buyer from each signed buyer, and likewise from each free buyer if there is no entry by the entrant.

Because prices to the free buyers will be lower with entry than without entry, the free buyers are always better off with entry. This gain can be quantified as follows. Let $q(\cdot)$ denote each buyer’s demand, with $q'(\cdot) < 0$. Let $p^m$ denote the monopoly price. Let $c < p^m$ denote the price that would arise under competition, and let $CS(p) = \int_p^\infty q(s)ds$ denote
the buyer’s consumer surplus at price \( p \). Then the extra surplus enjoyed by a free buyer when there is entry is given by the difference in consumer surplus, \( x^* = CS(c) - CS(p^m) \).

It is assumed throughout that the buyers have more to lose when the entrant is excluded than the incumbent stands to gain. That is, it is assumed that \( x^* > \pi \), where the difference is due to the deadweight loss from monopoly pricing. It is also assumed that there is an integer \( N^* < N \) such that because of scale economies it is profitable for the entrant to enter in stage two if and only if the number of signed buyers is less than \( N^* \). \(^6\)

**Two types of exclusion**

Given these assumptions, there are two types of exclusion that can arise in a subgame-perfect equilibrium when the incumbent’s offers are publicly observable: exclusion can arise from a coordination failure on the part of the buyers or from a divide-and-conquer strategy. In the first case, the incumbent is able to induce more than \( N^* \) buyers to sign its exclusive contract and does so relatively cheaply in the sense that the average payment signing buyers receive is less than \( x^* \). In the second case, the incumbent induces exactly \( N^* \) buyers to sign its exclusive contract and compensates each one fully for doing so.

To see this, let \( s_i \in \{0, 1\} \) denote buyer \( i \)'s acceptance decision, with \( s_i = 1 \) if buyer \( i \) accepts its offer and \( s_i = 0 \) if buyer \( i \) rejects its offer, and let \( S = \sum_i s_i \) denote the number of buyers who accept the incumbent’s offer on the equilibrium path. Then, Segal and Whinston (2000) have shown that subgame-perfect equilibria with exclusion can take the following forms: \(^7\)

**Proposition 1 (Segal and Whinston, 2000)** When the incumbent’s offers are publicly observable, exclusionary subgame-perfect equilibria in which

\[(A) \quad S > N^* \text{ always exist, and in all such equilibria, } \sum_i s_i x_i \leq \min\{N^* x^*, N \pi\} \text{ and } x_i = 0 \text{ if } s_i = 0;\]

\[(B) \quad S = N^* \text{ exist if and only if } N \pi \geq N^* x^*, \text{ and in all such equilibria, } x_i = x^* \text{ if } s_i = 1, \text{ and } x_i = 0 \text{ if } s_i = 0.\]

\(^6\)This assumption is needed because if the incumbent had to sign all \( N \) buyers to an exclusive contract to exclude the potential entrant in stage two, each would have to be offered at least \( x^* \) to sign. But this would make exclusion unprofitable because the incumbent can earn at most \( \pi < x^* \) from each buyer.

\(^7\)There are also subgame-perfect equilibria in which the entrant is not excluded if \( N \pi \leq N^* x^* \).
The incumbent finds exclusion to be profitable in the first case because the sum of its payments to the signing buyers, $\sum_i s_i x_i$, is less than or equal to the sum of the monopoly profits it receives, $N \pi$, when the entrant is excluded. Buyers suffer from a coordination failure in this instance. They are willing to sign for less than $x^*$ on average (i.e., less than their gain in consumer surplus if the entrant were to enter) because, given the set of offers from the incumbent, each buyer anticipates that enough other buyers will be signing the incumbent’s contract that the entrant will be excluded whether or not it also signs. Hence, knowing that it is not pivotal (i.e., that its acceptance is not needed to deter the entrant), each is willing to accept the payment it is offered by the incumbent. In the limiting case of $x_i = 0$ being offered and accepted, exclusion is costless to the incumbent.

The incumbent is able to exclude the entrant in this case only because the buyers who have been offered $x_i < x^*$ fail to coordinate on their most preferred continuation equilibrium. If they could instead make non-binding agreements among themselves concerning their accept or reject decisions, all buyers who are offered $x_i < x^*$ would reject their offer and the entrant would enter (since the number of buyers with $x_i \geq x^*$ is less than $N^*$).

The second type of exclusion (Case B) does not suffer from coordination failure, because the buyers who accept exclusivity are fully compensated. Exclusion is achieved by means of a divide-and-conquer strategy: the incumbent offers $x_i = x^*$ to $N^*$ buyers and $x_i = 0$ to the remaining $N - N^*$ buyers. Such a strategy can arise in equilibrium as long as the sum of the payments is less than or equal to the sum of the monopoly profits, which holds if the critical mass of buyers that need to be signed, $N^*$, is sufficiently small relative to $N$.

### 3 Privately Observable Offers

We now consider situations in which each buyer observes only its own offer. This case was touched on by Segal and Whinston (2000, p. 301), who say that “the only alterations

---

8The sum of the payments must also be less than or equal to $N^* x^*$ because otherwise the incumbent could profitably deviate by offering $x^*$ to exactly $N^*$ buyers, as in Case B in Proposition 1.

9This has been shown by Segal and Whinston (2000) for the case in which all equilibria in stage two are required to be coalition-proof (for more on this equilibrium refinement, see Bernheim et al., 1987).

