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Liquidity Measurement Problems in Fast, Competitive Markets: Expensive and Cheap Solutions

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ABSTRACT

Do fast, competitive markets yield liquidity measurement problems when using the popular *Monthly* Trade and Quote (MTAQ) database? Yes. MTAQ yields distorted measures of spreads, trade location, and price impact compared with the expensive *Daily* Trade and Quote (DTAQ) database. These problems are driven by (1) withdrawn quotes, (2) second (versus millisecond) time stamps, and (3) other causes, including canceled quotes. The expensive solution, using DTAQ, is first-best. For financially constrained researchers, the cheap solution—using MTAQ with our new Interpolated Time technique, adjusting for withdrawn quotes, and deleting economically nonsensical states—is second-best. These solutions change research inferences.

TWENTY-FIRST CENTURY EQUITY markets have become much faster (Jain (2005), Hendershott and Moulton (2011), Angel, Harris, and Spatt (2011, AHS)) and more competitive (AHS). On the speed dimension, AHS document a radical increase in the frequency of bid-ask quote updates. They report a nearly 20-fold increase in the frequency of quote updates for stocks in the S&P 500 from 0.17 per second in May 2003 to 3.3 per second in October 2009. Similarly, Chordia, Roll, and Subrahmanyam (2012) report a 33-fold increase in the value-weighted frequency of trades in New York Stock Exchange (NYSE) stocks from 0.13 per second in January 2003 to 4.3 per second in June 2008. On the competition dimension, AHS document that the NYSE's market share in NYSE-listed stocks has dropped from 80% in February 2005 to 25% in February 2009 and that NASDAQ's market share in NASDAQ-listed stocks has dropped from 53% in April 2005 to 30% in April 2009. Correspondingly, Wolfe (2010) documents that the NYSE's information share (i.e., its percentage contribution to price discovery) in Dow Jones Industrial Average stocks has dropped from a range of 91% to 95% in 2004 to a range of 8% to 53% in 2008. This shift from dominant players to many relatively coequal players means that, while it was once fine

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for researchers analyzing NYSE trading to rely on NYSE quotes only (e.g., Chordia, Roll, and Subrahmanyam (2000, 2001, 2002)), doing so is no longer sufficient. Researchers examining recent years (2009 and thereafter) must use National Best Bid and Offer (NBBO) quotes, where the national best bid (offer) is the highest bid (lowest offer) across all U.S. exchanges and market makers.

In this paper, we ask whether the liquidity of today's fast, competitive U.S. equity markets can be accurately measured using NYSE's *Monthly* Trade and Quote (MTAQ) database. MTAQ is the most popular intraday database for academic research in U.S. equities. It provides intraday trade and quote data time-stamped to the *second*. For a three to four times larger price,[1] the NYSE also sells a second database: the *Daily* Trade And Quote (DTAQ) database. DTAQ and MTAQ are identical, except for two critical differences.[2] First, DTAQ adds a "NBBO" file containing most[3] of the official NBBO quotes from the Securities Industries Processors (SIPs).[4] Second, all trades, quotes, and NBBO quotes in DTAQ are time-stamped to the *millisecond* (i.e., 1/1,000th of a second). In fast markets, the millisecond time stamp might well be important in matching trades and quotes and the official NBBO quotes may contain fewer errors than the raw quotes.

Our sample is 100 randomly selected firms from April 1, 2008 to June 30, 2008. This period is prior to the severe phase of the financial crisis that started in mid-September 2008.[5] We obtain data on 34 million trades and 351 million quotes.

We find large differences between liquidity measures computed from MTAQ and DTAQ. For MTAQ, when compared to DTAQ, we find that: (1) the percent effective spread is 54% larger, (2) the percent quoted spread goes negative 37 times more often, (3) the percent quoted spread is 47% smaller, (4) the effective spread is greater than the quoted spread 15% more often, (5) trades outside the NBBO happen eight times more often, (6) the percent realized spread is 12% larger, and (7) the percent price impact is 109% larger.

To determine *why* we observe these liquidity measurement problems, we conduct a three-way decomposition that examines three possible factors: (1) withdrawn quotes, where an exchange or market maker momentarily quotes nothing, (2) millisecond versus second time stamps of trades and quotes, and

[1] For pricing details, see www.nyxdata.com/Data-Products/Daily-TAQ or the Wharton Research Data Services (WRDS) website.

[2] There are two additional differences that are not critical for our purposes. First, DTAQ adds some extra quote condition fields. Second, DTAQ data can be downloaded the next day as opposed to on a monthly cycle for MTAQ.

[3] The NBBO file is *incomplete* by itself. Construction of the Complete Official NBBO requires combining the NBBO file and the Quote file. See footnote 24 for details.

[4] There are two SIPs. The Consolidated Tape Association (CTA) covers all NYSE-listed ("Tape A") and American Stock Exchange- and regional-listed ("Tape B") securities and the Unlisted Trading Privileges (UTP) Committee covers all NASDAQ ("Tape C") securities.

[5] During our sample period, the Volatility Index (VIX) ranged from 19 to 25, which is the same range that it had been in for the prior 12 months. During the severe phase of the financial crisis from mid-September 2008 to December 2008, the VIX ranged from 55 to 80.

(3) other causes. The last category includes canceled quotes where a limit sell (buy) setting the current ask (bid) is canceled and the exchange or market maker's quote is updated in the DTAQ NBBO file, but not in MTAQ. We find that each of these three factors is a statistically and economically significant source of liquidity measurement problems.

All three factors imply that MTAQ contains errors relative to DTAQ, which we attempt to capture with a simple errors-in-variables model. We model the MTAQ NBBO as being the DTAQ NBBO plus errors. Simulated trades are generated relative to the DTAQ NBBO with no errors. We compute the liquidity measures using the same trade prices but different NBBOs. We find that our simple errors-in-variables model can generate in a simulation nearly all of the liquidity measure differences between DTAQ and MTAQ with the same sign as the actual liquidity measure differences and with most of the magnitudes being roughly similar. This is strong evidence that the actual liquidity measure differences are driven by errors in the MTAQ NBBO that are not present in the DTAQ NBBO.

Next, we examine possible solutions. One possible solution is to purchase the expensive DTAQ database. With this database, one can construct the Complete Official NBBO, which we use as our benchmark.[6] Our empirical results show that this benchmark is credible as it yields a much lower frequency of negative quoted spreads and trades outside the NBBO than any MTAQ alternative that we consider. It is our first-best solution.

What if a researcher is financially constrained to using the relatively cheap MTAQ database? We consider several possible solutions. One possibility is to adjust for withdrawn quotes. When an exchange or market maker temporarily withdraws their bid and/or ask quote, MTAQ records the bid and/or the ask price as either zero or missing.[7] In this case, the researcher's correct adjustment is to set the bid and/or the ask quote for that exchange or market maker to missing, rather than throwing the current bid and/or ask quote away and using the prior bid and/or ask quote for that exchange or market maker. Correct adjustment avoids using stale quotes, which the three-way decomposition shows is a major source of liquidity measure differences. Since withdrawn quotes are directly observable and adjusting for withdrawn quotes eliminates a major problem, researchers using MTAQ should always adjust for withdrawn quotes.[8]

[6] In certain instances, when a single exchange has both the best bid and the best offer, the official SIP NBBO quote is recorded in the DTAQ Quotes file and not in the DTAQ NBBO file. We construct the Complete Official NBBO data set of official SIP NBBO quotes by adding these single-exchange NBBO quotes from the DTAQ Quotes file to the DTAQ NBBO file. See footnote 24 for a description of the construction of the Complete Official NBBO.

