Craig W. Holden  
*Indiana University*

Avanidhar Subrahmanyam  
*University of California, Los Angeles*

**News Events, Information Acquisition, and Serial Correlation***

I. **Introduction**

Jegadeesh and Titman (1993) find that momentum strategies, which buy stocks that have performed well in the past and vice versa, generate significant abnormal returns over a medium-term holding period of 3 to 12 months. Rouwenhorst (1998) analyzes momentum strategies on 12 European countries over a more recent period and finds strikingly similar results. He finds evidence of medium-horizon abnormal returns in all 12 countries and that abnormal returns are strongest for the smallest decile stocks in each country and decline in a smooth and nearly monotonic manner as firm size increases.

Paradoxically, the opposite strategy has been found to generate abnormal profits in long horizons. DeBondt and Thaler (1985) find that contrarian strategies (buying past losers and selling past winners) achieve significant abnormal returns.1 Specifically, they find

We develop a model that accounts for medium-term continuation (momentum) in asset returns by analyzing information acquisition about news events (such as earnings announcements) in a multiperiod setting. As more and more agents become informed about news events, temporal uncertainty is resolved endogenously through market prices over time, which leads to positive autocorrelations in asset returns. We empirically estimate serial correlations over medium-term horizons for portfolios sorted by firm size and past stock performance and find that calibration of serial correlations in our model spans the range of empirically estimated correlations.

---

1. At very short horizons of a day or a week, there is evidence of negative autocorrelation (Jegadeesh 1990; Lehmann 1990), but a large literature attributes this to microstructural biases and measurement problems (see, e.g., Kaul and Nimalendran 1990; Ahn et al. 1999).
this abnormal performance over a long-term holding period of 3 to 5 years. Fama and French (1996) find that their three-factor model is able to capture the long-term reversal in addition to many of the capital asset pricing model (CAPM) average-return anomalies, including the size, book-to-market equity, earnings/price, cash flow/price effect, and past sales effect. Their intuitive explanation for capturing the long-term reversal effect based on changing factor coefficients is that “stocks with low long-term past returns (losers) tend to have positive SMB [small minus big] and HML [high minus low] slopes (they are smaller and relatively distressed) and higher future returns. Conversely, long-term winners tend to be strong stocks that have negative slopes on HML and low future returns” (Fama and French 1996, p. 56). Significantly, however, Fama and French are unable to capture the medium-term continuation effect and leave open the possibility that momentum remains unexplained because their factors do not completely capture dynamic changes in risk premia (see Fama and French 1996, p. 82). Indeed, Chan, Jegadeesh, and Lakonishok (1996, p. 1682) lament the “woeful shortage of potential explanations for momentum.”

We provide a model that accounts for medium-term continuations by analyzing the dynamic behavior of asset price movements prior to significant news events such as earnings announcements. We build on existing models of information acquisition and study two settings: (1) we allow informed agents to trade prior to the time they receive private information, and (2) we consider sequential information acquisition, that is, we allow agents to expend resources to influence the timing of private information receipt. In both of the above settings, there is temporal resolution of uncertainty because private information is reflected in prices sequentially. This influences the dynamic behavior of asset prices prior to news events. We characterize agents’ optimal trades, information acquisition, and stock price behavior prior to the informational event. Our model essentially embeds information asymmetry into the work of Epstein and Turnbull (1980), who do not focus on serial correlation but analyze the effect of temporal resolution of uncertainty on risk premia. In contrast, our focus is on analyzing how dynamic changes in information asymmetry can influence serial correlation in asset returns.

As noted above, our first model recognizes that in reality, there are lags between the time an agent invests in resources to obtain information and the time he actually gets the information signal. This setting yields an analytic solution for the equilibrium in the trading stage and allows us to analyze continuations, volatility, and volume prior to news events. Our analysis suggests that total trading volume is highest for intermediate levels of information acquisition costs (where agent heterogeneity is highest), whereas there is no clear relationship between volatility and the cost of information acquisition.

We also find that stocks with low information acquisition costs will be characterized by continuations provided the variance of private information is sufficiently large.\(^3\)

The intuition for the last result is the following. Consider a standard model where liquidity shocks in each period are absorbed by risk-averse agents and where there is no trading on private information. In such a framework, price changes would exhibit reversal because of standard inventory considerations (see, e.g., Grossman and Miller [1988]). Now consider a structure similar to the one above, but suppose that a private information signal, concerning, say, a future earnings announcement or an annual report, is received sequentially by risk-averse agents. In this case, the risk borne by the market decreases over time, simply because the mass of better-informed agents increases over time. As a consequence, there is a gradual decrease in the conditional risk premia required to absorb liquidity shocks. This effect tends to lend positive autocorrelation to asset risk premia and thus leads to price continuations. If information costs are sufficiently low (so that the mass of informed agents is sufficiently large), and if the variance of information is sufficiently high, price changes will exhibit continuations.

In our extension of the basic setting, we allow agents to expend resources to influence the timing of information receipt. This model allows us to develop insight on how the costs of early versus late information acquisition influence price continuation versus reversal. The basic intuition in this case is that, if the costs of getting information early and late are very different, the mass of agents changes sharply over time, which leads to strong positive autocorrelation in risk premia and hence to momentum in asset prices.\(^4\) Thus tendency for momentum to obtain is increasing in the disparity between the cost of obtaining information early and late.\(^5\) Arbel (1985) argues that the cost of obtaining information early is larger for small stocks. Under this plausible assumption, our model predicts stronger continuations for small firms than large firms. Thus, our model is consistent with the empirical findings of Jegadeesh and Titman (1993) and Rouwenhorst (1998) that continuations are stronger in small firms.

3. Behavioral considerations such as those explored in Barberis, Shleifer, and Vishny (1998), Daniel, Hirshleifer, and Subrahmanyam (1998), and Hong and Stein (1999) may also play a role in continuations. In this article, however, we focus on whether continuations can be obtained by strictly rational agents.

4. Sequential private information arrival of the type considered in this article is also a feature of Hirshleifer, Subrahmanyam, and Titman (1994). However, in that model, the masses of early and late-informed agents are exogenous; further, there is no predictability in asset returns, because prices are set by risk-neutral uninformed agents (interpreted as market makers). In our model, uninformed agents are risk averse.

5. Kim and Verrecchia (1991) show how the information acquisition about a private signal is altered as a result of a separate public signal. In contrast, we analyze information acquisition about the public signal itself. Also, Kim and Verrecchia (1991) do not analyze price continuation vs. reversal. Wang (1993a, 1993b) develops important continuous time models with long horizon investors in which information arrives smoothly. In his work, asset price changes can exhibit positive serial correlation because of persistence in risk aversion or asset supplies. Our model instead considers short-horizon traders and is intended to apply to sporadic news events.
To get a feel for the ability of the model to generate empirically relevant results, we estimate serial correlations over quarterly horizons using the Center for Research in Security Prices (CRSP) Monthly Size Decile portfolios for the combined New York Stock Exchange (NYSE)–American Stock Exchange (AMEX)–NASDAQ universe of stocks and for decile portfolios sorted by past stock performance. All but one of our serial correlation estimates are positive, and the majority of them are significantly so. Our calibrated theoretical model covers the full range of empirically estimated serial correlations. In addition, our estimated serial correlations are highest for small firms, which is consistent both with our theoretical model and with the evidence of Jegadeesh and Titman (1993) and Rouwenhorst (1998). We also develop several other testable implications of our model that relate the magnitude of the serial correlation to the information content of the order flow in a stock.

In related work, Jones and Slezak (1999) provide a multiasset dynamic rational expectations model to explain cross-sectional patterns in asset returns. Their model provides insights into book-to-market effects and can generate reversals arising from the reversion of risk premia due to liquidity trading shocks. However, their model does not generate continuations because they assume that the mass of informed agents is constant over time. Thus, our consideration of the sequential nature of information acquisition is the key to generating positive serial correlation within a rational expectations model.

This article is organized as follows. Section II presents the economic setting. Section III describes the equilibrium of the model in which agents trade in advance of private information receipt, performs an empirical calibration, and then considers endogenous information acquisition. Section IV discusses the extension in which different agents receive information at different times. Section V concludes.

II. The Economic Setting

Our initial purpose is to analyze information acquisition and the dynamics of price movements prior to a single major news event. We imbed this news event as one in a series of informational events. Thus, we consider a risky security that pays cash flows at regular intervals. In period \( t, t = 1, 2, \ldots, \infty \), the security’s cash flow is given by

\[ F_t = \bar{F}_t + \theta_t + \epsilon_t. \] (1)

The term \( \bar{F}_t \) is nonstochastic, whereas \( \theta_t \) and \( \epsilon_t \) are mutually and serially independent, and multivariate normally distributed with mean zero and common variance \( \nu_t \) and \( \nu_e \), respectively. We also normalize the security’s supply to be zero. 6

6. This assumption is without loss of generality. Introducing a positive mean supply causes the unconditional risk premium to be nonzero but does not affect the rest of the analysis that we perform.
The time between \( t - 1 \) and \( t \) is subdivided into three dates \( t_1, t_2, \) and \( t_3 \), with \( t_3 = t \). The dates \( t \) can be interpreted as the date of a significant informational event such as an earnings announcement. The subperiods \( t_i, i = 1, 2, 3 \) represent dates on which trading takes place between informational events occurring on the dates \( t - 1 \) and \( t \).