10It is inconsequential whether $x_i = 0$ is interpreted as a zero offer or as a “no offer.”
are that we must have \( x_i = 0 \) for all \( i \) in [Case A]” when offers are privately rather than publicly observable. They refer the interested reader to their working paper (Segal and Whinston, 1996) for the formal analysis.

To support the play in Case A where \( S > N^* \) (after setting \( x_i = 0 \) for all \( i \)), Segal and Whinston (1996, p. 30) assume that buyers have passive beliefs, which they define as: “even after observing an out-of-equilibrium offer, buyer \( i \) continues to believe that [the incumbent] makes its equilibrium offer \( x_j \) to every buyer \( j \neq i \).” However, to support the play in Case B, Segal and Whinston (1996, p. 31) assume that buyers have zero-offer beliefs, where upon receiving an out-of-equilibrium offer, each buyer \( i \) believes that every other buyer \( j \neq i \) has received a “zero offer” (i.e., either no offer or an offer of \( x_j = 0 \)).

The out-of-equilibrium beliefs used by Segal and Whinston are thus not consistent across the two cases. In the first instance, with passive beliefs, each buyer who receives an out-of-equilibrium offer believes that its offer is the incumbent’s only deviation. In the second instance, with zero-offer beliefs, each buyer who receives an out-of-equilibrium offer believes that its offer is part of a multilateral deviation. Moreover, while the beliefs in the first instance have the interpretation that buyers view unexpected offers as trembles or mistakes, the beliefs in the second instance have the unappealing feature that a buyer with an unexpected offer believes that the incumbent’s offers to the other buyers, although part of the same multilateral deviation, are not optimal given the offer it has received. We address these concerns in what follows.

**Case A: \( S > N^* \)**

We begin with equilibria in which there is a coordination failure on the part of the buyers. Segal and Whinston (1996) have shown that \( x_i = 0 \) for all \( i \) in any such equilibrium when offers are privately observable, and that such equilibria exist when buyers have passive beliefs. As we will show, they exist for any out-of-equilibrium beliefs.

The intuition for why there is no equilibrium with a strictly positive payment to at least one of the buyers who accepts the incumbent’s offer is simply that the seller could deviate profitably by offering zero to that buyer, regardless of the buyer’s out-of-equilibrium beliefs.\(^{11}\) Because \( S - 1 \geq N^* \) and the other buyers, who do not observe

---

\(^{11}\) The offer to a buyer who rejects exclusivity on the equilibrium path cannot be strictly positive either, as otherwise the buyer would strictly prefer to accept its offer, a contradiction.
the deviation, continue to make the same decisions, exclusion still happens after such a deviation. The beliefs of the buyer who gets the deviation offer do not matter because that buyer’s acceptance is not needed for the deviation to be profitable for the incumbent.

Out-of-equilibrium beliefs also do not matter for *existence* of such equilibria. Intuitively, because the incumbent obtains exclusion for free on the equilibrium path, it has no incentive to deviate from its equilibrium offers, regardless of whether deviant offers would be accepted (and hence regardless of out-of-equilibrium beliefs). More formally, suppose that every buyer’s strategy is to accept all non-negative offers. It is then a best-response for the incumbent to offer zero to all buyers, and for each buyer to accept its equilibrium offer of zero. Moreover, if buyer $i$ receives an unexpected offer $x_i > 0$, acceptance is indeed optimal for $i$ (given the strategies of the other buyers), regardless of $i$’s beliefs about the offers that the other buyers have received. Hence, exclusionary equilibria with coordination failure exist for any out-of-equilibrium beliefs when offers are privately observable.\footnote{The exclusion of negative offers (i.e., offers where the incumbent asks the buyer to pay it) from the action space does not matter for this result. If the incumbent could make negative offers, rejection is always the unique best response for a buyer who receives such an offer. It follows that the incumbent has no incentive to deviate from the equilibrium offers if they allow it to obtain exclusion for free. This holds regardless of the acceptance decisions that the buyers’ strategies prescribe for positive offers. And, given that $S > N^*$, none of the buyers who accepts the incumbent’s zero offer on the equilibrium path can gain by means of a unilateral deviation to rejection.}

**Case B: $S = N^*$**

In contrast, whether the divide-and-conquer strategy of Case B, where the incumbent offers full compensation (i.e., $x^*$) to $N^*$ buyers and zero to the remaining buyers, can arise in equilibrium does depend on the buyers’ out-of-equilibrium beliefs. Because exclusion by means of a divide-and-conquer strategy is costly for the incumbent, it may be tempted to deviate to obtain exclusion more cheaply. However, the profitability of such a deviation depends on buyers’ beliefs because a buyer accepts less than $x^*$ only if it believes that exclusion happens regardless of its own acceptance decision. A buyer who believes that it is pivotal, as are the $N^*$ signed buyers on the equilibrium path, rejects any compensation below $x^*$.

The two most commonly used beliefs in the vertical-contracting literature are passive beliefs and wary beliefs. We will now discuss each of these beliefs and their implications
for the sustainability of a divide-and-conquer exclusion strategy in turn.

**Passive beliefs**

Passive beliefs are commonly assumed in the literature and have a natural interpretation in which deviations are seen as mistakes or trembles by the incumbent. We now show that the play in Case B cannot arise in equilibrium if buyers have passive beliefs.