[7] The DTAQ Quote file does *not* make any adjustment (same as the MTAQ Quote file), but the DTAQ NBBO file *does* adjust appropriately. So, using the DTAQ Complete Official NBBO correctly deals with withdrawn quotes.

[8] Although withdrawn quotes are directly observable, we are the first (to our knowledge) to address them, to document their substantial importance, and to recommend a fix for them.

Taking the adjustment for withdrawn quotes as given, we consider additional possible MTAQ solutions. First, we consider quote timing rules. One quote timing rule is Prior Second as recommended by Henker and Wang (2006), which matches a trade to the NBBO quotes that are in-force in the *prior* second. A second quote timing rule is Same Second as recommended by Bessembinder (2003) and Peterson and Sirri (2003), which matches a trade to the NBBO quotes that are in-force during the *same* second. We introduce a third and potentially more accurate quote timing rule, which we call Interpolated Time. This rule uses the ordering of trades and quotes within a second to make an educated guess about the millisecond in which the events occurred and then matches each trade at the inferred millisecond to the NBBO quotes inferred to have been in-force in the prior millisecond.

Finally, we consider deleting NBBO quotes during economically nonsensical states and deleting any trades that occur under those states. The NBBO of a market is defined as being "crossed" if the national best bid is greater than the national best offer. Similarly, the NBBO of a market is defined as being "locked" if the national best bid is equal to the national best offer. We conjecture that many of these nonsensical states arise as a result of quotes that have been canceled, but no quote update has been recorded. Therefore, we delete NBBO quotes whenever the NBBO is crossed or locked because, in market microstructure theory, the offer price must always be greater than the bid price.[9] We also delete any trades that occur while the NBBO is crossed or locked because, if there are no legitimate NBBO quotes to benchmark the trade, then it is impossible to compute several standard liquidity measures for those trades, to type the trade as a buy or sell, to compute the Probability of Informed trading (PIN) using those trades, etc. For example, we would delete the NBBO quotes and trades for as long as the National Best Bid quoted by the Chicago Stock Exchange (CHX) is greater than or equal to the National Best Offer quoted by the National Stock Exchange. With many competitive players, it is important to address crosses and locks *between* exchanges/market makers, not just *within* a given exchange/market maker (e.g., when NYSE bid $\geq$ NYSE offer).

In addition to adjusting for withdrawn quotes, we find that the best overall MTAQ solution is to use our new Interpolated Time technique and to delete NBBO quotes and trades when the NBBO is crossed or locked. The combination of all three techniques goes the furthest distance possible in reducing liquidity measurement problems. Specifically, MTAQ with all three techniques, when compared to MTAQ with no adjustments, has the following benefits: (1) the percent quoted spread difference (relative to DTAQ) is completely eliminated, (2) the percent effective spread difference is reduced by 91%, (3) the outside-the-NBBO difference is reduced by 90%, (4) the crossed NBBO absolute difference is reduced 97%, and (5) the percent price impact difference is

[9] There are three well-established reasons why the offer price must always be greater than the bid price: (1) adverse selection (Glosten and Milgrom (1985), Kyle (1985), Easley and O'Hara (1987)), (2) order processing costs (Roll (1984)), and (3) compensation for bearing inventory risk (Amihud and Mendelson (1980), Ho and Stoll (1981, 1983)).

reduced by 97%. Despite these important improvements, significant liquidity measurement problems remain. Thus, we conclude that using MTAQ with these three techniques is second-best and not as good as the first-best solution of using DTAQ.

A complete, commented SAS file to implement our MTAQ recommendations is available on our individual web sites.[10] The September 2013 version of this program is provided in the Internet Appendix that accompanies this article.[11] The SAS code does the following: downloads MTAQ data from WRDS, applies these three techniques (adjusts for withdrawn quotes, uses interpolated time, and eliminates NBBO quotes and trades when the market is crossed or locked), and computes standard liquidity measures (time-weighted percent quoted spread, volume-weighted percent effective spread, volume-weighted percent realized spread, and volume-weighted percent price impact).

Next, we consider whether different methods yield different research inferences. We first reexamine the analysis of Hendershott and Moulton (2011, HM). They conduct an event study around the NYSE's Hybrid Market reform, which significantly increased the exchange's automation and speed. HM match trades *to NYSE quotes only*, which is sufficient because the NYSE still held a dominant market share during the 2006 to 2007 Hybrid reform period. We replicate the HM study in three ways: (i) using the "conventional" MTAQ method (NBBO across all markets, no adjustments for withdrawn quotes, etc.); (ii) using the first-best DTAQ approach; and (iii) using the second-best adjusted MTAQ approach. We find that using our first- and second-best solutions yields the identical inference that HM find using NYSE quotes only, namely, an increase in percent effective spreads around the event date. By contrast, using the conventional MTAQ method yields an incorrect inference, namely, no change in percent effective spreads, because the spread measures are so noisy that they have large standard errors. Although HM find the same result using NYSE quotes only, that is no longer a viable approach. Thus, we demonstrate that, among the approaches that include all markets (important for the competitive markets of 2009 and forward), the conventional MTAQ method yields a flawed inference, while the first-best DTAQ and second-best adjusted MTAQ methods yield correct inference.

Next, we examine exchange performance based on relative effective spread rankings. We find that MTAQ with no adjustments yields different rankings from DTAQ the majority of the time and yields biased conclusions about which exchanges have superior versus inferior performance. Further, we find that using our second-best solution reduces the frequency of rankings different from DTAQ and reduces the bias about exchange performance, but falls short of using our first-best solution. Finally, we conduct a firm trading costs sort that is common in the corporate finance and asset pricing

[10] See www.kelley.iu.edu/cholden and www.cox.smu.edu/web/sjacobsen. We appreciate any feedback on the SAS code and plan to update it over time.

[11] The Internet Appendix is located in the online version of this article on the *Journal of Finance* web site.

literature. We find that using MTAQ with no adjustments, the majority of dollar effective spread quintiles differ from our first-best solution, whereas using our second-best solution, the vast majority are the same as the first-best solution.

Regarding research that studies 2008 and years thereafter and that is based on NBBO quotes using MTAQ with no adjustments, our 2008 evidence leads us to conclude that any estimates of the quoted spread, effective spread, realized spread, price impact, frequency of trades outside the NBBO, frequency of locked and crossed markets, and buy/sell classification are likely to be strongly biased, whereas estimates of depth and absolute order imbalance are likely to be unbiased. The decline of the NYSE and NASDAQ's market shares through 2008 means that using only NYSE or NASDAQ quotes to study 2009 and later is no longer an option.