We consider three types of agents who trade at the dates \( t_i \): informed agents, uninformed agents, and liquidity traders with exogenous demands. These agents form the cohort for any period \( t \). Motivated by recent literature on the implications of investors with short investment horizons and the notion that much effort appears to be directed at forecasting and speculating on earnings announcements in the short-term, we assume that the cohort associated with an announcement at time \( t \) trades only in the subperiods corresponding to that cohort. This implies that all members of the cohort corresponding to time \( t \) completely reverse their positions at time \( t \).

The assumptions that news events are serially independent and that a cohort of agents trades only in the subperiods corresponding to a particular event imply that we can analyze the equilibrium corresponding to a particular set of subperiods \( t_i \) separately from all other sets of subperiods. Note that these assumptions do not affect our analysis of price behavior prior to a particular news event. The assumptions are made solely to allow a tractable analysis of price behavior without conditioning on a specific news announcement; this conditioning exercise is performed in subsection E of Section III. For now, we analyze a generic cohort \( t_i \), suppress time subscripts from all variables for convenience, and relabel the dates \( t_1, t_2, \) and \( t_3 \) to 1, 2, and 3, respectively.

In the basic setting considered in the next section, we assume that there are two types of utility-maximizing traders: informed traders who learn precisely the realization of \( \theta \) just prior to trade at date 2 and uninformed traders who have no knowledge of \( \theta \). Each informed and uninformed trader has an endowment of \( B_0 \) units of the riskless bond. Further, the informed agent is required to make a decision to acquire information prior to trade at date 1. This setting captures the notion that, if one wishes to acquire information, resources must be committed to do so well in advance of the actual receipt of the signal. In Section IV, we consider dynamic information acquisition, wherein agents can endogenously choose whether to (1) become informed at dates 1 or 2 or (2) be uninformed at both dates.

Thus, the existing cohort reverses its position in the stock after the dividend is paid at time \( t - 3 = t \), whereas the arriving cohort trades the stock ex-dividend at times \( t + 1 \), \( i = 1, 2, 3 \). Note that we normalize the supply of the stock to zero, so no agent has to carry the supply from one cohort to the next. Short-horizon behavior has previously been assumed, e.g., in Froot, Scharfstein, and Stein (1992), Dow and Gorton (1994), McNichols and Trueman (1994), Vives (1995), and Bhushan, Brown, and Mello (1997).

An alternative would be to consider overlapping generations models, where the existing cohort overlaps for one or two periods with the next cohort. A preliminary analysis on our part suggests that similar results obtain in such a setting, but the model becomes very complicated and does not promise any gain in understanding the phenomena we consider. Details of this exploratory analysis are available from the authors upon request.
We consider a competitive framework and thus assume that there is a continuum of each type of trader.\footnote{Unfortunately, we find that the modeling of strategic behavior is intractable in our dynamic framework.} We also assume that exogenous liquidity trades of \( z_1 \) and \( z_2 \), which are mutually independent and normally distributed with zero mean and common variance \( \sigma \), arrive at the market at dates 1 and 2, respectively. The liquidity demand shocks are independent of each other and of \( \theta \) and \( \epsilon \).\footnote{If we were to assume that there is only one demand shock, there would exist an equilibrium in which the dates 1 and 2 prices would, taken together, fully reveal the private information—as in Grundy and McNichols (1989). Modeling two independent demand shocks precludes this type of equilibrium from obtaining and thereby enables us to examine serial correlation in asset price changes.} In the basic setting, the mass of informed traders is \( M \), and the mass of uninformed traders is \( 1 - M \), so that the total mass of all informed and uninformed traders is normalized to unity. All traders have negative exponential utility over final wealth with a common risk aversion coefficient \( R \).

### III. Equilibrium with One Stage of Information Acquisition

In this section, we consider the equilibrium of a setting where some agents are informed at date 2, all agents are uninformed at date 1, and agents can trade in advance of the receipt of private information. We initially fix the mass of informed agents, \( M \), and characterize linear equilibria in the trading stage (subsections A and B of this section) and then consider a setting where \( M \) is determined endogenously (subsection C of this section).

Let \( P_1 \) and \( P_2 \) denote the date 1 and date 2 equilibrium prices for the security. We will consider linear equilibria implied by the model. Thus, let us postulate that \( P_1 \) and \( P_2 \) are linearly related to the observables at each date such that

\[
P_2 = P + a\theta + b z_1 + c z_2, \tag{2}\]

\[
P_1 = f z_1. \tag{3}\]

In the ensuing analysis we verify that these conjectures are consistent with the equilibrium we derive.

#### A. The Analytic Solution for the Equilibrium

This section presents the unique linear equilibrium. The complete solution is given in the following lemma (which is proved in the appendix).

**Lemma 1.** The unique linear equilibrium of the model is given by

\[
a = \frac{M[M\nu + R^2\nu_1\nu_2(\nu_1 + \nu_2)]}{D}, \tag{4}\]

where

\[
D = M + R^2\nu_1\nu_2(\nu_1 + \nu_2).
\]
Comparative Statics

Our primary focus in performing comparative statics is on volume, volatility, and serial correlation. For building insight, the present subsection focuses on the case of exogenous $M$, while the next subsection, which endogenizes information acquisition, considers comparative statics with respect to the cost of information.

We first analyze trading volume. Let $x_{i1}$ and $x_{i2}$ denote the demands (holdings) of the informed agents and $x_{u1}$ and $x_{u2}$ the holdings of uninformed agents, at dates 1 and 2. Note that $x_{u2}$ denotes the holdings, rather than the trades, of agent $i$ ($i = I$ or $U$). Since the expectation of the absolute value of a normally distributed variable is proportional to its standard deviation, the total expected trading volume at date 2 is proportional to

$$TV_2 = M \text{ std}(x_{i2} - x_{i1}) + (1 - M) \text{ std}(x_{u2} - x_{u1}) + v_z^{1/2}. \quad (8)$$

The measure of total expected volume thus has three components: volume from the informed, uninformed, and liquidity traders. Note that the date 1 trading volume is proportional to $2v_z^{1/2}$ and is invariant to $M$, since the date 1 demand shock is split in the ratio $M$ to $1 - M$ between the informed and uninformed. The following proposition on how the date 2 trading volume varies with the mass of informed traders is proved in the appendix.

**Proposition 1.** Locally around $M = 0$, the total volume at date 2 is increasing in $M$, and locally around $M = 1$, is decreasing in $M$.

The intuition for the above proposition is that, when most of the agents are uninformed, adding an informed agent adds to volume because the degree of heterogeneity in the market increases, but when most of the agents are informed, adding another agent decreases volume because the reverse happens. Since the demands of the informed and uninformed agents are continuous functions of $M$, equation (8) implies that the total trading volume as a function of $M$ is a continuous function of $M$. From proposition 1, this function is increasing in $M$ for $M = 0$ and decreasing in $M$ for $M = 1$. This suggests that volume as a function of $M$ has at least one maximum for $0 \leq M \leq 1$. The

11. The constant of proportionality is $(2/\pi)^{1/2}$. 

\[ b = \frac{Rv[M^2v_g + R^2v_g(v_e + v_g)]}{D}, \quad (5) \]

\[ c = \frac{Rv a}{M}, \quad (6) \]

\[ f = R(v_e + v_g), \quad (7) \]
intuition is simply that total volume should be maximized when there is sufficient heterogeneity in the market, which should happen for an interior value of $M$. Under the plausible assumption that the mass of informed agents is correlated with firm size and hence with the level of institutional ownership in a company, intriguing evidence consistent with this result is provided by Utama and Cready (1997). They show that the empirical functional form of volume prior to earnings announcements across firms is quadratic and is maximized at an institutional ownership level of about 50%.

We next consider the volatility of price changes. In addition to the quantities $\text{Var}(P_1 - P_2)$ and $\text{Var}(P_2 - P_3)$, it is also of interest to calculate the model’s overall volatility, that is, volatility that does not rely on picking any specific pair of dates. Consider an econometrician who calculates price change variances by sampling price changes repeatedly but does not condition on any particular pair of dates. He will calculate variances placing equal weights on the price changes $P_1 - P_2$ and $P_3 - P_2$. By the law of iterated expectations, the unconditional volatility is then just the arithmetic mean of $\text{Var}(P_1 - P_2)$ and $\text{Var}(P_2 - P_3)$. Our next proposition describes some results related to the above volatility measures.

**Proposition 2.** (1) The variance of the price change across dates 2 and 3, $\text{Var}(P_2 - P_3)$, is decreasing in $M$. (2) Locally around $M = 0$, the variance of the price change across dates 1 and 2, $\text{Var}(P_2 - P_1)$, is decreasing in $M$ if and only if $R^2v_1(v_1 + v_0) > 1$, and locally around $M = 1$, is decreasing in $M$ if and only if $v_2 > v_0$. (3) Locally around $M = 0$, the average of the two variances above is decreasing in $M$ if and only if $3R^2v_3(v_3 + v_0) > 1$, and locally around $M = 1$, is decreasing in $M$ if and only if $R^2v_3(3v_3 - v_0) + 1 > 0$.