To see this, suppose (in negation) that there is an equilibrium with stage one play as described in Case B: \( x_i = x^* \) if \( s_i = 1 \), and \( x_i = 0 \) if \( s_i = 0 \). Consider the following deviation by the incumbent: select a buyer \( j \) who accepts exclusivity \( (s_j = 1) \) in the candidate equilibrium and a buyer \( k \) who rejects exclusivity \( (s_k = 0) \) in the candidate equilibrium, offer 0 to buyer \( j \), \( x_k \in (0,x^*) \) to buyer \( k \), and make the candidate equilibrium offers to all buyers other than \( j \) and \( k \). With passive beliefs, buyer \( k \) believes that the incumbent continues to offer \( x^* \) to the \( N^* \) buyers who accept exclusivity in the candidate equilibrium. Buyer \( k \) therefore expects exclusion to occur regardless of its own acceptance decision, which implies that accepting any \( x_k > 0 \) is optimal for buyer \( k \). With this deviation, the incumbent has bought exclusivity at a cost of \( (N^* - 1)x^* + x_k \). Since this is less than the \( N^*x^* \) that the exclusivity would have cost in the candidate equilibrium, a divide-and-conquer strategy is not sustainable in equilibrium.\(^\text{13}\)

This result is surprising in the sense that the offers in the candidate equilibrium are pair-wise proof and thus immune to profitable single deviations.\(^\text{14}\) Nevertheless, as we have shown, the incumbent can profitably deviate by changing its offer to more than one buyer at a time, taking care to ensure that it reduces its offer to one buyer by more than it raises its offer to another buyer. Because buyers have passive beliefs, they do not anticipate the change in the other buyer’s offer after seeing the change in their own offer.

**Wary beliefs**

The restriction to passive beliefs is not as compelling when buyers recognize that the incumbent’s best offer to any one buyer will depend on its contracts with the other buyers.

\(^{13}\)The multilateral deviation described here is profitable regardless of whether buyer \( j \) accepts its zero offer. In particular, it is profitable even if \( j \) rejects any offer below \( x^* \) (which, in fact, is necessary to rule out profitable unilateral deviations in which the incumbent offers less to one of the buyers who accept exclusivity on the equilibrium path).

\(^{14}\)Some authors impose pair-wise proofness as a primitive solution concept (this holds in the contract equilibrium in Cremer and Riordan, 1987; and O’Brien and Shafer, 1992). In contrast, when buyers have passive beliefs, a perfect Bayesian equilibrium must also be resistant to multilateral deviations.
In this case, if buyers interpret deviations as deliberate choices by the incumbent, it may be more natural to assume that they have wary beliefs (introduced by McAfee and Schwartz, 1994, pp. 221-222, in the context of an upstream monopolist offering possibly different supply contracts to competing downstream firms). McAfee and Schwartz state that with wary beliefs “each firm thinks that others received offers that are the monopolist’s best choices given the offer it made to that firm.” In our context, this means that a buyer who receives an out-of-equilibrium offer from the incumbent believes that all other buyers have received offers that maximize the incumbent’s expected profit given the observed offer.

To make the meaning of wary beliefs more precise, it is useful to summarize buyer i’s equilibrium strategy by an acceptance set \( A_i \) that denotes all offers that buyer \( i \) is willing to accept. For every possible offer \( x_i \), the set \( A_i \) must be optimal given \( i \)’s beliefs about the offers the incumbent makes to the other buyers \( j \neq i \) when buyer \( i \) receives the offer \( x_i \) (and given the strategies of the other buyers). Using this notation, we can then define wary beliefs in our setting as follows:\(^{15}\)

**Definition (wary beliefs):** We say that buyer \( i \) has “wary beliefs” if, after receiving an offer \( x_i \) from the incumbent, buyer \( i \) believes that

1. the incumbent expects it to accept the offer if and only if \( x_i \in A_i \);

2. the incumbent’s offers to the remaining \( N - 1 \) buyers are best for the incumbent, given condition 1 and the acceptance sets of the remaining \( N - 1 \) buyers.

One might think that wary beliefs would make it difficult for the incumbent to engage in opportunistic behavior. But this turns out not to be the case. As with passive beliefs, when buyers have wary beliefs, a divide-and-conquer strategy cannot arise in equilibrium.

To see this, suppose (in negation) that there is an equilibrium with stage one play as described in Case B: \( x_i = x^* \) if \( s_i = 1 \), and \( x_i = 0 \) if \( s_i = 0 \). In such an equilibrium, each buyer’s acceptance set must exclude all offers below \( x^* \), otherwise the incumbent could profitably deviate to offers that deter entry at a lower total cost to the incumbent.

\(^{15}\)McAfee and Schwartz’s (1994) also assume that (i) if a firm is offered a contract that is in its acceptance set, then the firm believes that the upstream seller expects it to accept the contract and offers the other firm the best contract conditional on its acceptance, and (ii) if a firm is offered a contract that is not in its acceptance set, then the firm believes that the upstream seller expects it to reject the contract and offers the other firm the best contract conditional on its rejection.
Now suppose the incumbent offers $\tilde{x}_i \in (0, x^*)$ to a buyer $i$ with $s_i = 0$ in the candidate equilibrium (and simultaneously offers $\tilde{x}_j = 0$ to a buyer $j$ with $s_j = 1$ in the candidate equilibrium). Since $\tilde{x}_i \notin A_i$, wary beliefs imply that buyer $i$ believes that the incumbent (i) expects it to reject the deviation offer and (ii) makes optimal offers to the remaining $N - 1$ buyers given its rejection. It follows that when buyer $i$ has wary beliefs and $\tilde{x}_i \notin A_i$, it believes that the incumbent offers $x^*$ to $N^*$ other buyers for whom $x^*$ is in their acceptance sets and 0 to the remaining $N - N^* - 1$ buyers (since $N^*$ buyers accept $x^*$ on the equilibrium path and $s_i = 0$, there always exist at least $N^*$ other buyers who are willing to accept $x^*$). Buyer $i$ hence believes that exclusion will occur regardless of its own decision, which implies that accepting $\tilde{x}_i$ is optimal for the buyer, a contradiction.