Looking to the future, we consider what happens when the trading process accelerates into microseconds (10^{-6} seconds) in the late 2010s and nanoseconds (10^{-9} seconds) in the 2020s. If bid and offer update messages could travel arbitrarily fast, then it would be possible to maintain a common NBBO for all economic agents in all locations. However, bid and offer update messages *cannot* travel faster than the speed of light (186,282 miles per second) and so high-speed traders face immutable lag times in receiving bid and offer updates from remotely located exchanges. As a replacement for the NBBO, we propose a Relative Best Bid and Offer (RBBO) that accounts for the theoretical minimum lag time in communicating at the speed of light and is different for each market center.

The paper is organized as follows. Section I describes the institutional setting. Section II describes the liquidity measures. Section III describes the data. Section IV presents the overall liquidity measure differences. Section V presents a three-way decomposition of the liquidity measure differences. Section VI develops a simple errors-in-variables model to test how well simulated liquidity measure differences match actual liquidity measure differences. Section VII discusses alternative solutions. Section VIII presents empirical results for alternative MTAQ solutions. Sections IX and X analyze whether methodology affects research inferences. Specifically, Section IX analyzes the case of Hybrid Market reform and Section X analyzes order routing decisions. Section XI analyzes the impact on other research areas. Section XII discusses the ultimate breakdown of the NBBO and our proposed replacement concept of RBBOs. Section XIII concludes.

I. The Institutional Setting

Figure 1 illustrates the information flows in Tape A (NYSE-listed) and Tape B (AMEX- and regionally listed) securities. On the left side, we see that there are N market centers, where a market center is defined as an exchange, market maker, or broker-dealer. For convenience, we designate the N^{th} market center as the NYSE. Each market center has a matching engine that arranges trades by matching and/or recording matches of liquidity-demanding orders

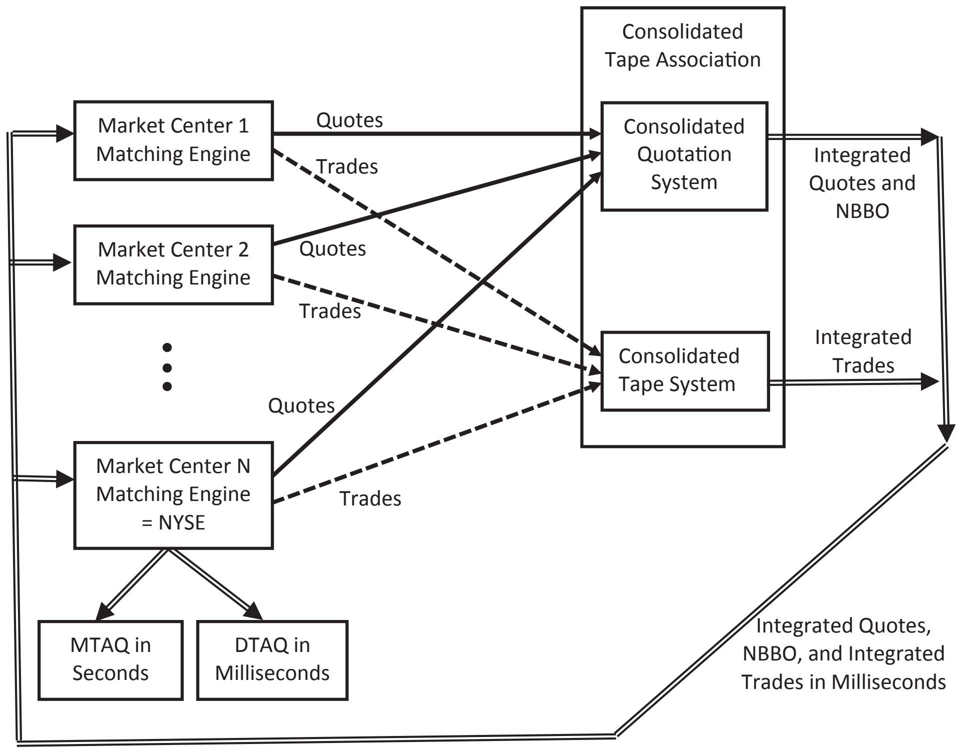

Figure 1. Information flows in Tape A (NYSE-listed) and Tape B (AMEX- and regionally listed) securities.

with liquidity-supplying orders and/or dealers, and updates quotes as appropriate. Trades and quotes from each market center are sent to the Consolidated Tape Association (CTA), which is the SIP for Tapes A and B. Operating out of a data center in Brooklyn, the CTA's Consolidated Quotation System (CQS) integrates the quotes from all market centers and computes the NBBO. Operating out of a data center in lower Manhattan, the CTA's Consolidated Tape System (CTS) integrates the trades from all market centers. In the moment in which the corresponding information is processed by the two systems, an official time stamp is added, which is recorded to the millisecond. From there, the integrated quotes, NBBO, and integrated trades are broadcast by IP Multicast back to all of the Market Centers, including the NYSE. Finally, the NYSE warehouses the CQS and CTS data feed into the DTAQ and MTAQ databases.

The process works in an analogous manner for Tape C (NASDAQ-listed) securities. The substitutions are: (1) UTP Committee replaces CTA, (2) UTP Quote Data Feed replaces CQS, and (3) UTP Trade Data Feed replaces the CTS.

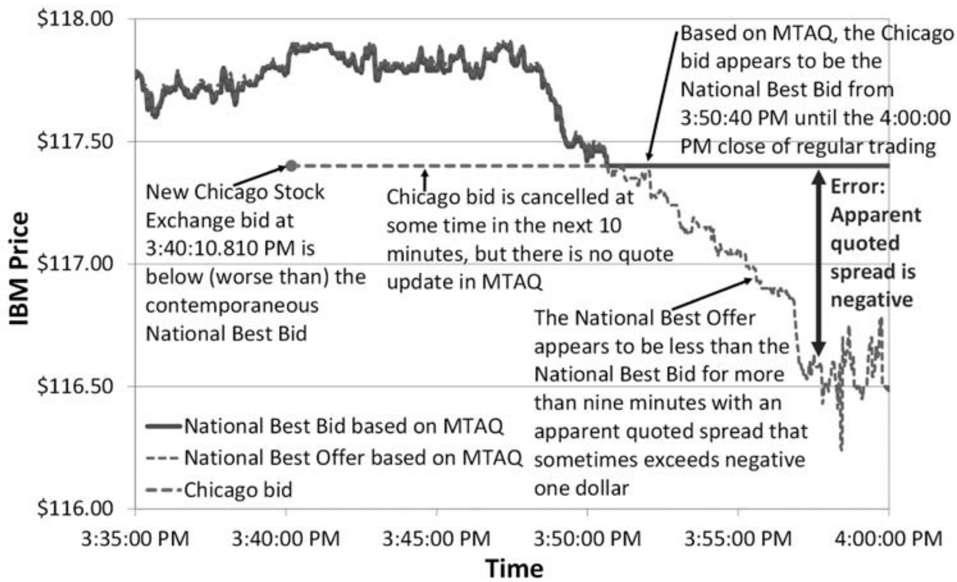

Figure 2. Example of a canceled quote in IBM on April 1, 2008 between 3:40 and 4:00 p.m.