The above proposition indicates that, in conformance with intuition, informed trading attenuates volatility across dates 2 and 3. The reason simply is that informed traders make the price more informative, while making the date 2 price move closer to the fundamental variable $v$ that is revealed at date 3, thereby reducing price fluctuations across dates 2 and 3.

The date 2 volatility is not monotonic in the mass of informed traders. The broad intuition for this is the following. The volatility of the price change $P_2 - P_1$ is determined by two aspects. First, $P_2$ is sensitive to the mass of informed agents, but $P_1$ is not, which makes fluctuations in $P_2 - P_1$ depend on the mass of informed agents. Second, the risk premium associated with the liquidity shocks declines at date 2, because the entry of informed agents causes the overall level of risk borne by the market to fall. As $M$ increases, the first effect adds to the volatility of the price change $P_2 - P_1$, while the second tends to reduce it. The overall effect is determined by these opposing considerations.

Finally, we analyze the serial covariance $\text{Cov}(P_1 - P_2, P_2 - P_3)$. The proposition that describes the behavior of this covariance follows:

12. Note that $E(P_3 - P_2) = E(P_2 - P_1) = 0$, so that the variance equals the expectation of the squared price change. The overall variance is the arithmetic mean of $E(P_3 - P_2)^2$ and $E(P_2 - P_1)^2$; assuming each pair of dates is equally likely to be picked.
Proposition 3. The analytic expression for the equilibrium value of the serial covariance is given by

\[
\text{Cov}(P_3 - P_2, P_2 - P_1) = \frac{R^2v_1[Rv_0 + R^2v_1(v_0 + v_3)](Mv_0 - v_1)}{(M^2v_0 + MR^2v_0v_3 + R^2v_3v_3)^2},
\]

so that this serial correlation is positive if and only if \(Mv_0 > v_1\).

The broad intuition for the above result is the following. There are two opposing effects influencing the serial covariance. The first effect is that the risk premium required to bear the supply shock \(z_1\) is decreasing over time because, as agents receive private information gradually, they bear less risk and consequently reduce the risk premium. However, the shock \(z_2\) is reversed by date 2, and this tends to cause a negative autocorrelation (as in a standard inventory model). If the mass of informed agents is sufficiently high, the first effect dominates, and we get positive serial correlation.

To obtain a further understanding of the above result, note that the serial covariance can be written in terms of the price coefficients as

\[
\text{Cov}(P_3 - P_2, P_2 - P_1) = a(1 - a)v_0 - b(b - f)v_1 - c^2v_2.
\]

Suppose first that \(M = 0\). In this case, \(b = f = R(v_0 + v_3)\), and \(a = 0\), so that we are left with the last term in equation (10), and the serial correlation in this case is always negative. The notion is simply that the conditional risk premium owing to the demand shock \(z_2\) reverses by date 3, causing a price reversal on average. Note, however, that the risk borne by the agents across dates 1 and 2 does not change, so the conditional risk premium related to \(z_1\) does not change and thus does not influence the serial correlation.

Now suppose that \(M = 1\). Then \(a = 1, b = Rv_0, c = Rv_3, and f = R(v_0 + v_3)\) so that the first term in equation (10) is zero, the second term is \(Rv_3\), and the last term is \(-Rv_2^2\). Thus, positive serial correlation obtains if \(v_2 > v_1\). The idea here is that the conditional risk premium related to \(z_1\) is decreasing across dates 1 and 2 and is also decreasing across dates 2 and 3 as prices approach full revelation. This is because the average risk borne by agents is smaller at date 2, simply because agents become informed at this date. The positive autocorrelation in the conditional risk premium tends to cause a continuation. If the gradual decrease in the conditional risk premium related to \(z_1\) dominates the reversal of an additional risk premium due to \(z_2\), the overall serial correlation is positive, so that we obtain positive serial correlation. As \(M\) is increased starting from \(M = 0\), the tendency for positive serial correlation to obtain increases. This is because as \(M\) becomes larger, the decrease in the conditional risk premium related to \(z_1\) is strong (because

---

13. The expression for the serial covariance below follows immediately from the date 3 liquidation value (1), and the two price definitions (2) and (3).
the mass of agents receiving information at date 2 is increased) and the tendency for the serial covariance to be positive becomes stronger.

It is worth noting that, though we consider a stylized setting to facilitate analytic solutions, the results on serial correlation around news events illustrate the following intuitive point. In standard models of symmetric information, liquidity shocks generally lend a negative serial correlation. However, conditional risk premia related to liquidity shocks gradually decline if private information about a future public announcement is received sequentially by agents, as greater knowledge of asset values means less risk. As proposition 3 suggests, if the mass of informed agents is sufficiently large and private information is sufficiently valuable, the first effect dominates and positive serial correlation obtains prior to the news event. Overall, our finding is consistent with the momentum effect documented by Jegadeesh and Titman (1993). In Section IV, we analyze this phenomenon in a richer setting where agents can receive information at both dates 1 and 2, and can influence the timing of information receipt by spending additional resources. As a preamble to this exercise, the next subsection endogenizes information acquisition in the context of the present model.

C. Equilibrium with Endogenous Information Acquisition

We now endogenize the mass of informed agents $M$ by considering a scenario where the information about $\theta$ must be purchased at a cost $C$. This cost is incurred prior to trade at date 1. We thus assume that there is lag between commitment of resources to obtain the signal and the receipt of the signal. Endogenizing information acquisition allows us to obtain conditions for positive serial correlation in terms of parameters such as the variance of liquidity shocks and the cost of information that were not represented in proposition 3. As will be seen in subsection D of this section, this exercise leads to additional empirical predictions as well.

Denote the informed by $I$ and the uninformed by $U$. Since the wealth levels $W_i(i = I, U)$ of the informed and uninformed agents are quadratic forms of multivariate normal random variables, the ex ante utilities of each agent can be evaluated by using standard results on the moment-generating function of such quadratic forms; details are provided in the appendix. There it is shown that the ex ante utility of agent $i$ takes the simple form

$$EU_i = -|2A_i \Sigma + I|^{-1/2} \exp(-RB_{i0}),$$

where $A_i$ is the square, symmetric matrix such that $RW_i = \lambda A_i \lambda$ and where, in turn, $\lambda$ is the normal vector ($\theta \in z_1 z_2$). Note that the expected utility of an agent can be transformed into the certainty equivalent $CE_i$ by way of the following relation: $CE_i = -(1/R) \log(-EU_i)$. We define the difference in informed and uninformed certainty equivalents as $CE_i(M) - CE_i(0) = \Gamma(M)$. Let $C$ be the cost of receiving the signal. For an equilibrium with
endogenous information acquisition, in addition to the market clearing conditions (A5) and (A6), the condition $\Gamma(M) = C$ must be satisfied.

Our next proposition describes the behavior of the expected utilities of the two types of agents.

**Proposition 4.** The difference in the certainty equivalents of wealth for the informed and uninformed is given by

$$
\Gamma(M) = \frac{1}{2R} \ln \left[ \frac{M^2 v + R^2 v v (v_i + v_f)}{M^2 v + R^2 v^2 v_i} \right],
$$

and is therefore positive and is decreasing in $M$.

The above proposition indicates that the utility of being informed exceeds the benefit of being uninformed for all values of $M$ and that the benefit to being informed relative to being uninformed is monotonically decreasing in $M$. Using propositions 3 and 4, a definitive result relating serial correlation to the cost of information acquisition can be derived:

**Proposition 5.** The serial correlation of asset price changes prior to the news event is positive if and only if

$$
C < \frac{1}{2R} \ln \left[ \frac{v + R^2 v (v_i + v_f)}{v + R^2 v v_i} \right].
$$

(11)

Overall, proposition 5 indicates that the threshold level of the cost below which positive serial correlation obtains is increasing in the variance of information ($v_i$) and decreasing in the variance of the component of price movements not due to the public announcement ($v_f$) and the variance of liquidity trading ($v$).

We analyze how serial correlation, volume, and volatility change as a function of the cost of information acquisition, using figures 1–4. These figures
use the parameter values $R = v_p = 2, v_i = v_z = 1$. These parameter value choices are an attempt to calibrate our model to real data. Thus, the orders of magnitude of $v_i$ and $v_p$ are consistent with an annual return standard deviation of 20% (for realistic ranges of stock prices), as reported by Mehra and Prescott (1985) and several others. The value for risk aversion is identical to that assumed by Leland (1992) in his calibration and implies a risk premium over the risk-free rate of that twice the payoff variance (i.e., about 8%).

The range of the cost of information is chosen such that it spans the feasible range of $M$ from 0 to 1. Figure 1 shows how the serial correlation goes from positive to negative as the cost of information acquisition is increased.