These findings, along with our insights on the existence of equilibria with a coordination failure between buyers, can be summarized as follows:

**Proposition 2** When the incumbent’s offers are privately observable, exclusionary (weak) perfect-Bayesian equilibria in which

(A) $S > N^*$ exist for any out-of-equilibrium beliefs, and in all such equilibria, $x_i = 0$ for all $i$;

(B) $S = N^*$ do not exist when buyers have passive or wary beliefs.

Proposition 2 contains our main results. It implies that (i) equilibria in which exclusion is the result of a coordination failure on the part of the buyers exist for any out-of-equilibrium beliefs, and (ii) exclusionary equilibria in which the incumbent uses a divide-and-conquer strategy do not exist with passive or with wary beliefs. Hence, of the two types of exclusion that can arise when the incumbent’s offers are publicly observable, only one type survives when the offers are privately observable and buyers have passive or wary beliefs. Moreover, in the equilibria that survive, $x_i = 0$ for all buyers. There is no exclusionary equilibrium in which $S > N^*$ buyers accept the incumbent’s offer and some buyer $i$ receives a strictly positive payment, regardless of beliefs, and there is no exclusionary equilibrium in which exactly $N^*$ buyers accept the incumbent’s offer when buyers have passive or wary beliefs.
Equilibria in which there is a coordination failure on the part of the buyers arise because of the beliefs buyers hold on the equilibrium path. If each buyer who receives its anticipated offer believes that enough other buyers will be accepting their offers to ensure that the entrant will be excluded, then it is a best response for each buyer also to accept. Moreover, the incumbent has no incentive to deviate when the offers entail no payment.

A divide-and-conquer strategy cannot arise in equilibrium when buyers have passive or wary beliefs because, in both cases, when a buyer who would receive no compensation in the candidate equilibrium receives an unexpected offer from the incumbent that is below $x^*$, that buyer believes that the entrant will be excluded regardless of its own acceptance decision. Profitable deviations then have the characteristic that the incumbent offers (i) $x < x^*$ to some buyer who accepts the offer under the candidate equilibrium, and (ii) $x \in (0, x^*)$ to some unsigned buyer. The former offer will be rejected, but the latter will be accepted, no matter how small it may be, which implies that the incumbent can profitably deviate.

More generally, it is easy to show that there are no exclusionary equilibria with divide-and-conquer offers when there is at least one buyer who remains unsigned in the candidate equilibrium, but who would accept an out-of equilibrium offer below $x^*$ due to its beliefs. It follows that not all buyers need to have passive or wary beliefs for the divide-and-conquer exclusionary strategy to fail.\footnote{Martin et al (2001) consider the case of privately-observable offers in non-exclusionary vertical settings. In their experimental work, they find that no single restriction on beliefs (passive, wary, or other) is consistent with their data. They suggest this implies that buyers may be heterogeneous in their beliefs.} Suppose, for example, that only $N^* + m$ buyers have either wary or passive beliefs, where $m \in [1, N - N^*]$. Then, in any candidate equilibrium that is supported by a divide-and-conquer strategy, there will be at least one unsigned buyer who has either passive or wary beliefs, which means that the incumbent will be able to deviate profitably to offers that achieve exclusion at a lower overall cost.

**Discussion**

If an equilibrium in which the incumbent relies on a divide-and-conquer strategy to deter entry is to exist for some beliefs (other than passive or wary), any buyer who receives an unexpected offer below $x^*$ must believe that the entrant will enter if it rejects the offer, which makes it indeed optimal for the buyer to reject the offer. More formally, any buyer who receives an offer $x_i \in (0, x^*)$ must believe that at most $N^* - 1$ other buyers have
received offers in their acceptance sets, so that the buyer believes either that its decision
is pivotal or that entry will happen regardless of its decision.

In fact, whenever any buyer who receives an offer in \((0, x^*)\) does believe that at most
\(N^* - 1\) other buyers have received offers in their acceptance sets, an equilibrium with play
as described in Case B of Proposition 1 indeed exists (given \(N\pi > N^*x^*\)). For any beliefs
that satisfy this property, rejecting any offer in \([0, x^*)\) is optimal for buyers. Moreover,
accepting any offer greater than or equal to \(x^*\) is optimal (regardless of beliefs). Hence, an
optimal strategy for all buyers in this case is to accept any offer in \([x^*, \infty)\) and to reject
all other offers. The incumbent’s best-response to the buyers’ strategy is then to adopt a
divide-and-conquer strategy, offering \(x^*\) to \(N^*\) buyers and zero to the remaining buyers.

We can summarize this discussion as follows: an equilibrium in which \(S = N^*\) can
arise if and only if a buyer who receives an offer out of equilibrium believes that few
enough other buyers have received an acceptable offer. Proposition 3 states this precisely.