The SIPs are not fooled by canceled quotes[12] or withdrawn quotes. Figure 2 provides an extreme example of a canceled quote. On April 1, 2008 at 3:40:10.810 p.m., the CHX quotes a new IBM bid price of $117.40. At this point in time, the bid price is more than $0.40 below (worse than) the contemporaneous National Best Bid. The Chicago bid is canceled some time in the next 10 minutes, but there is no Chicago quote update in MTAQ before the 4:00 p.m. close of regular trading. Over the next 10 minutes, the bid prices from other exchanges drift lower and then at 3:50:39.011 p.m. drop below $117.40. A researcher computing the IBM NBBO based on MTAQ would conclude that the Chicago bid price of $117.40 sets the National Best Bid from 3:50:40 p.m. until the 4:00 p.m. close of regular trading. In the last nine minutes of the regular trading day, the National Best Offer declines by approximately a dollar. Figure 2 shows the error that results. The National Best Offer appears to be less than the National Best Bid for more than nine minutes with an apparent quoted spread that sometimes exceeds negative one dollar. However, the SIP knows that the Chicago quote has been canceled. The Complete Official NBBO shows *both* the National Best Bid and National Best Offer declining in tandem over the last nine minutes. Over the last nine minutes of regular trading, the

[12] Tantalizingly, the DTAQ Quotes file adds a field called "Quote Cancel/Correction." According to the Daily TAQ Client Specification Version 1.0 documentation, this variable is supposed to take on a value of "A," "B," or "C," where "B" means "Cancel Quote/Cancel Price Indication/Cancel Trading Range Indication." Unfortunately, this variable is always blank in our sample.

time-weighted average of the dollar quoted spread[13] is −44.7 cents based on MTAQ versus 1.5 cents based on DTAQ.

A key distinction is that withdrawn quotes are *directly observable*, whereas canceled quotes are *not*. According to the NYSE TAQ User Guide,[14] when a bid and/or ask quote is withdrawn during normal market hours, the bid and/or ask price are set equal to zero or missing. A withdrawn quote can be one-sided, where the bid (ask) is withdrawn, but the ask (bid) is still valid, or two-sided, where both the bid and ask are withdrawn. A withdrawn quote is recorded as a regular quote update in MTAQ. The researcher can properly account for a withdrawn bid and/or ask quote by treating that particular bid and/or ask quote as missing until a new bid and/or ask quote is displayed by that exchange or market maker. By contrast, we do not directly observe canceled quotes because by definition a canceled quote is when the limit order underlying a bid or ask quote is canceled, but *no quote update is recorded in MTAQ*. In other words, it is an *error* in the MTAQ data set. The existence of canceled quotes can only be inferred indirectly in extreme cases, such as in Figure 2. In this case, it is obvious that the CHX bid price of $117.40 was canceled at some point because we see such an extreme outcome, namely, that the apparent quoted spread exceeds negative one dollar. But, when using MTAQ, we do not know when the quote was canceled because there is *no quote update recorded in MTAQ*. While the indirect inference is obvious in extreme cases, it is not clear when or if a quote has been canceled in moderate cases. Since a canceled quote is not directly observable, nor can it be reliably inferred in a moderate case, there is no cure for this problem. Since we cannot cure the problem, it will turn out that the only thing we can do is treat its worst symptom, namely, by throwing out all cases of NBBO crossed and locked markets.

II. Liquidity Measures

The liquidity measures that we analyze are standard measures of market quality. Our first category of liquidity measures evaluates trade location. Specifically, we determine the percentage of trades that are at, inside, and outside the NBBO and the percentage of trades that occur when a market is experiencing the economically nonsensical conditions of being crossed or locked. The k^{th} trade at price P_k is considered *At the NBBO* when $P_k = A_k$ or $P_k = B_k$, where A_k is the National Best Ask and B_k is the National Best Bid assigned to the k^{th} trade by a particular technique. A trade is considered *Inside the NBBO* when $A_k > P_k > B_k$ and *Outside the NBBO* when $P_k > A_k$ or $P_k < B_k$. The more a particular technique misaligns trades and quotes, the more the apparent percentage of *Outside the NBBO* trades will be elevated and so we focus on this metric, rather than *At the NBBO* or *Inside the NBBO*. A market observes a *Crossed NBBO* when the National Best Ask is strictly less than the National Best Bid, $A_k < B_k$, and the market observes a *Locked NBBO* when the National

[13] See equation (2) below.

[14] The NYSE TAQ User Guide can be found on the WRDS website.

Best Ask is equal to the National Best Bid, $A_k = B_k$. A crossed market is a more severe condition than a locked market because the former represents an arbitrage opportunity, whereas the latter does not. Thus, we focus on the frequency of a crossed market.

Our second category of liquidity measures evaluates the quoted and effective spread. For a given time interval s, the dollar and percent quoted spread are defined as

$$\textit{Dollar Quoted Spread}_s = A_s - B_s, \tag{1}$$

$$\textit{Percent Quoted Spread}_s = \frac{A_s - B_s}{M_s}, \tag{2}$$

where A_s is the National Best Ask and B_s is the National Best Bid assigned to time interval s by a particular technique and M_s is the midpoint, which is the average of B_s and A_s. Aggregating over the sample period, a stock's *Dollar (Percent) Quoted Spread* is the time-weighted average of *Dollar (Percent) Quoted Spread*$_s$ computed over all time intervals. For a given stock, the dollar and percent effective spread on the k^{th} trade is defined as

$$\textit{Dollar Effective Spread}_k = 2D_k(P_k - M_k), \tag{3}$$

$$\textit{Percent Effective Spread}_k = \frac{2D_k(P_k - M_k)}{M_k}, \tag{4}$$

where D_k is an indicator variable that equals $+1$ if the k^{th} trade is a buy and -1 if the k^{th} trade is a sell and M_k is the midpoint of the NBBO quotes assigned to the k^{th} trade by a particular technique. Aggregating over the sample period, a stock's *Dollar (Percent) Effective Spread* is the dollar-volume-weighted average of *Dollar (Percent) Effective Spread*$_k$ computed over all trades.

Our third category of liquidity measures considers the realized spread and price impact. The dollar realized spread is the temporary component of the dollar effective spread. For a given stock, the dollar realized spread on the k^{th} trade is defined as

$$\textit{Dollar Realized Spread}_k = 2D_k(P_k - M_{k+5}), \tag{5}$$

$$\textit{Percent Realized Spread}_k = \frac{2D_k(P_k - M_{k+5})}{M_k}, \tag{6}$$

where M_{k+5} is the midpoint five minutes after the midpoint M_k. Aggregating over the sample period, a stock's *Dollar (Percent) Realized Spread* is the dollar-volume-weighted average of the *Dollar (Percent) Realized Spread*$_k$ computed over all trades. The dollar price impact is the permanent component of the dollar effective spread. For a given stock, the dollar price impact on the k^{th} trade is defined as

$$Dollar\ Price\ Impact_k = 2D_k(M_{k+5} - M_k), \tag{7}$$

$$Percent\ Price\ Impact_k = \frac{2D_k(M_{k+5} - M_k)}{M_k}. \tag{8}$$

Aggregating over the sample period, the *Dollar (Percent) Price Impact* is the dollar-volume-weighted average of *Dollar (Percent) Price Impact*$_k$ computed over all trades.