It is interesting to note that at a cost level, $C = 0.2554$ (found computationally), price changes are serially uncorrelated, so that prices look like martingales, even though both informed and uninformed agents in the market are risk averse. In addition, the magnitude of the serial correlation is quite respectable; for example, at a cost level of 0.21, the serial correlation is 0.30, a substantial number.

Turning now to trading volume, proposition 1 suggests that trading volume is increasing in $M$ for small $M$ and decreasing in $M$ for large $M$. This, in turn, suggests that under endogenous information acquisition, trading volume should be nonmonotonic in the cost of information acquisition. This intuition is borne out in figure 2, which shows that trading volume is maximized for an intermediate level of the cost of information acquisition. Note that trading volume drops off toward the right of the graph, simply because the mass of informed agents decreases relative to uninformed agents, and uninformed agents trade less aggressively than informed ones. However, even though the mass of informed agents approaches zero toward the right of the graph (see fig. 1), trading volume does not approach zero, because uninformed agents do trade some nonzero amount for risk-sharing purposes.

Figure 3 plots the variances, $\text{Var} (P_1 - P_2)$ and $\text{Var} (P_2 - P_3)$, and shows that
the variance of price change across dates 2 and 3 is monotonic in the cost of information acquisition, while that of the price change across dates 1 and 2 is not. It is also interesting to note that the volatility across dates 2 and 3 increases sharply as the cost of information becomes very high, because prices become virtually uninformative so that risk premia become large. It can also be seen from figure 4 that the average level of volatility is decreasing in the cost of information acquisition. These results are consistent with those in proposition 2. In addition, our results are consistent with that of Chari, Jagannathan, and Ofer (1988) who find that volatility prior to earnings announcements is higher relative to normal levels of volatility. They also find that this volatility ratio is greater for small firms than for large firms, which is consistent with our results under the additional assumption that it is more costly to obtain information about small firms.

D. Additional Empirical Implications

Our theory addresses the issue of how asset price changes behave near significant public informational events such as earnings announcements. Our basic premise is the notion that information events occur in a lumpy fashion and lags in acquisition (or processing) of information lead to serial correlation in asset returns prior to the news event. There are other models, such as Foster and Viswanathan (1993, 1996) and Back, Cao, and Willard (1998), in which volatility and volume persist and can depend on public information arrival because of nonnormal distributions and/or diversely informed traders. Our model is complementary to these papers as it demonstrates that returns prior to public events can show persistence owing to the entry of agents with event-related information as the event approaches. Note that in a Glosten and Mil-

14. This finding is confirmed by Pope and Inyangete (1992) for U.K. stocks.
Fig. 4.—Average volatility versus cost of information. The parameter values are $R = v_o = 2, v_r = v_z = 1$.

grom- (1985) or a pure Kyle- (1985) type setting, asset price changes do not show persistence. Under such a setting, returns on earnings announcement dates and returns preceding the announcement should be unrelated for any cross-sectional subsample. In contrast, our study predicts a positive correlation between announcement date returns and preannouncement returns of regularly scheduled events such as earnings announcements for those cross-sectional subsamples with a high mass of privately informed traders. Since the mass of informed traders is endogenous in our model, we can obtain implications for public event-induced return persistence in terms of exogenous parameters such as earnings volatility and the variance of liquidity trading.

To obtain empirical implications, we suggest using a proxy for the mass of informed agents, such as a measure of the informativeness of trades (e.g., using the methodology of Hasbrouck [1991] or Huang and Stoll [1996]). For convenience, we refer to this proxy as $I$. Then, our analysis suggests a number of predictions related to volume, volatility, and the serial covariance of price changes prior to earnings announcements that have not been suggested previously in the literature.

Proposition 1: Suppose one stratifies a sample of stocks by trade informativeness ($I$). Our analysis suggests that trading volume near earnings announcements should be increasing in trade informativeness for the high $I$ sample and decreasing in $I$ for the low $I$ sample.

Proposition 2: Price change volatility near earnings announcements

15. For irregular events such as stock splits and takeover announcements, a positive correlation between preevent returns and announcement-date returns could simply mean that such events tend to occur when the firm is doing well. Our theory provides a rationale for preevent return persistence for regularly scheduled events such as earnings announcements for which the problem of the run-up causing the event does not apply.
should be decreasing in \( I \). The relationship of volatility to \( I \) during periods that are distant from earnings announcements is ambiguous.

**Proposition 3**: The tendency for the serial covariance of returns near earnings announcements to be positive should be stronger for large \( I \) stocks and, controlling for \( I \), for stocks with greater earnings volatility (a proxy for the model parameter \( v \)).

**Proposition 5**: The tendency for the serial covariance of asset returns to be positive near earnings announcements should be stronger for stocks with high variance of liquidity trading (a proxy for which can be a measure of trading activity such as the average number of trades per day) and stocks with greater earnings volatility.

### E. Unconditional Serial Correlation

Let us now turn briefly to the more general setting with an infinite sequence of news events. Our serial correlation is calculated for the subperiods corresponding to each period \( t \). The question naturally arises as to the behavior of the overall (unconditional) serial correlation implied by the model. The sign of the unconditional serial correlation of contiguous price changes, however, is the same as that of the covariance calculated in proposition 3, so long as one makes the additional assumptions that liquidity trades are all serially uncorrelated across all times \( t \), the variance of liquidity trades in subperiods \( t_1 \) and \( t_2 \) is constant across all \( t \), and the cost of acquiring information is constant for all \( t \) as well. To see this, note first that the serial correlation across contiguous time periods corresponding to successive cohorts is zero (i.e., \( \text{Cov}(P_{t_1+1_i} - P_{t_j}, P_{t_2} - P_{t_j}) = 0 \)), because successive innovations are serially uncorrelated. Further, each cohort is symmetric owing to the fact that noise trade variances and innovation variances are equal across different times \( t \), so the sign of the serial correlation \( \text{Cov}(P_{t_1} - P_{t_2}, P_{t_2} - P_{t_1}) \) does not switch across subperiods corresponding to different \( t \)'s. Now, while sampling from a large set of asset price changes, the econometrician is equally likely to pick the pair \( P_{t_1} - P_{t_2}, P_{t_2} - P_{t_1} \) and \( P_{t_2+1_i} - P_{t_1}, P_{t_2} - P_{t_1} \). So long as the conditions for positive serial correlation detailed in propositions 3 and 5 obtain for a particular news event at time \( t \), the above arguments imply that the covariance corresponding to the first pair is positive, while that corresponding to the second pair is zero. Thus, the overall serial correlation calculated by the econometrician is also positive. These results are formalized in the appendix and summarized in the following proposition.

**Proposition 6**: In the model with an infinite sequence of news events, where the econometrician is equally likely to sample from price changes surrounding a news event and price changes preceding the news event, the overall autocorrelation of price changes will be positive under the conditions for positive autocorrelation described in propositions 3 and 5.

Thus, the results of propositions 3 and 5 and the empirical implications in
subsection D of this section carry over to a setting that does not condition on the arrival of any given public event.

In an intuitive sense, sufficient conditions for our result on positive serial correlation are as follows: at least some information arrives in form of discrete news events rather than smoothly and continuously, agents have finite horizons and speculate on near-term announcements, and information about these announcements is acquired with a lag. Under these conditions, the reduction in risk borne by the short-term informed and uninformed agents as more and more agents become informed about an impending information event gives rise to positive serial correlation. The assumption of short horizons appears reasonable and, as pointed out in the introduction, has been considered previously in DeLong et al. (1990), Froot, Scharfstein, and Stein (1992), Dow and Gorton (1994), and Vives (1995), among others. Such short horizons can be motivated by agency problems in the money management industry (e.g., from money managers being evaluated too frequently [Holmström and Ricart i Costa 1986]) or, as DeLong et al. (1990) and Shleifer and Vishny (1990) point out, aversion to the unpredictability of noise trader sentiment. While we do not analyze the problem of long-horizon agents in this article for reasons of tractability, we believe our results would generalize to such a setting, so long as the mass of such agents is small so that their risk-bearing capacity is limited. Our result on positive serial correlation in proposition 3 is consistent with the momentum result of Jegadeesh and Titman (1993). As noted above, our model also produces momentum effects of a respectable magnitude for reasonable parameter values.

We have attempted to argue above that the intuition behind our results generalizes to richer settings. However, there still may be a question about the realism of our assumption that agents trade in advance of receiving an information signal. We do not believe this is an unreasonable assumption. Indeed, it requires time to synthesize information gathered about a firm’s management, its products, its suppliers, into an overall signal about the future earnings of the firm. Therefore, lags between expenditure of resources and receipt of information are quite likely.

Aside from lags of the type mentioned above, it also is plausible that in financial markets some agents may be able to obtain private information about a future informational event (say, an earnings announcement) earlier than some other set of agents by expending additional resources. In the next section, we allow for this by considering a setting with dynamic information acquisition, wherein we allow agents to influence the timing of information receipt. Unfortunately, this model does not permit analytical solutions, so that we have to resort to numerical simulations. However, the model deepens our understanding of the conditions needed to obtain positive serial correlation. In

16. The presence of agents with finite horizons and a terminal date is also crucial in Froot, Scharfstein, and Stein (1992) and Hirshleifer, Subrahmanyan, and Titman (1994).
particular, by allowing informed trading at both dates 1 and 2, we are able to obtain insights into how the change in the mass of informed agents over time relates to the equilibrium serial correlation in asset price changes.