**Proposition 3** When the incumbent’s offers are privately observable, (weak) perfect-
Bayesian equilibria in which the entrant is excluded and \(S = N^*\) exist if and only if
\(N\pi \geq N^*x^*\) and any buyer \(i\) who receives an offer \(x_i \in (0, x^*)\) believes that at most
\(N^* - 1\) other buyers have received offers in their acceptance sets.

Whether any given buyer’s beliefs satisfy the condition in Proposition 3 depends not
only on the type of beliefs that are assumed (e.g., passive beliefs) but also on the accept-
ance sets (i.e., the strategies) of the other buyers. In any equilibrium in which \(S = N^*\),
however, we know that all buyers with \(s_i = 1\) on the equilibrium path must have accep-
tance set \([x^*, \infty)\) and all other buyers with \(s_i = 0\) on the equilibrium path must have
acceptance set \([x^*, \infty)\) or \((x^*, \infty)\). The question thus becomes whether the type of beliefs
that are assumed satisfy the condition in Proposition 3 for each buyer given that the
other buyers’ acceptance sets are consistent with a divide-and-conquer equilibrium. Segal
and Whinston’s (1996) zero-offer beliefs satisfy this condition, as do other beliefs, some
of which we describe below. Passive beliefs and wary beliefs, on the other hand, do not.

An immediate implication of this is that any out-of-equilibrium beliefs that satisfy the
condition in Proposition 3 when the buyers’ acceptance sets are consistent with a divide-
and-conquer equilibrium must violate (at least) one of the conditions in the definition
of wary beliefs: either the buyer does not believe that the incumbent’s offers to the
other buyers are best choices given the offer it has received, and/or the buyer believes 
that the incumbent has incorrect expectations about which offers will be accepted off the 
equilibrium path (e.g., the buyer must think that the incumbent expects it to accept offers 
that are not in its acceptance set, when in fact it would optimally reject any such offer).

Segal and Whinston’s (1996) zero-offer beliefs can support an equilibrium in which 
\( S = N^* \) because any buyer who receives an out-of-equilibrium offer believes that no other 
buyer has received an acceptable offer. The buyer thus believes that the entrant will enter 
if it rejects its offer, which implies that rejecting any offer below \( x^* \) is indeed optimal.

Zero-offer beliefs are not compelling, however, because, given its offer to one buyer, 
making offers that will be rejected to all other buyers is never a best choice for the 
incumbent when \( N\pi > N^*x^* \). To see this, suppose first that buyer \( i \) believes that the 
incumbent expects it to reject its out-of-equilibrium offer \( \tilde{x}_i \). In this case, the incumbent’s 
expected profit if it makes offers that will be rejected to all other buyers is zero. However, 
the incumbent could obtain a payoff equal (or arbitrarily close) to \( N\pi - N^*x^* > 0 \) by 
making an offer of \( x^* \) to exactly \( N^* \) other buyers. Now suppose the buyer believes the 
incumbent expects it to accept its offer of \( \tilde{x}_i < x^* \). In this case, if the incumbent makes 
offers that will be rejected by all other buyers, its expected profit is \( \pi - \tilde{x}_i \). By instead 
offering \( x^* \) to \( N^* - 1 \) other buyers among those that accept exclusivity in the candidate 
equilibrium, the incumbent would expect exclusion, which would yield an expected profit 
of \( N\pi - (N^* - 1) x^* - \tilde{x}_i \). The incumbent then strictly prefers the latter if \( N\pi > N^*x^* \).

Another example of beliefs that have been used in the literature and that can support 
an equilibrium in which \( S = N^* \) are “symmetric beliefs.” Under symmetric beliefs (first 
proposed by McAfee and Schwartz, 1994, see also Pagnozzi and Piccolo, 2012), a buyer 
who receives an unexpected offer believes that all other buyers must have received the 
same out-of-equilibrium offer as well. Suppose all buyers have acceptance set \([x^*, \infty)\). 
With symmetric beliefs, a buyer who receives an out-of-equilibrium offer that is below \( x^* \) 
believes that all other buyers will reject their offers and will therefore indeed reject its own 
offer. Hence, given these beliefs, the incumbent cannot profitably deviate if \( N\pi > N^*x^* \).\(^{17} \)

\(^{17}\)Because a divide-and-conquer strategy involves discrimination on the equilibrium path, this notion 
of symmetric beliefs is not particularly satisfying in our context. Perhaps a more natural interpretation 
of symmetric beliefs in our setting would be: if buyer \( i \) receives an unexpected offer, it believe that (i) 
all buyers who receive the same offer as buyer \( i \) on the equilibrium path have received the same out-of-
equilibrium offer as buyer \( i \), and (ii) all buyers who receive a different offer than buyer \( i \) on the equilibrium
However, these beliefs, like zero-offer beliefs, are not compelling in our context. Unlike buyers with wary beliefs, a buyer who has symmetric beliefs does not believe that the incumbent’s offers to the other buyers are best choices given the offer it has received. For example, suppose all buyers have acceptance set $[x^*, \infty)$ and buyer $i$ receives an offer from the incumbent that is below $x^*$. Then, buyer $i$ will believe that all other buyers have received offers that will be rejected. Given its offer to buyer $i$, however, making offers that will be rejected to all other buyers is never a best choice for the incumbent if $N\pi > N^*x^*$.