There are three popular trade-typing conventions for determining whether a given trade is a liquidity-demander "buy" or liquidity-demander "sell," which, in turn, determines whether D_k is $+1$ or -1. Using the Lee and Ready (1991, LR) convention, a trade is a buy when $P_k > M_k$, a sell when $P_k < M_k$, and the tick test is used when $P_k = M_k$. The tick test specifies that a trade is a buy (sell) if the most recent prior trade at a different price was at a lower (higher) price than P_k. Using the Ellis, Michaely, and O'Hara (2000, EMO) convention, a trade is a buy when $P_k = A_k$, a sell when $P_k = B_k$, and the tick test is used otherwise. Using the Chakrabarty et al. (2006, CLNV) convention, a trade is a buy when $P_k \in [0.3B_k + 0.7A_k, A_k]$, a sell when $P_k \in [B_k, 0.7B_k + 0.3A_k]$, and the tick test is used otherwise.[15] We consider three versions of dollar realized spread and three versions of dollar price impact based on these three trade-typing conventions.

Our fourth category of liquidity measures evaluates dollar and share bid and ask depth. The dollar (share) ask depth is the dollar (share) amount available at the best ask quote from the exchange or market maker with the largest size quoted at that price. In the benchmark DTAQ NBBO, depth is also the largest size based on price priority and then size priority. The dollar (share) bid depth is computed analogously.

Our final liquidity measure is absolute order imbalance, defined as

$$Absolute\ Order\ Imbalance = \left|\frac{Buys - Sells}{Buys + Sells}\right|, \tag{9}$$

where *Buys* and *Sells* are the number of buys and number of sells, respectively, based on a particular technique and based on the three trade-typing conventions. Easley et al. (2008) and Kaul, Lei, and Stoffman (2008) show that absolute order imbalance is an alternative measure of the Probability of Informed trading (PIN). Absolute order imbalance has two advantages over PIN. It can be computed over relatively short periods of time and it does not have the numerical overflow problems that are often encountered when computing the PIN log-likelihood function using large numbers of buys or sells per period.

[15] The economic rationale for the three conventions only makes sense in normal markets (when the National Best Offer is greater than the National Best Bid). If the market is locked or crossed, then we ignore the three different rules and apply the tick test exclusively using all three conventions.

III. Data

We use the DTAQ and MTAQ data sets. Because of the high price of the DTAQ data, we purchase a limited sample from April 1, 2008 to June 30, 2008. Due to computational limitations, we select a random sample of traded stocks. Following the methodology of Hasbrouck (2009), a stock must meet five criteria to be eligible: (1) it must be a common stock; (2) it must be present on the TAQ master file for the first and last date of the sample period; (3) it must have a primary listing on the NYSE, AMEX, or NASDAQ; (4) it cannot change primary exchange, ticker symbol, or CUSIP code during the sample period; and (5) it must be listed in the Center for Research in Security Prices (CRSP) database.

Starting with eligible firms, we first divide them into five quintiles based on number of trades per day, and then randomly select 20 firms from each quintile. This yields a random sample of 100 traded stocks, which results in 34 million trades and 351 million quotes over the sample period. In the base case of MTAQ with no adjustments, we then apply the following screens to the trade and quote data. Only quotes/trades during normal market hours (quotes between 9:00 a.m. and 4:00 p.m. and trades between 9:30 a.m. and 4:00 p.m.) are considered. For each exchange or market maker, we delete cases in which the bid of one exchange or market maker is greater than or equal to the ask of the *same* exchange or market maker. If the quoted spread is greater than \$5.00 and the bid (ask) price is less (greater) than the previous midpoint − \$2.50 (previous midpoint + \$2.50), then the bid (ask) is not considered. The quote condition must be normal, which excludes cases in which trading has been halted.[16] We delete bid (ask) quotes that have a bid (ask) price or bid (ask) size that is set to zero or a missing value.

When we consider withdrawn quotes, we slightly alter the screens described above. First, we delete cases in which the bid of one exchange or market maker is greater than or equal to the ask of the same exchange or market maker only if both the bid and the ask are greater than zero. Specifically, if an observation is crossed because the bid > 0 and the ask $= 0$, we assume that the bid is valid and the ask has been withdrawn. Second, we delete cases in which the quoted spread is greater than \$5.00 only if both the bid and the ask are greater than zero. Therefore, if the spread is greater than \$5.00 because the bid $= 0$ and the ask > 0, we assume that the ask is valid and the bid has been withdrawn. Third, when a bid (ask) price or bid (ask) size equals zero or a missing value, we assume that the bid (ask) quote has been withdrawn and set it to missing. See the Internet Appendix for a more detailed description of our data screening process.

We calculate the NBBO across all exchanges and across all market makers for any given millisecond in DTAQ or second in MTAQ. If there are multiple quote updates from a given exchange or market maker within a given millisecond (second), then the last quote update within that millisecond (second) is what we

[16] In DTAQ, we exclude quotes with nonnormal quote conditions (A, B, H, O, R, and W). Similarly, in MTAQ, we exclude quotes with nonnormal quote conditions (4, 7, 9, 11, 13, 14, 15, 19, 20, 27, and 28).

take to be in-force from that exchange or market maker.[17] For DTAQ, we match trades and quotes with a one millisecond lag (i.e., a given trade is matched to the NBBO that was in-force one millisecond earlier).

IV. Overall Liquidity Measure Differences

Table I reports the overall liquidity measure differences between MTAQ and DTAQ. Liquidity measures are calculated using firm-day averages. Asterisks represent differences between MTAQ and DTAQ that are statistically significant at the 1% level. *T*-tests are calculated using standard errors that are robust to clustering along both the firm and the day dimension as described in Thompson (2011).[18] The key comparison is between column (1), which reports liquidity measures based on MTAQ in seconds with no adjustments (i.e., the standard way that most researchers currently compute them), and column (4), which reports liquidity measures based on DTAQ in milliseconds with the NBBO file.

Panel A reports trade location statistics. The frequency of *Outside the NBBO* trades is 26.3% for MTAQ versus 3.3% for DTAQ, which is eight times larger using MTAQ. The frequency of *Crossed NBBO* is 18.3% versus 0.5%. Since a crossed NBBO is equivalent to a negative quoted spread, this implies that *Percent Quoted Spread* goes negative 37 times more often. Both differences are statistically significantly at the 1% level. Both differences are economically important because frequent trades outside the NBBO using MTAQ suggest that trading is expensive and a frequently crossed NBBO would represent a frequent opportunity to make arbitrage profits.

Panel B reports quoted and effective spreads. The *Percent Quoted Spread* is 0.213% for MTAQ versus 0.399% for DTAQ, which is 47% lower using MTAQ. The lower *Percent Quoted Spread* is consistent with the higher frequency of a crossed NBBO because a crossed NBBO yields a negative percent quoted spread. The *Dollar Quoted Spread* is 73% lower using MTAQ. The *Percent Effective Spread* is 0.580% for MTAQ versus 0.377% for DTAQ, which is 54% higher using MTAQ. The *Dollar Effective Spread* is more than twice as large using MTAQ and the *Percent Effective Spread* is greater than the *Percent Quoted Spread* 15% more often.