IV. An Extension to Multiple Stages of Information Acquisition

We revert to analyzing a cohort corresponding to a generic time $t$ and suppress time subscripts. We now consider a setting in which there are three types of utility-maximizing traders: early-informed traders learn precisely the realization of $v$ just prior to trade at date 1, while late-informed traders learn the realization of $v$ just prior to trading at date 2. There also are uninformed traders who have no knowledge of $v$ at either date. All traders have negative exponential utility with a common risk aversion coefficient $R$. The respective masses of the early-informed, late-informed, and uninformed agents are given by $E$, $L$, and $U$, with $E + L + U = 1$. In the general setting, these masses are endogenized by postulating that the cost of receiving the signal late (at date 2) is a number $C_1$, while the total resources required to receive the signal early (at date 1) is a number $C_2$, with $C_1 > C_2$.

Let $CE_i (i = E, L, U)$ denote the certainty equivalent of an agent of a particular type (early-informed, late-informed, and uninformed). The masses $E$, $L$, and $U$ are such that no early-informed, late-informed, or uninformed agent wishes to change his information acquisition strategy. This implies that in equilibrium, the difference in certainty equivalents of obtaining information early and late should equal $C_1 - C_2$, that is, the additional cost required to obtain information early, and the difference in certainty equivalents of obtaining information late and not acquiring information should equal $C_2$, that is, the cost of obtaining information late. Thus, in equilibrium, $E$, $L$, and $U$ satisfy:

$$CE_e - CE_i = C_1 - C_2,$$  \hspace{1cm} (12)

$$CE_{i} - CE_u = C_2.$$  \hspace{1cm} (13)

A. Equilibrium

Let the subscripts $e$, $l$, and $u$ denote the early-informed, the late-informed, and the uninformed, respectively. Further, let $x_{ij}$ denote the demand of agent $i, i = e, l, u$, at time $j, j = 1, 2$.

**Proposition 7.** The equilibrium with multiple stages of information acquisition is characterized by agent demands on date 2

$$x_{12} = x_{22} = \frac{P + \theta - P_2}{R_i},$$  \hspace{1cm} (14)
\[ x_{a2} = \frac{F + E(\theta|P_1, P_2) - P_2}{\text{Var}(\theta + \epsilon|P_1, P_2)}, \]  

(15)

by agent demands on date 1,

\[ x_{a1} = \frac{E(P_1|\theta, P_1) - P_1}{R S_{1i}} + k_i \frac{F + \theta - E(P_2|\theta, P_1)}{R V_i}, \]  

(16)

\[ x_{ii} = \frac{E(P_2|P_1) - P_i}{R S_{1i}} + k_{ii} \frac{F + E(\theta|P_1) - E(P_2|P_1)}{R V_i}, \]  

(17)

\[ x_{a0} = \frac{E(P_2|P_1) - P_1}{R S_{1i}} + k_{a0} \frac{F + E[(E(\theta|P_1, P_2)|P_1) - E(P_2|P_1)]}{R \text{Var}(\theta + \epsilon|P_1, P_2)}, \]  

(18)

where the \( S \) and \( k' \) terms are exogenous constants described in the appendix,

by market-clearing conditions

\[ Ex_{a1} + Lx_{a1} + UX_{a1} + z_1 = 0, \]  

(19)

\[ Ex_{a2} + Lx_{a2} + UX_{a2} + z_1 + z_2 = 0, \]  

(20)

by agent ex-ante utilities

\[ EU_i = -|2A_i, \Sigma + l|^{-1/2} \exp(-RB_i) \quad i = e, I, u, \]  

(21)

by the corresponding certainty-equivalent utilities

\[ CE_i = -(1/R) \log(-EU_i) \quad i = e, I, u, \]  

(22)

where \( A_i \) denotes the quadratic form of agent \( i \)'s wealth \( W_i \) in terms of the normal vector \( (\theta \in z_i, z_{12}) \) and by the entry indifference equations (12)–(13).

As the proposition suggests, the model with multiple stages of information acquisition is exceedingly complex and does not permit a closed-form solution, so we provide some numerical simulations. We are unable formally to address issues of equilibrium uniqueness, though changing parameters in the neighborhood of our base case does not yield evidence of equilibria other than the one we find in our simulations. For brevity, we do not analyze the model in complete detail but focus on the behavior of the serial correlation of price changes.\(^\text{17}\)

\(^\text{17}\) The trading behavior of early-informed agents in this model is similar to that in Hirshleifer, Subrahmanyam, and Titman (1994). In particular, these agents trade aggressively in the initial round and, to reduce their exposure to the shock \( \epsilon \), they partially reverse their trades at date 2 when the late-informed move the price in the direction of their information. Specifically, we have verified that, for the parameter values we consider (see subsection C of Sec. III), the covariance between their date 1 trade and the date 2 price move is negative, whereas the covariance between their date 1 trade and date 1 price move is positive for all feasible ranges of \( E, U, \) and \( L \). The trades of the other classes of agents are typically positively correlated with the contemporaneous price move.
Fig. 5.—Serial correlation by the cost differential of early versus late information. The parameter values are mass of uninformed = .25, \( R = 2, v_p = 3, v = v_1 = 1 \).

The discussion following proposition 3 suggests that what matters for the serial correlation prior to the news event is the increase in the mass of informed traders across dates 1 and 2, rather than the absolute masses at each date. To investigate this conjecture further, we analyze the case where decision to acquire information early, as opposed to late, is endogenized, and the mass of uninformed is fixed at 0.25.\(^{18}\) Thus, the relevant equilibrium condition is \( CE_e - CE_e = C_1 - C_2 \), with \( U = 0.25 \).

Figure 5 plots the serial correlation versus the cost differential for obtaining information early versus late.\(^{19}\) The serial correlation is negative for low levels of the cost of early information acquisition but becomes positive for higher levels of the cost. The figure illustrates the intuition that, since the magnitude of changes in the conditional risk premium related to \( z \), is governed by the change in the mass of informed agents across dates 1 and 2, increasing the cost of acquiring information early relative to the cost of acquiring information late actually increases the tendency for markets to exhibit positive serial correlation prior to the news event. This is because increasing this cost decreases

\(^{18}\) We assume that each agent is required to make a decision to acquire information either early or late or to acquire no information prior to trade at date 1. This assumption is made for simplicity but captures the notion that even if one wishes to acquire information late, resources must be committed to do so well in advance of the actual receipt of the signal. It also is possible to analyze cases in which all three masses \( E, L, \) and \( U \) are endogenously determined by the costs of early and late information acquisition. Such cases do not lend any more insight on serial correlation than the ones presented here, so we omit them for brevity.

\(^{19}\) The parameter values used are \( R = 2, v_p = 3, v_1 = v = 1 \).
the mass of informed agents at date 1, thus causing a relatively larger increase in the mass of informed agents across dates 1 and 2.\footnote{20}

B. Discussion

The results above indicate that, with sequential information acquisition, what matters for the sign of the serial correlation prior to news events is how the mass of informed agents changes over time and not the absolute mass levels at each date. Thus, if informed agents receive information at different times, then, for positive serial correlation to occur prior to the news event, not only is it necessary that the cost of late information acquisition be low (as we argued in subsection C of Sec. III) but also that the cost of receiving information early relative to late be sufficiently high. When these conditions are satisfied, there will be large changes in the mass of informed agents over time. In these cases, the risk premia associated with liquidity shocks will reduce gradually over time, lending a positive serial correlation to asset returns. Conversely, when these conditions are not satisfied, there will be no sharp changes in the mass of informed agents across dates 1 and 2, and therefore the positive autocorrelation in changes in the conditional risk premia will be small, and price changes will exhibit reversals prior to public news events.

Arbel (1985) argues that the cost of obtaining information early is larger for small stocks. One would expect that it would be more costly to obtain information early for small firms. Indeed, Arbel (1985) makes a compelling case that the main source of information for small firms is the accounting data released at the turn of the year, whereas information is accessible throughout the year for larger firms. Thus, the disparity between the costs of obtaining information early and late is likely to be greater for small firms. Under this plausible premise, our model implies stronger continuations for small firms than large firms. Thus, our model is consistent with the empirical evidence of Jegadeesh and Titman (1993) and Rouwenhorst (1998), that continuations are stronger in small firms. It is noteworthy that the 3–12-month lags over which continuation obtains are consistent with the time period between occurrences of important events such as earnings announcements and the release of annual reports.

C. Empirical Calibration

To get a feel for the ability of the model to generate empirically relevant results, we estimate serial correlations over medium-term horizons. The first set of data we use is the CRSP Monthly Stock Indices file for the combined NYSE-AMEX-NASDAQ universe of stocks over the period 1960–98. Spe-
specifically, we used the size decile portfolios that are formed by dividing all stocks into 10 categories based on market capitalization and are rebalanced annually. For each size decile, we calculated serial correlations between the current return and the cumulative return over the past 3 months. For robustness, we also present these correlations for portfolios sorted by the cumulative return over the past 12 months.