Zero-offer beliefs and symmetric beliefs violate the condition in the definition of wary beliefs that each buyer believes that the incumbent’s offers to the other buyers are best choices given the offer it has received. However, some beliefs that can support an equilibrium in which $S = N^*$ are not subject to this criticism. Consider the following beliefs:

1. the incumbent expects it to accept the offer regardless of whether $x_i \in A_i$;
2. the incumbent’s offers to the remaining $N - 1$ buyers are best for the incumbent, given condition 1 and the acceptance sets of the remaining $N - 1$ buyers.

With these beliefs, an equilibrium in which the incumbent deters entry by means of a divide-and-conquer strategy exists if $N\pi > N^*x^*$. To see this, suppose each buyer’s acceptance set is $A = [x^*, \infty)$ and the incumbent offers $x^*$ to $N^*$ buyers and zero to the remaining buyers on the equilibrium path. If a buyer $i$ receives an out-of-equilibrium offer $\tilde{x}_i \in (0, x^*)$, it believes that the incumbent expects it to accept its offer and makes optimal offers to the remaining buyers given its acceptance, i.e., offers $x^*$ to $N^* - 1$ other buyers and zero to the remaining buyers. (The beliefs thus satisfy the condition in Proposition 3.) Buyer $i$ therefore wants to reject $\tilde{x}_i$, which implies that its acceptance strategy is optimal. Hence, the incumbent cannot profitably deviate from its divide-and-conquer strategy.

In contrast to the case of zero-offer beliefs (and also of symmetric beliefs), these beliefs do not violate the second condition in the definition of wary beliefs, and thus do not assume that each buyer is naive regarding the incumbent’s offers to other buyers (given the offer it has received). Nevertheless, these beliefs are also unappealing because they violate the path have received their equilibrium offers. With such beliefs, an equilibrium in which $S = N^*$ no longer exists. This can be shown using the same arguments we used for the case of passive and wary beliefs.
first condition in this definition. Buyers who have these beliefs believe that the incumbent has incorrect expectations about which offers will be accepted off the equilibrium path.\footnote{As in Segal and Whinston (2000), we have assumed that making offers is costless for the incumbent. If instead making an offer involves even an arbitrarily small transaction cost, a profit-maximizing incumbent should never make any offer that it expects to be rejected. In this case, if a buyer receives an out-of-equilibrium offer $\tilde{x}_i \notin A_i$, sensible beliefs are difficult to pin down. The buyer might believe that the incumbent incorrectly anticipated acceptance of $\tilde{x}_i$ (which would justify incurring the transaction cost), or the buyer might believe that the incumbent made the offer by mistake. The latter belief may be more natural here, which would lead us back to our conclusion for the case of passive beliefs (Proposition 2).}

Our discussion so far has focused on the intuitive appeal of different types of beliefs. An alternative approach, advocated by Eguia et al. (2015), would be to select the action profiles that can be supported in equilibrium by the largest collection of beliefs. In our setting, such an approach would lead one to select exclusionary equilibria in which $S > N^*$, because as we have shown such equilibria exist for all out-of-equilibrium beliefs.\footnote{As noted earlier, if $N \pi \leq N^* x^*$, there can also be equilibria with entry. However, while it is easy to check that these equilibria continue to exist with unobservable offers when buyers hold passive or wary beliefs, they do not exist for all out-of-equilibrium beliefs, unlike exclusionary equilibria in which $N > N^*$.}

\section{Price Commitments}

To facilitate comparison with the earlier literature, we have assumed that contracts in stage one may contain an exclusivity provision and an offer of a lump-sum payment, but no agreement on the future price of the good. It has been argued that the lack of a price commitment can be justified in circumstances in which the precise nature of the good to be delivered in period three is not known in period one (see, for example, footnote 7 in Segal and Whinston, 2000). Nevertheless, there are many settings in practice in which agreements on price are possible. We now extend the model to consider these settings.

As in the base model, there is an incumbent seller, a potential entrant, a set of $N$ buyers, and a game that has three stages. However, in stage one, instead of offering each buyer $i$ a lump-sum payment of $x_i$ in exchange for exclusivity, we now assume the incumbent can commit to a per-unit price $p_i \geq 0$ at which it will sell its product if the buyer accepts exclusivity.\footnote{Similar results hold if we allow the incumbent to offer $x_i \geq 0$ and commit to a per-unit price $p_i \geq 0$.} As before, the offers are made simultaneously, as are all acceptance decisions in stage one. In stage two, the potential entrant decides whether or
not to enter. In stage three, the incumbent honors the price commitments it has made, and competes with the entrant (if it is active) for the buyers who are unsigned. All other aspects of the model are the same.

With these modifications, is straightforward to show that, as in the base model, there are two types of exclusion that can arise in equilibrium when the incumbent’s offers are publicly observable: exclusion can arise from a coordination failure on the part of the buyers or from a divide-and-conquer strategy. In the latter case, for example, the incumbent commits to charging $p_i = c$ to exactly $N^*$ buyers to induce them to sign, but makes no offer or offers only the monopoly price $p_i = p^m$ to the other $N - N^*$ buyers.

One noteworthy difference is that the incumbent does not cause a deadweight loss for each signed buyer when it uses a divide-and-conquer strategy, and thus equilibria of this type exist when the offers are publicly observable regardless of how close $N^*$ is to $N$. However, for the same reasons as in the base model, it is still the case that only the first type of exclusion (i.e., a coordination failure) can survive in equilibrium when the incumbent’s offers are privately observable and the buyers have wary or passive beliefs.