To summarize the results in Panels C and D, using MTAQ: (1) *Percent Realized Spread* is 3% to 27% larger using all three conventions, (2) *Percent Price Impact* is 68% to 131% larger using all three conventions, and (3) the dollar depth and share depth measures show very minimal differences. To summarize the table, there are large overall differences in liquidity measures, including *Outside the NBBO*, *Crossed NBBO*, *Percent (Dollar) Quoted Spread*, *Percent (Dollar) Effective Spread*, frequency with which the *Percent Effective Spread* is

[17] For the Interpolated Time technique described in Section VII, all quotes within a second are utilized.

[18] Unless otherwise stated, all significance tests throughout this paper are based on standard errors that are robust to clustering along both the firm and the day dimensions.

Table I

Overall Liquidity Measure Differences between MTAQ and DTAQ and a Three-Way Decomposition

In this table, we provide overall liquidity measure differences between the MTAQ and DTAQ data sets (column (4) versus (1)). We then provide a three-way decomposition determining how much of the liquidity measure differences are due to: withdrawn quotes (column (2) versus (1)), millisecond time stamps (column (3) versus (2)), and other National Best Bid and Offer (NBBO) problems (column (4) versus (3)). The sample spans April to June 2008 inclusive and consists of 100 randomly selected stocks, resulting in 33,754,779 trades. Liquidity measures are calculated using firm-day averages. * indicates statistically different from column (4) at the 1% level. Standard errors are robust to clustering along both the firm and the day dimensions.

	MTAQ in Seconds, with No Adjustments (1)	MTAQ in Seconds, with Withdrawn Quotes Accounted for (2)	DTAQ in Milliseconds, with Withdrawn Quotes Accounted for, without NBBO File (3)	DTAQ in Milliseconds, with NBBO File (4)
Panel A: Trade Location				
At the NBBO	61.0%*	67.8%*	65.6%*	71.0%
Inside the NBBO	12.7%*	14.6%*	23.2%*	25.7%
Outside the NBBO	26.3%*	17.6%*	11.2%*	3.3%
Locked NBBO	3.0%*	1.7%	2.4%*	1.7%
Crossed NBBO	18.3%*	9.9%*	8.1%*	0.5%
Panel B: Quoted and Effective Spreads				
Percent Quoted Spread	0.213%*	0.287%*	0.312%*	0.399%
Dollar Quoted Spread	2.03¢*	4.76¢*	5.13¢*	7.47¢
Percent Effective Spread	0.580%*	0.506%*	0.453%*	0.377%
Dollar Effective Spread	13.39¢*	9.62¢*	8.68¢*	6.46¢
Time%:% Eff Spd >% Quo Spd	49.4%*	52.1%*	39.5%*	34.5%
Panel C: Realized Spread and Permanent Price Impact				
Percent Realized Spread: LR	0.159%	0.142%*	0.175%*	0.154%
Percent Realized Spread: EMO	0.105%*	0.089%	0.103%*	0.083%
Percent Realized Spread: CLNV	0.147%	0.132%*	0.158%*	0.140%
Percent Price Impact: LR	0.516%*	0.365%*	0.280%*	0.223%
Percent Price Impact: EMO	0.396%*	0.351%*	0.292%*	0.235%
Percent Price Impact: CLNV	0.507%*	0.357%*	0.284%*	0.224%
Panel D: Depth Measures				
Dollar Ask Depth (000's)	$13.5	$14.1	$14.0	$14.1
Dollar Bid Depth (000's)	$13.2*	$13.5*	$13.5*	$14.0
Share Ask Depth	531*	561	559	561
Share Bid Depth	529*	535*	538*	567
# of Trades (Millions)	33.8	33.8	33.8	33.8

greater than the *Percent Quoted Spread*, *Percent Realized Spread*, and *Percent Price Impact*.

V. Three-Way Decomposition

What are the causes of the large liquidity measure differences? Table I reports a three-way decomposition to determine how much of the liquidity measure differences is due to the following three factors: (1) withdrawn quotes, (2) millisecond time stamps, and (3) other NBBO problems. Column (2) reports liquidity measures based on MTAQ in seconds with withdrawn quotes accounted for, so the difference between columns (2) and (1) shows the impact of withdrawn quotes. Column (3) reports liquidity measures based on DTAQ in milliseconds with withdrawn quotes accounted for and without the NBBO file, so the difference between columns (3) and (2) shows the impact of millisecond versus second time stamps. Finally, the difference between columns (4) and (3) shows the impact of any remaining NBBO problems. The third factor includes canceled quotes where a limit sell (buy) setting the current ask (bid) is canceled and the exchange or market maker's quote is updated in the DTAQ NBBO file, but not updated in the DTAQ Quote file or in MTAQ.

Examining the row *Outside the NBBO* from right to left, we see an increase due to remaining NBBO problems (3.3% to 11.2%), due to millisecond time stamps (11.2% to 17.6%), and due to withdrawn quotes (17.6% to 26.3%). So, each of the three factors has a significant impact. Similarly, each of the three factors causes an increase in *Crossed NBBO* (0.5% to 8.1% due to remaining NBBO problems to 9.9% due to millisecond time stamps to 18.3% due to withdrawn quotes). Again, from right to left, each of the three factors causes a decrease in *Percent (Dollar) Quoted Spread*, an increase in *Percent (Dollar) Effective Spread*, and an increase in *Percent Price Impact* using all three conventions. Lack of millisecond time stamps and remaining NBBO problems yield an increased frequency of *Percent Effective Spread* being greater than *Percent Quoted Spread*, although the impact of withdrawn quotes is unclear. Further, withdrawn quotes and remaining NBBO problems cause an increase in *Percent Realized Spread* using all three conventions, but the impact of millisecond time stamps is ambiguous. Finally, all three factors have very little impact on the dollar and share depth measures. In summary, all three factors have a significant impact on liquidity measure differences.

VI. A Simple Errors-in-Variables Model

All three factors analyzed above imply that MTAQ with no adjustments contains errors. To assess if this is what is really going on, we develop a simple errors-in-variables model. Specifically, we represent the MTAQ NBBO as being the DTAQ NBBO plus errors.[19] We run a simulation of this model and compute

[19] We do not mean to suggest that DTAQ is the unvarnished "truth"—just that there are additional errors in the MTAQ with no adjustments.

the same liquidity measures as above. The goal of this calibration test is to see whether we can reverse engineer the observed results in Table I. That is, can a simple error-in-variables model yield simulated liquidity measure differences that have the same sign as the actual liquidity measure differences in Table I? Even more challenging, can the model yield simulated liquidity measures that are *roughly* of the same magnitude as the actual liquidity measures in Table I? If so, then the results would bolster our confidence that we truly understand the problem.