The serial correlations start at 0.14 for the smallest size decile and decline monotonically through the largest size decile. All but one of the serial correlations are positive. In addition, all the serial correlations for the winner-loser sorted portfolios are positive. We test if these serial correlations are statistically greater than zero. Swinscow (1997, ch. 11), gives the appropriate test statistic for a correlation as

\[ t = r \sqrt{\frac{n-2}{1-r^2}} \]

where \( r \) is the correlation estimate and \( n \) is the number of observations. The cutoff for a one-tailed test at the 5% significance level observations is 1.65. The correlation between current return and past 3-month return is significant for the smallest through size 6 decile portfolio. This evidence is consistent with our prediction and the Jegadeesh and Titman (1993) and Rouwenhorst (1998) finding that continuations are stronger for small firms. In addition, observe that eight out of the 10 serial correlations for the winner-loser portfolios are significant. It is also worth noting that our theoretical model generates positive serial correlations up to 0.20 (see the right side of fig. 5), which covers the entire range of estimated serial correlations in table 1.

D. Additional Empirical Implications

In subsection D of Section III, we considered additional empirical implications of the basic model. To distinguish empirically between the basic model and the model of this section, we propose stratifying the sample by analyst following. Based on Arbel (1985), firms with low levels of analyst following would be more likely to conform to the model of the previous section, where private information is only received at date 2. However, firms with wider analyst coverage would have more accessible sources of information so there would be more opportunities to receive information earlier than others.

If one is willing to accept the premise that our analysis in this section is more likely to apply to the firms with high levels of analyst following, the analysis in this section suggests the following implications for the sample with high analyst following.

1. Divide the sample into high I and low I stocks. Within the high I
TABLE 1  Serial Correlation of Current Return with 3-Month Past Return for Size Deciles and Past Return Deciles

<table>
<thead>
<tr>
<th>Size Deciles</th>
<th>Serial Correlation</th>
<th>Past Return Deciles</th>
<th>Serial Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smallest</td>
<td>.14* (3.00)</td>
<td>Past winner 2</td>
<td>.11* (2.17)</td>
</tr>
<tr>
<td>2</td>
<td>.13* (2.73)</td>
<td>3</td>
<td>.10* (2.06)</td>
</tr>
<tr>
<td>3</td>
<td>.10* (2.25)</td>
<td>4</td>
<td>.10* (2.08)</td>
</tr>
<tr>
<td>4</td>
<td>.10* (2.10)</td>
<td>5</td>
<td>.08 (1.94)</td>
</tr>
<tr>
<td>5</td>
<td>.10* (2.10)</td>
<td>6</td>
<td>.10* (1.64)</td>
</tr>
<tr>
<td>6</td>
<td>.08* (1.82)</td>
<td>7</td>
<td>.10* (2.10)</td>
</tr>
<tr>
<td>7</td>
<td>.06 (1.22)</td>
<td>8</td>
<td>.09* (1.95)</td>
</tr>
<tr>
<td>8</td>
<td>.05 (1.11)</td>
<td>9</td>
<td>.08* (1.73)</td>
</tr>
<tr>
<td>9</td>
<td>.04 (0.92)</td>
<td>Past loser</td>
<td>.06 (1.70)</td>
</tr>
<tr>
<td>Largest</td>
<td>−.00 (−.11)</td>
<td></td>
<td>−.00 (1.27)</td>
</tr>
<tr>
<td>N</td>
<td>464</td>
<td>405</td>
<td></td>
</tr>
</tbody>
</table>

*Significant at the 5% level.

Sample, the tendency for the serial covariance of asset returns near earnings announcements to be positive will be stronger for the firms with high analyst following.

2. Further, as more and more agents become informed, agent heterogeneity decreases, so that we should see a drop in trading volume as we approach the announcement date.

3. A feature of our model is that the mass of agents acquiring information about an impending news event changes over time. Our model thus suggests that the informativeness measure I should gradually increase as we approach the date of a significant public announcement, as sequential information acquisition causes the mass of informed agents to increase over time.

V. Conclusion

Fama and French (1996) indicate that their three-factor model can account for many asset-market regularities, but they are unable to explain the medium-term continuation effect (momentum) in equity returns. They leave open the possibility (Fama and French 1996, p. 82) that their factors are unable to fully capture dynamic changes in risk premia. The continuation effect has also been difficult to explain using traditional models of asset pricing. In this article, we develop a model that accounts for this effect by analyzing information
acquisition prior to significant news events. Our analysis considers the temporal resolution of uncertainty through market prices as the mass of investors who receive information about the news event increases over time. Essentially, we enrich the approach of Epstein and Turnbull (1980) by allowing for asymmetric information and show how continuation is a natural consequence of autocorrelation in risk premia as the mass of informed agents increases over time.

We consider two dynamic models: one in which agents are allowed to trade in advance of receiving an information signal and another in which agents can influence the timing of information receipt by expending resources. The first setting permits an analytic solution for the equilibrium serial covariance as a function of the mass of informed traders. In this setting, our analysis is consistent with momentum if information acquisition costs are sufficiently low. The intuition is that conditional risk premia related to early liquidity shocks reduce gradually as agents sequentially receive private information about a future public announcement; this lends unconditional positive autocorrelation to stock price changes. However, risk aversion naturally lends negative serial correlation to asset prices because of standard inventory considerations. If the mass of informed agents is sufficiently high because of a low cost of information acquisition, positive serial correlation obtains.

When different agents receive information at different times, how the informed mass changes over time is the key determinant of whether markets exhibit positive serial correlation prior to news events. This is because a large disparity in the costs of early versus late information acquisition causes the mass of informed agents to change sharply over time, thus leading to a gradual reversal in the conditional risk premium owing to early liquidity shocks. This creates a stronger tendency for positive serial correlation to obtain. Thus, if agents can influence the timing of information arrival by expending resources, our analysis suggests that necessary conditions for positive serial correlation are that the cost of late information acquisition be low and that the cost of receiving information early relative to late be sufficiently high. Based on the work of Arbel (1985), these conditions are particularly likely to be satisfied for relatively smaller firms.

We estimate empirical serial correlations over quarterly horizons for decile portfolios sorted by size and past performance. All but one of the serial correlations are positive and the majority of them are significantly so. In addition, the serial correlations monotonically decline as one moves from the small firm decile to the large firm decile, which is consistent both with our model and with the evidence of Jegadeesh and Titman (1993) and Rouwenhorst (1998). Using reasonable parameter values, our model covers the full range of empirically estimated serial correlations. Our analysis also suggests several untested empirical implications which relate the serial correlation to the information contained in a stock’s order flow.
Appendix

Proof of lemma 1. We begin by conjecturing that all trader demands and both date 1 and date 2 prices are normally distributed. In the linear equilibrium we derive, this conjecture is confirmed to be correct.

Using mean variance analysis, it can be shown that

$$x_{i2} = \frac{\bar{F} + \theta - P_2}{R\bar{v}},$$  \hspace{1cm} (A1)$$

$$x_{t2} = \frac{\bar{F} + E(\theta|P_1, P_2) - P_2}{R\text{Var}(\theta + \epsilon|P_1, P_2)},$$  \hspace{1cm} (A2)$$

The date 1 demands of the informed and uninformed agents are given by

$$x_{i1} = \frac{E(P_1|\theta) - P_1}{R\bar{v}} + k_iE(x_{i2}|P_1),$$  \hspace{1cm} (A3)$$

$$x_{t1} = \frac{E(P_1|\theta) - P_1}{R\bar{v}} + k_iE(x_{t2}|P_1),$$  \hspace{1cm} (A4)$$

where $S$ and the $k$ coefficients are exogenous constants. Market clearing implies

$$Mx_{i1} + (1 - M)x_{t1} + z_1 = 0, \hspace{1cm} (A5)$$

$$Mx_{i2} + (1 - M)x_{t2} + z_1 + z_2 = 0. \hspace{1cm} (A6)$$

Next we state the following lemma, which is a standard result on multivariate normal random variables (see, e.g., Brown and Jennings [1989]).