**Proposition 4** When the incumbent’s offers are privately observable, and the incumbent can commit to prices, exclusionary (weak) perfect-Bayesian equilibria in which

(i) $S > N^*$ exist for any out-of-equilibrium beliefs, and in all such equilibria, $p_i = p^m$ for all $i$;

(ii) $S = N^*$ do not exist when buyers have passive or wary beliefs.

**Proof.** See appendix. ■

Of the exclusionary equilibria that can arise when the incumbent’s offers are privately observable, Proposition 4 implies that $p_i = p^m$ for all buyers. There is no exclusionary equilibrium in which $S > N^*$ buyers accept the incumbent’s offer and some buyer $i$ receives a per-unit price that is less than the monopoly price, regardless of beliefs, and there is no exclusionary equilibrium in which $S = N^*$ buyers accept the incumbent’s offer.

The intuition for these results is analogous to the case in which no price commitments are possible. The intuition for why there is no equilibrium in which there is a coordination failure on the part of the buyers and a per-unit price that is strictly less than $p^m$ for
a buyer who accepts the incumbent’s offer, regardless of the buyer’s out-of-equilibrium beliefs, is that the seller could profitably deviate by not making any offer to that buyer. And the intuition for why there is no equilibrium in which exactly $N^*$ buyers accept the incumbent’s offer when the buyers have passive or wary beliefs is because signed buyers know that they are pivotal and will only accept a price of $c$ or less. Profitable deviations then have the characteristic that the incumbent offers (i) $p > c$ to some buyer who accepts the offer under the candidate equilibrium, and (ii) $p \in (c, p^m)$ to some unsigned buyer.

5 Conclusion

When a seller’s contract offers are privately observable, equilibrium outcomes typically depend on how a buyer who receives an unexpected offer updates its belief about the contracts offered to the other buyers. In a perfect-Bayesian equilibrium, there is no restriction on what these out-of-equilibrium beliefs can be. Nevertheless, some beliefs are more sensible than others, and thus not all equilibria are equally deserving of attention.

It is common in the literature on vertical relations to restrict attention either to (i) passive beliefs (Hart and Tirole, 1990), where after receiving an unexpected offer, each buyer continues to believe that other buyers receive their equilibrium offers, or (ii) wary beliefs (McAfee and Schwartz, 1994), where after receiving an unexpected offer, each buyer anticipates that the incumbent adjusts its offer to other buyers in order to maximize its profit. In the first case, buyers interpret the incumbent’s deviation as a mistake, and in the second case, buyers interpret the incumbent’s deviation as being deliberate. We have shown that in both cases, exclusion cannot be induced in equilibrium with a divide-and-conquer strategy when contracts are privately observable. Exclusionary equilibria that rely on a coordination failure on the part of the buyers continue to exist, however, and we have shown that these equilibria exist regardless of the buyers’ out-of-equilibrium beliefs.

To put our results in context, the Rasmusen et al. (1991) response to the Chicago-school view of exclusive contracts has been criticized by Segal and Whinston (2000) and others as being not fully satisfying because it relies on the buyers failing to coordinate on their most preferred continuation equilibrium. The latter showed that if the buyers could make non-binding agreements among themselves concerning their acceptance decisions,
then this type of exclusion (i.e., one that is based on coordination failure) would not arise. All buyers would reject the incumbent’s offer and the entrant would enter. To this end, Segal and Whinston’s divide-and-conquer strategy seemed to remove the importance of expectations, and thus further undermine the Chicago-school view that exclusive contracts are harmless. However, we have shown that expectations, in the form of out-of-equilibrium beliefs, do matter — because when offers are privately observable, as is realistic in many settings, the divide-and-conquer strategy fails to deter the entrant under plausible beliefs.

Understanding the kinds of behavior that can lead to anti-competitive exclusion is important for antitrust practice. Typically, in an exclusive dealing case, the issue for the courts and competition authorities is not whether exclusion has occurred, but whether it was pro- or anti-competitive. Previous findings, which were obtained assuming observable offers, suggest that discriminatory payments (to otherwise similar buyers) are consistent with a dominant firm achieving naked exclusion by means of a divide-and-conquer strategy. Our findings, on the other hand, suggest that an anti-competitive theory based on evidence of discriminatory contracts is less convincing when contracts are privately observed. Competition authorities might therefore want to give pro-competitive justifications for exclusive contracts more weight in such cases.

In the present model, buyers do not compete and the incumbent’s offers are made simultaneously. Extending the model to allow buyers to compete (as in Fumagalli and Motta, 2006, Simpson and Wickelgren, 2007, Abito and Wright, 2008, and Wright, 2009) would likely reduce the gains from exclusion when the buyers are differentiated (because of the well known inability of an incumbent monopolist to support monopoly pricing downstream when buyers have passive or wary beliefs) but not fundamentally alter our results. As long as buyers are differentiated enough, so that the entrant’s profit from dealing with a single unsigned buyer does not suffice to cover the cost of entry, an equilibrium with divide-and-conquer offers would still exist with publicly observable offers (under certain conditions on the parameters), but not with privately observable offers when buyers have passive or wary beliefs.