Let NBO_k^i, NBB_k^i, OD_k^i, and BD_k^i be the National Best Offer, National Best Bid, Offer Depth, and Bid Depth, respectively, in-force at the time of the k^{th} trade as computed from the i^{th} data set, where $i = M$ refers to MTAQ and $i = D$ refers to DTAQ. The DTAQ NBBO is assumed to be measured with no error based on the midpoint plus or minus the half-spread,

$$NBO_k^D = M_k + 0.5^* S_k, \tag{10}$$

$$NBB_k^D = M_k - 0.5^* S_k, \tag{11}$$

where M_k is the midpoint and S_k is a random spread at the time of the k^{th} trade; S_k is assumed to have a positive mean ($E[S_k] > 0$), but occasional negative spreads (crossed markets) are permitted. We assume that the MTAQ NBBO is the DTAQ NBBO plus errors as given by

$$NBO_k^M = NBO_k^D + \varepsilon_k^O, \tag{12}$$

$$NBB_k^M = NBB_k^D + \varepsilon_k^B, \tag{13}$$

$$OD_k^M = OD_k^D + \varepsilon_k^{OD}, \tag{14}$$

$$BD_k^M = BD_k^D + \varepsilon_k^{BD}, \tag{15}$$

where ε_k^O, ε_k^B, ε_k^{OD}, and ε_k^{BD} are independent errors. Price priority tends to censor the way that individual exchange errors translate into NBBO errors.[20] Therefore, we assume that the mean of the *NBO* error is negative (i.e., $E[\varepsilon_k^O] < 0$), and in the mirror image case, that the mean of the *NBB* error is positive (i.e., $E[\varepsilon_k^B] > 0$). Price priority has no influence on depth errors, so we assume that these errors have zero mean (i.e., $E[\varepsilon_k^{OD}] = E[\varepsilon_k^{BD}] = 0$).

Trades are generated relative to the DTAQ NBBO with no errors. The simulated results are computed using the same trade prices, but different NBBOs. For the simulation, we analyze 100,000 realizations of the set of random

[20] Consider an MTAQ error in the offer price of an individual exchange. If a positive error causes that exchange's offer price to be too high, then price priority will cause other exchanges to set the National Best Offer. Conversely, if a negative error that causes the individual exchange's offer price to be too low, then price priority will cause that exchange to set the National Best Offer. Thus, positive errors tend to be censored for the National Best Offer and negative errors tend to be censored for the National Best Bid.

Table II

Actual versus Simulated Liquidity Measures for MTAQ and DTAQ

In this table, we provide actual and simulated liquidity measure differences between the MTAQ and DTAQ data sets. The simulation is based on a simple errors-in-variables model, where the MTAQ NBBO is the DTAQ NBBO plus errors. Trades are generated relative to the DTAQ NBBO with no errors. The simulated results are computed using the same trade prices, but different NBBOs. The simulation draws 100,000 realizations for the set of random variables. The actual results are based on a data sample that spans April to June 2008 inclusive and consists of 100 randomly selected stocks, resulting in 33,754,779 trades.

	MTAQ Actual (1)	DTAQ Actual (2)	MTAQ Simulated (3)	DTAQ Simulated (4)
Panel A: Trade Location				
At the NBBO	61.0%	71.0%	47.9%	70.3%
Inside the NBBO	12.7%	25.7%	27.9%	26.4%
Outside the NBBO	26.3%	3.3%	24.1%	3.3%
Locked NBBO	3.0%	1.7%	1.9%	1.7%
Crossed NBBO	18.3%	0.5%	21.7%	0.5%
Panel B: Quoted and Effective Spreads				
Percent Quoted Spread	0.213%	0.399%	0.150%	0.377%
Dollar Quoted Spread	2.03¢	7.47¢	3.00¢	7.55¢
Percent Effective Spread	0.580%	0.377%	0.518%	0.278%
Dollar Effective Spread	13.39¢	6.46¢	10.38¢	5.57¢
Time%:% Eff Spd >% Quo Spd	49.4%	34.5%	17.6%	3.1%
Panel C: Realized Spread and Permanent Price Impact				
Percent Realized Spread: LR	0.159%	0.154%	0.086%	0.104%
Percent Realized Spread: EMO	0.105%	0.083%	0.096%	0.104%
Percent Realized Spread: CLNV	0.147%	0.140%	0.089%	0.104%
Percent Price Impact: LR	0.516%	0.223%	0.432%	0.174%
Percent Price Impact: EMO	0.396%	0.235%	0.193%	0.174%
Percent Price Impact: CLNV	0.507%	0.224%	0.388%	0.174%
Panel D: Depth Measures				
Dollar Ask Depth (000's)	$13.5	$14.1	$11.2	$11.2
Dollar Bid Depth (000's)	$13.2	$14.0	$11.2	$11.2
Share Ask Depth	531	561	560	560
Share Bid Depth	529	567	560	560
# of Trades (Millions)	33.8	33.8	33.8	33.8

variables. Further details of the simple model are discussed in the Internet Appendix for this paper.

Table II provides actual versus simulated liquidity measures for MTAQ and DTAQ.[21] In Panel A, *Outside the NBBO*, *Locked NBBO*, and *Crossed NBBO* have the same sign (MTAQ > DTAQ) for the simulated results as the actual

[21] For convenience, columns (1) and (2) in Table II reproduce columns (1) and (4) in Table I.

results, and the magnitudes are roughly similar as well.[22] In Panel B, *Percent Quoted Spread* and *Dollar Quoted Spread* have the same sign (MTAQ < DTAQ), *Percent Effective Spread* and *Dollar Effective Spread*, and percent of time that *Percent Effective Spread* is greater than *Percent Quoted Spread* have the same sign (MTAQ > DTAQ), and all but the last of these magnitudes are roughly similar.[23] In Panel C, *Percent Realized Spread* has the opposite sign for all three conventions (MTAQ < DTAQ), *Percent Price Impact* has the same sign for all three conventions (MTAQ > DTAQ), and many of the magnitudes are roughly similar. In Panel D, all of the dollar and share depth measures have the same sign (MTAQ = DTAQ) and the magnitudes are roughly similar. Essentially, depth levels are unbiased.

In summary, we find that a simple errors-in-variables model can generate nearly all of the simulated liquidity measure differences between DTAQ and MTAQ with the same sign as the actual liquidity measure differences and with most of the magnitudes being roughly similar. This is strong evidence that the actual liquidity measure differences are driven by errors in the MTAQ NBBO that are not present in the DTAQ NBBO.

VII. Alternative Solutions

Now we turn to possible solutions to eliminate or mitigate the liquidity measure differences. One possible solution is to purchase the expensive DTAQ database, which includes the NBBO file. This allows researchers to construct the Complete Official NBBO.[24] Table I shows that DTAQ with the NBBO file has the lowest frequency of *Crossed NBBO* (which potentially represents arbitrage opportunities) at 0.5% and the lowest frequency of *Outside the NBBO* (which is elevated when there is a misalignment of trade and quotes) at 3.3%.[25] Hence, DTAQ with the NBBO file is very credible as the best representation

[22] By reverse engineering, a large majority of the probability of the offer error and the bid error is on zero. This is necessary in order to generate reasonable values for *Outside the NBBO* and *Crossed NBBO*.