Lemma 2. Let $Q(\chi)$ be a quadratic function of the random vector $\chi$: $Q(\chi) = C + B^t\chi - \chi^tA\chi$, where $\chi \sim N(\mu, \Sigma)$, and $A$ is a square, symmetric matrix whose dimensions correspond to that of $\chi$. We then have

$$E[\exp(Q(\chi))] = |\Sigma|^{-1/2}|2A + \Sigma^{-1}|^{-1/2}$$

$$\times \exp[C + B^t\mu + \mu^tA\mu + \frac{1}{2}(B' - 2\mu'\Sigma^{-1})(B - 2A\mu)].$$

Let $\phi_i$ and $x_i$ denote the information set and demand, respectively, of an agent $ii(i = I, U)$, at date $j$. The date 2 demand of the agent (from maximization of the mean-variance objective) is given by

$$x_{i2} = \frac{E(F|\phi_{i2}) - P_2}{R\text{Var}(F|\phi_{i2})},$$  \hspace{1cm} (A7)$$

Let $\mu_2 = E(F|\phi_{i2})$. Note that in period 1, the trader maximizes the derived expected utility of his time 2 wealth, which is given by

$$E[\left(-\exp(-R[B_0 - x_1P_1 + x_1P_2 + (\mu_2 - P_2)^2]/[2R\text{Var}(F|\phi_{i2})])]\right)\phi_{i1}].$$  \hspace{1cm} (A8)$$

Let $\overline{P}_2$ and $\mu$ denote the expectations of $P_2$ and $\mu_2$, and $\Sigma$ the variance-covariance
matrix of $P_t$ and $\mu_t$, conditional on $\phi_t$. Then, the expression within the exponential above (including terms from the normal density) can be written as

$$ - \left[ \frac{1}{2} y'Gy + h'y + w \right], $$

where

$$ y' = [\mu_t - \mu, P_t - \bar{P}_t], $$

$$ h' = \left[ -Rx_t + \frac{(P_t - \mu)}{\text{Var}(F|\phi_z)} \frac{(\mu - \bar{P}_t)}{\text{Var}(F|\phi_z)} \right], $$

$$ G = \left[ \Pi^{-1} + \left[ s^{-1} - s^{-1} \right] \right], $$

$$ w = Rx_t(P_t - \bar{P}_t) + g, $$

where $s = \text{Var}(F|\phi_z)$, and where $g$ is an expression that does not involve $x_t$. From lemma 2 and Bray (1981, app.), (A8) is given by

$$ - \frac{1}{(\text{det}(\Pi)^{1/2} | \text{det}(A)|^{1/2})} \exp \left( \frac{1}{2} h'G^{-1}h - w \right). $$

(A9)

Thus, the optimal $x_{i_t}$ solves

$$ \left[ \frac{dh}{dx_{i_t}} \right] G^{-1}h - \frac{dw}{dx_{i_t}} = 0. $$

Substituting for $h$ and $w$, we have

$$ x_{i_t} = \frac{\bar{P}_t - P_t}{RG_{i_t}} + \frac{\mu - \bar{P}_t}{R \text{Var}(F|\phi_z)} \frac{G_{i_t} - G_z}{G_{i_t}}, $$

(A10)

where $G_{i_t}$ and $G_z$ are the elements in the first row of the matrix $G^{-1}$. It follows that the demands $x_{i_t}$ and $x_{i_t}$ are given by (A3) and (A4), respectively, with the $S$ coefficients being the $G_{i_t}$ coefficient above and the $k$ coefficients being the term $(G_{i_t} - G_z)/G_{i_t}$. We thus obtain (A3) and (A4).

We can rewrite the market clearing condition at date 2, (A6), as

$$ M \bar{F} + \theta - P_t \frac{R \text{Var}(F|\phi_z)}{R \text{Var}(F|\phi_z)} + (1 - M) \bar{F} + E(\theta|\phi_z) - \frac{P_t}{R \text{Var}(\theta + e|\phi_z)} + z_{i_t} + z_2 = 0, $$

(A11)

where $\phi_z$ is the date 2 information set of the uninformed. Now, the uninformed observe $P_t$ at date 2, which is equivalent to observing

$$ \tau = \theta + \frac{R \text{Var}(F|\phi_z)}{M}(z_{i_t} + z_2). $$

In addition, since there is no private information at date 1, the uninformed also observe the date 1 demand shock $z_{i_t}$. Thus, we have

$$ E(\theta|\phi_z) = E(\theta|\tau, z_{i_t}) = \frac{v_\theta}{v_\theta + k^2 z_2} (\theta + k z_2) $$

(A12)
and

\[
\text{Var}(\theta + \epsilon | \varphi) = \nu + \text{Var}(\theta | \varphi, \zeta) = \nu + \frac{k^2 \nu \nu}{\nu + k^2 \nu}, \quad \text{(A13)}
\]

where \( k = R_v / M \). Substituting for the above moments into the market clearing condition (A11), for the price \( P_2 \) from (2), and equating coefficients of the variables \( \theta, \zeta, \) and \( z \), we can obtain a closed-form expression for the date 2 price.

Now, from the market clearing condition at date 1, (A5), we can solve for \( K \) in terms of the \( k \) and \( S \) coefficients in (A3) and (A4). This exercise yields

\[
f = \frac{b[MRS_v + (1 - M)/RS_v - [Mk_v/RS_v + (1 - M)k_v/RS_v + 1]]}{MRS_v + (1 - M)/RS_v}, \quad \text{(A14)}
\]

The \( G \) coefficients for the informed agents are given by the first row of the matrix

\[
\begin{pmatrix}
(a^2 \nu + c^2 \nu + \nu^2) v + (a^2 \nu + c^2 \nu + \nu^2) v \\
(a^2 \nu + c^2 \nu + \nu^2) v + (a^2 \nu + c^2 \nu + \nu^2) v
\end{pmatrix}^{-1} + \begin{pmatrix}
(v^{-1} - v^{-1})^{-1} \\
(v^{-1} - v^{-1})^{-1}
\end{pmatrix}, \quad \text{(A15)}
\]

and those for the uninformed agents are given by the first row of the matrix

\[
\begin{pmatrix}
(a^2 \nu + c^2 \nu + \nu^2) v + (a^2 \nu + c^2 \nu + \nu^2) v \\
(a^2 \nu + c^2 \nu + \nu^2) v + (a^2 \nu + c^2 \nu + \nu^2) v
\end{pmatrix}^{-1} + \begin{pmatrix}
(v^{-1} - v^{-1})^{-1} \\
(v^{-1} - v^{-1})^{-1}
\end{pmatrix}, \quad \text{(A16)}
\]

where \( \nu = \text{Var}(F|P_1, P_2) \). Substituting for \( a, b, \) and \( c, \) we find that

\[
S_i = S_v = \frac{\nu[Mv_v + R^2v_v(v_i + \nu)](M + R^2v_v)}{M^2v_v + 2MR^2v_vv + R^2v_vv[R^2v_v(v_i + \nu) + 1]},
\]

\[
k_i = \frac{R^2v_v}{Mv_v + R^2v_v(v_i + \nu)}.
\]

and

\[
k_u = k_i \frac{M^2v_v + R^2v_v(v_i + \nu)}{M^2v_v + R^2v_v(v_i + \nu)}.
\]

Note that \( k_u > k_i \). Using (A13), it is easy to show that \( k_i / \nu = k_i / \nu \). Substituting for \( S_i, S_v, k_i, \) and \( k_u \) into (A14) and performing some tedious algebra yields (7). Q.E.D.

This completes the proof of lemma 1, which started on page 24 above.

Proof of proposition 1. The total trading volume at date 2 in terms of the price coefficients can be written as

\[
M \sqrt{(1 - a_i)^2 v + \left(1 - \frac{b}{R_v}\right) v + \frac{c^2 v}{R^2 v^2}} +
\]

\[
(1 - M) \sqrt{(1 - a_2)^2 v + \left(1 - \frac{b}{R_v}\right) v + \frac{(a_2 - c)^2}{R^2 v^2}} v + v^{1/2}
\]

where \( a_i = M^2v_v/(M^2v_v + R^2v_vv) \) and \( a_2 = (Rv_v/a_i / M) \). Substituting for the price
coefficients for lemma 1 and differentiating, we find that the derivative of trading volume with respect to $M$ at $M = 0$ is

$$\frac{\sqrt{R^2 v(v^2 + 2v_0 + 2v_0^2) + v_0}}{Rv} - \frac{v_0^{1/2}}{v} - v_0^{1/2}.$$  

The above term is positive, as can be seen by comparing the first and second terms in the square root to the last two negative terms following the square root. Similarly, the derivative at $M = 1$ is

$$\frac{Rv/vv^{1/2} - \sqrt{R^2 v(v^2 + v_0^2) + v_0}}{R^2 v(v + v_0)},$$

which is negative. Q.E.D.

**Proof of proposition 2.** The variance

$$\text{Var}(P - P_0) = (1 - c) v + v + (d^2 + e^2)v = R^2 v[v(M^2 v_0^2 + M^2 v_0 [(R^2 v v (3v_0 + 2v_0) + v)] + 2MR^2 v_0 v v_0 + R^2 v_0 v_0 [2R^2 v_0 (v + v_0) + v_0])]

\implies \frac{(M^2 v_0 + MR^2 v_0 v_0 + R^2 v_0 v_0)^2}{v_0 - R^2 v_0 v_0 (v + v_0) + v_0}.$$  

It is easy to verify that the above expression is decreasing in $M$ for $0 \leq M \leq 1$, completing the proof of (1).

$$\text{Var}(P - P_0) = a^2 v_0 + [(b - f)^2 + c^2]v.$$  

Substituting for the price coefficients from lemma 1, we have

$$\text{Var}(P - P_0) = \frac{2v_0 (v_0 + v_0) [1 - R^2 (v + v_0)]}{v_0},$$

and at $M = 1$, it is

$$\frac{2R^2 v_0^2 v_0^2 (v_0 - v_0)}{R^2 v_0 v_0 (v + v_0) + v_0},$$

thus completing the proof of (2).