---

21 Abito and Wright (2008) analyze a four-stage model in which (i) exclusive offers are made and accepted or rejected in stage one, (ii) the entrant makes its entry decision in stage two, (iii) the incumbent and the entrant (if active) propose terms of trade to each available retailer in stage three, and (iv) price competition takes place in stage four. In their model, the terms of trade in stage three are assumed to be privately observable, but the exclusive offers, which are the focus of our paper, are assumed to be public.
Extending the model to allow offers to be made sequentially, as Segal and Whinston (2000) do for the case of publicly observable offers, is conceptually more problematic when offers are only privately observable because it begs the question whether buyers know their position in the sequence. Moreover, even if buyers are aware of their positions, a buyer that receives an unexpected offer may not be able to infer whether the offer resulted from a deviation by the incumbent (the only possibility in the simultaneous offer game) or in reaction to a deviation (say, an unexpected rejection decision) by a buyer that was approached earlier. We leave these extensions for future research.
Proof of Proposition 4:

To show that an equilibrium in which \( S > N^* \) and \( p_i = p^m \) for all \( i \) exists, suppose that \( p^m \in A_i \) for all \( i \), where \( A_i \) denotes the set of prices that buyer \( i \) accepts in stage one. By the definition of the monopoly price, the incumbent’s offering \( p_i = p^m \) to every buyer \( i \) is then a best-response, regardless of what other prices each buyer accepts or rejects given its strategy. Moreover, if buyer \( i \) receives the equilibrium offer \( p_i = p^m \), acceptance is a best-response for \( i \) (given that \( p^m \in A_j \) for all \( j \neq i \)). Hence, an equilibrium in which, on the equilibrium path, the incumbent offers \( p_i = p^m \) to all \( i \) and all \( i \) accept this offer exists for any out-of-equilibrium beliefs. Buyer \( i \)’s out-of-equilibrium beliefs matter for \( i \)’s best-response only if \( i \) receives an unexpected offer \( \tilde{p}_i < p^m \); however, given that the incumbent sells to all buyers at the monopoly price on the equilibrium path, a deviation that involves one or multiple offers below the monopoly price is unprofitable regardless of whether these offers would be accepted (and thus regardless of out-of-equilibrium beliefs).

Next, we show that in all equilibria in which \( S > N^* \), \( p_i = p^m \) for all \( i \). Note first that there cannot be an equilibrium in which a buyer accepts the incumbent’s offer and agrees to pay more than \( p^m \), because this buyer would have been better off rejecting the offer. Now suppose (in negation) that there is an equilibrium in which some buyer \( i \) accepts the incumbent’s offer and receives \( p_i < p^m \). Then the incumbent can profitably deviate by offering \( p_i = p^m \) to this buyer, while keeping the offers to all other buyers the same. Since the other buyers do not observe the deviation and hence continue to make the same decisions, exclusion still happens (because \( S > N^* \)). It follows that the incumbent will earn the monopoly profit from this buyer whether or not the deviation offer is accepted.

To show that equilibria in which \( S = N^* \) do not exist when buyers have passive beliefs, suppose (in negation) that such an equilibrium exists, and consider the following deviation by the incumbent: select a buyer \( j \) who accepts exclusivity \( (s_j = 1) \) in the candidate equilibrium and a buyer \( k \) who rejects exclusivity \( (s_k = 0) \) in the candidate equilibrium, offer \( p_j = p^m \) to buyer \( j \), \( \tilde{p}_k \in (c, p^m) \) to buyer \( k \), and make the candidate equilibrium

---

22 If buyer \( i \) receives an out-of-equilibrium offer \( \tilde{p}_i > p^m \), rejection is always the unique best response because in this case, whether or not the entrant were to enter, \( i \) would expect to receive a lower price by not signing.
offers to all buyers other than $j$ and $k$. With passive beliefs, buyer $k$ accepts its offer because it believes that the incumbent continues to offer the same price commitments to the $N^*$ buyers who accept exclusivity in the candidate equilibrium and thus expects exclusion to occur regardless of its own acceptance decision. The deviation therefore continues to induce exclusion, but now the incumbent earns positive profit from buyer $k$ whereas before it was earning zero profit from buyer $j$ (we know that buyer $j$ was receiving $p_j = c$ because otherwise it would have rejected its offer and induced entry).

To show that equilibria in which $S = N^*$ do not exist when buyers have wary beliefs, note first that in any such equilibrium, each buyer’s acceptance set must exclude all offers above $c$. This holds for any buyer $i$ such that $s_i = 1$ because the buyer can receive a price of $c$ simply by rejecting the offer and causing the entrant to enter. And it holds for any buyer $i$ such that $s_i = 0$ because otherwise the incumbent could profitably deviate by signing this buyer instead of signing a buyer with $s_i = 1$ in the candidate equilibrium.

Now suppose (in negation) that such an equilibrium exists, and consider the following deviation by the incumbent: offer $\tilde{p}_i \in (c, p^m)$ to a buyer $i$ with $s_i = 0$ in the candidate equilibrium, and simultaneously offer $\tilde{p}_j = p^m$ to a buyer $j$ with $s_j = 1$ in the candidate equilibrium. Since $\tilde{p}_i \notin A_i$, wary beliefs imply that buyer $i$ believes that the incumbent (i) expects it to reject the deviation offer and (ii) makes optimal offers to the remaining $N - 1$ buyers given its rejection. It follows that when buyer $i$ has wary beliefs and $\tilde{p}_i \notin A_i$, $i$ believes that the incumbent offers $c$ to $N^*$ other buyers for whom $c$ is in their acceptance sets and $p^m$ to the remaining $N - N^* - 1$ buyers. Buyer $i$ hence believes that exclusion will occur regardless of its own decision, which implies that accepting $\tilde{p}_i < p^m$ is optimal for $i$, a contradiction. Q.E.D.
References