Similarly, it can be shown that the derivative of the sum $\text{Var}(P_2 - R) + \text{Var}(P_3 - P_2)$ with respect to $M$ at $M = 0$ is

$$\frac{4v_0 (v_0 + v_0) [1 - 2R^2 v_0 (v_0 + v_0)]}{v_0},$$

and at $M = 1$ is

$$\frac{2R^2 v_0^2 v_0^2 [2R^2 v_0 (2v_0 - v_0) + 1]}{R^2 v_0 v_0 (v + v_0) + v_0}.$$
thus completing the proof of (3). Q.E.D.

Proof of proposition 3. The serial covariance

\[ \text{Cov}(P_2 - P_1, P_1) = a(1 - a)\nu_k - b(b - f)v - c^2v. \]

Again, substituting for the price coefficients from lemma 1, the equilibrium serial covariance becomes

\[ \text{Cov}(P_2 - P_1, P_1) = \)

\[ \frac{(R^2v(\nu_k + \nu))[(M^2v_k + R^2v)(\nu_k + \nu)](Mv_k - \nu)}{(M^2v_k + MR^2v)(\nu_k + \nu)} \]

When \( M = 0 \), the serial covariance is \( -R^2v(\nu_k + \nu)^2 \), which is negative, and when \( M = 1 \), the serial covariance becomes \( R^2v(\nu_k - \nu) \), which is positive if and only if \( \nu > \nu \). Q.E.D.

Derivation of the ex ante utilities of each informed agent. The ex ante utility of each type of agent is derived by an application of lemma 2. Define \( \lambda = (\theta \epsilon z_i z_j) \).

Given that the terminal wealth of agent \( i \) (where \( i = I, U \)) is given by

\[ W_i = x_i (F - P_1) - x_i (P_2 - P_1) + B_0, \]

we can construct the square, symmetric matrix \( A \), such that \( RW_i = \lambda A \). Noting then that the ex ante expected utility is given by \( EU_i = E[-\exp(-RW_i)] \), we can apply lemma 2 with \( \mu = 0, C = -RB_{ii}, B = 0 \), and \( A = A \). The agent’s ex ante utility thus becomes

\[ EU_i = E[-\exp(-\lambda A \epsilon)] \]

\[ = -|\Sigma|^{-1/2}[2A_i + \Sigma^{-1}]^{-1/2}\exp(-RB_{ii}) \]

\[ = [2A_i \Sigma + I]^{-1/2}\exp(-RB_{ii}). \]

Proof of proposition 4. The determinant of the matrix \( [2A_i \Sigma + I] \) is monotonically related to the certainty equivalent and the expected utility of being informed, and is given by

\[ \frac{(R^2v(\nu_k + \nu))[(M^2v_k + R^2v)(\nu_k + \nu)](Mv_k + 2MR^2v)(\nu_k + \nu)}{(M^2v_k + MR^2v)(\nu_k + \nu)} \]

Similarly, the determinant of the matrix \( [2A_i \Sigma + I] \) is monotonically related to the certainty equivalent and the expected utility of being uninformed, and is given by

\[ \frac{(R^2v(\nu_k + \nu))[(M^2v_k + R^2v)(\nu_k + \nu)](Mv_k + 2MR^2v)(\nu_k + \nu)}{(M^2v_k + MR^2v)(\nu_k + \nu)} \]

It follows that the ratio of the above two quantities is given by

\[ \frac{M^2v_k + R^2v(\nu_k + \nu)}{M^2v_k + R^2v(\nu_k + \nu)}. \]

which is greater than unity and is decreasing in \( M \). Q.E.D.

Proof of proposition 5. Proposition 3 indicates that a necessary and sufficient condition for positive serial correlation to obtain is \( M > v/\nu \), this condition is equivalent
to $C < \Gamma(v/v_0)$. Condition (11) immediately obtains from substituting $M = v/v_0$ in (A17). Q.E.D.

Proof of proposition 6. First, note that

$$P_{t+1} = \bar{P}_e + b z_{t+1},$$

$$P_t = \bar{P}_e + \theta_t + \epsilon_t,$$

$$P_t = \bar{P}_e + a \theta_t + b z_{t} + c z_{t+1}.$$ Since $z_{t+1}$ is uncorrelated with $z_t, \theta_t$ and $\epsilon_t$, this implies that $Cov(P_{t+1} - P_t, P_t - P_0) = 0$. If the condition $M_{t+1} > v_t$ or condition (11) (under endogenous information acquisition) is satisfied then we have $Cov(P_{t+1} - P_t, P_{t+1} - P_0) > 0$. By the law of iterated expectations, if the pairs $P_t - P_0$, $P_{t+1} - P_t$, and $P_{t+1} - P_0$ are equally likely to be selected, the overall autocorrelation is a simple arithmetic mean of the two covariances above and is also positive. Q.E.D.

Proof of proposition 7. Equations (14)–(18) follow from standard mean variance analysis. Next, we supply the definitions of the $S$ and the $k'$ terms in (16)–(18). From (A10), $k_1 = (S_1 - S_2)S_1$, $k_2 = (S_1' - S_2')S_1'$, and $k_\alpha = (S_1' - S_2')S_1'$; where $S_1$ and $S_2$ are the elements in the first row of the matrix

$$\left[\begin{array}{c}
\text{Cov} (P_2, \theta | P_1) -1 + \left( \begin{array}{c}
v_1' - v_2' \\
v_2' - v_2' \\
v_2' - v_2' \\
v_3' - v_3'
\end{array} \right)
\end{array}\right]^{-1},$$

$S_1'$ and $S_2'$ are the elements in the first row of

$$\left[\begin{array}{c}
\text{Cov} (P_2, \theta | P_1) ^{-1} + \left( \begin{array}{c}
v_1' - v_2' \\
v_2' - v_2' \\
v_2' - v_2' \\
v_3' - v_3'
\end{array} \right)
\end{array}\right]^{-1},$$

and, defining $v' = \text{Var}(\theta + \epsilon | P_1, P_2)$, $S_1'$ and $S_2'$ are the elements in the first row of

$$\left[\text{Cov} [P_2, E(\theta, P_1, P_2) | P_1] ^{-1} + \left( \begin{array}{c}
v_1' - v_2' \\
v_2' - v_2' \\
v_2' - v_2' \\
v_3' - v_3'
\end{array} \right)
\end{array}\right]^{-1}.$$ We again consider linear equilibria implied by the model. Thus, let us postulate that $P_t$ and $P_{t+1}$ are linearly related to the observables at each date such that

$$P_{t+1} = \bar{P}_e + a \theta_t + b z_{t+1},$$

$$P_t = \bar{P}_e + a \theta_t + b z_{t+1},$$

Note that the variable has a coefficient of unity in the form for $P_{t+1}$. Given the initial assumption that prices are normally distributed, one can substitute for the moments in the demand equations in terms of the price coefficients in (A18) and (A19). Then, from the market clearing conditions (19) and (20), one can confirm that prices are indeed normally distributed in equilibrium, and obtain a system of nonlinear equations in the unknowns, that is, $a$, $b$, $c$, $e$, and $f$. Let us define the following quantities:

$$E(\theta | P_1, P_2) = \alpha \theta + \alpha_2 z_1 + \alpha_3 z_2,$$

$$E(\theta | P_1) = l_1 \theta + l_2 z_1,$$
\[ E(P_t | R_t) = m_1 \theta + m_2 z_t, \quad (A22) \]

\[ E[E(\theta | P_t, P_{t+1}) | R_t] = n_1 \theta + n_2 z_t. \quad (A23) \]

(Expressions for the coefficients on the right-hand side of the above equations and for \( v' \), in terms of the price coefficients \( a, b, c, e, \) and \( f \) are easy to derive and are omitted for brevity.) From the market clearing condition (A6), we then have

\[ (E + L) \frac{1 - a}{R_v} + U \frac{\alpha - a}{R_v} = 0, \quad (A24) \]

\[ -(E + L) \frac{b}{R_v} + U \frac{\alpha - b}{R_v} + 1 = 0, \quad (A25) \]

\[ -(E + L) \frac{c}{R_v} + U \frac{\alpha - c}{R_v} + 1 = 0, \quad (A26) \]

and, from (19), we have

\[ E \frac{a - e}{RS_1} + E \frac{1 - a}{RS_1} S_i - S_i + L \frac{m_i - e}{RS_1} + L \frac{l_i - m_i S_i}{R_v} S_i - S_i
\]

\[ + U \frac{m_i - e}{RS_i} + U \frac{n_i - m_i S_i}{R_v} S_i = 0, \quad (A27) \]

\[ E \frac{b - f}{RS_1} - E \frac{b}{RS_1} S_i - S_i + L \frac{m_i - f}{RS_i} + L \frac{l_i - m_i S_i}{R_v} S_i
\]

\[ + U \frac{m_i - f}{RS_i} + U \frac{n_i - m_i S_i}{R_v} S_i - S_i = 1. \quad (A28) \]

The equilibrium in the securities market is described by the values of \( a, b, c, e, \) and \( f \), which satisfy equations (A24)–(A28). Expression (21) follows by a straightforward application of lemma 2. Q.E.D.

References