[23] By reverse engineering, the nonzero realizations of the offer errors and the bid errors are relatively large in absolute value. This is necessary to generate the large differences in *Percent Effective Spread* and *Dollar Effective Spread*. See the Internet Appendix for this paper for more details.

[24] In certain instances, when a single exchange has both the best bid and the best offer, then the official SIP NBBO quote is recorded in the DTAQ Quotes file, not in the DTAQ NBBO file. When this happens, the field "National BBO Ind" is set equal to 1 (for NYSE, AMEX, and regional stocks) or else the field "NASDAQ BBO Ind" is set equal to 4 (for NASDAQ stocks). The DTAQ NBBO file is therefore incomplete because it is missing these records. We construct the Complete Official NBBO by adding these single-exchange NBBO quotes from the DTAQ Quotes file to the DTAQ NBBO file. Specifically, we interweave these records by Symbol, Date, Time, and Sequence Number.

[25] The Internet Appendix considers matching DTAQ trades to NBBO quotes that are lagged by 100 milliseconds versus by one millisecond. We find that the differences between a 100 millisecond lag and a one millisecond lag are very small. We recommend a one millisecond lag (e.g., matching a DTAQ trade to the NBBO that is in-force one millisecond earlier).

of the market. By definition, purchasing the DTAQ database completely eliminates any differences compared to itself. So, this is our first-best solution.

In the event a researcher is financially constrained to using the relatively cheap MTAQ database, we next consider several alternative approaches to mitigate the differences in liquidity measures. One solution is to adjust for withdrawn quotes. The MTAQ Quotes file often shows a zero or a missing value as the bid price, ask price, bid depth, or ask depth. Current research practice treats a zero or missing value in the quote as an error and throws the quote away.[26] In this case, the prior quote by that exchange or market maker is still considered to be valid and might enter the NBBO.

To characterize the frequency of a prior quote before a zero or missing value quote entering the NBBO, we examined 10 stocks on April 1, 2008 and treated the zero or missing value quote as an error. We identify 9,553 instances in which the prior quote before the zero or missing value quote set the NBBO out of a total of 63,818 NBBO quotes. These prior quotes set the NBBO for a time-weighted median of 10.7% of the trading day, which is equivalent to 42 minutes per trading day.

However, the NYSE (2008), page 26 TAQ 3 Users Guide, says that a zero in the bid and/or the ask price represents an exchange or market maker *withdrawing* their previously established bid and/or ask quote. Under this interpretation, there is a moment during which there is no bid and/or ask quote for that exchange or market maker and thus the prior bid and/or ask quote is *invalid*. Thus, the correct approach is to treat the bid and/or ask quote for that exchange or market maker as missing until a new bid and/or ask quote is made by that exchange or market maker. The large differences between columns (2) and (1) in Table I show that failing to correctly treat withdrawn quotes is a major source of liquidity measure differences. Since researchers can directly observe instances in which the quoted bid and/or ask price is zero or missing and since correct treatment of withdrawn quotes eliminates a major problem, researchers using MTAQ should always do so.

Taking the adjustment for withdrawn quotes as given, we consider additional possible MTAQ solutions. In particular, we consider three quote timing techniques: (1) Prior Second, (2) Same Second, and (3) Interpolated Time. Prior Second matches a trade in second s to the calculated NBBO quotes that are in-force in the *prior* second $s-1$. Same Second matches a trade in second s to the calculated NBBO quotes that are in-force during the *same* second s.[27]

[26] To document current research practices, we surveyed all articles published in the *Journal of Finance* (JF), *Journal of Financial Economics* (JFE), and *Review of Financial Studies* (RFS) from 2006 to 2011 and identified 28 articles that used MTAQ data and none that used DTAQ data. Four articles explicitly state that they delete quotes that contain a zero or missing value. The other 24 articles do not explicitly say what they do about such quotes, but presumably they filter such observations out as well, since allowing an ask price equal to zero to be treated as valid would have a large effect on the NBBO. None of the 28 articles mentions treating a quote containing a zero or missing value as a withdrawn quote.

[27] In the JF-JFE-RFS survey described earlier, we find significant variation in the quote timing techniques used. Seven articles use Prior Second, three articles use Same Second, five articles use the quote five seconds earlier, and the rest provide no information on quote timing.

We introduce a new, potentially more accurate, quote timing method that we call Interpolated Time. Suppose that the MTAQ data set lists I trades and J quotes as occurring in second s. We do not know in which millisecond those trades or quotes occurred, but we do know the *order of the trades* and the *order of the quotes* in MTAQ.[28] Interpolated Time takes advantage of that ordering to make an educated guess about in which millisecond each event happened through a process of simple interpolation. Specifically, we assume a priori that trades and quotes are each uniformly distributed over the second. Based on this assumption, the i^{th} trade in second s is assigned an interpolated trade time of

$$s + \frac{2i - 1}{2I}, \quad i = 1, 2, \ldots, I. \tag{16}$$

This formula assigns a time gap of $\frac{1}{I}$ of a second between each trade, a time gap of $\frac{1}{2I}$ of a second from the beginning of the second s to the first trade, and a time gap of $\frac{1}{2I}$ of a second from the last trade to the end of the second s. Similarly, the j^{th} quote in second s is assigned an interpolated quote time of

$$s + \frac{2j - 1}{2J}, \quad j = 1, 2, \ldots, J. \tag{17}$$

Similarly, this formula assigns a time gap of $\frac{1}{J}$ of a second between each quote, a time gap of $\frac{1}{2J}$ of a second from the beginning of the second s to the first quote, and a time gap of $\frac{1}{2J}$ of a second from the last quote to the end of the second s. The j^{th} quote is presumed to have occurred at the interpolated quote time and the usual NBBO computation across all exchanges and all market makers is updated at that time. The i^{th} trade is presumed to have occurred at the interpolated trade time and is matched to the NBBO quotes that were in-force one millisecond earlier.

Figure 3 provides a timeline illustrating how Interpolated Time assigns times within the second $s = 9{:}45{:}23$ in the case of $I = 4$ trades and $J = 5$ quotes. Applying equation (16), the four trades are assigned interpolated trade times of 9:45:23$\frac{1}{8}$, 9:45:23$\frac{3}{8}$, 9:45:23$\frac{5}{8}$, and 9:45:23$\frac{7}{8}$. Applying equation (17), the five quotes are assigned interpolated quote times of 9:45:23$\frac{1}{10}$, 9:45:23$\frac{3}{10}$, 9:45:23$\frac{5}{10}$, 9:45:23$\frac{7}{10}$, and 9:45:23$\frac{9}{10}$. Consider the third trade. It is presumed to have occurred at the interpolated trade time of 9:45:23$\frac{5}{8}$ = 9:45:23.625 and it is matched to the NBBO presumed to be in-force one millisecond earlier at 9:45:23.624. The time 9:45:23.624 is after the third quote's interpolated quote time of 9:45:23$\frac{5}{10}$ = 9:45:23.500, but before the fourth quote's interpolated quote time of 9:45:23$\frac{7}{10}$ = 9:45:23.700. Thus, the third trade is matched to the NBBO presumed to be in-force at 9:45:23.624 based on the currently active quote at

[28] The order of the trade records and the order of the quote records are identical in DTAQ and MTAQ.

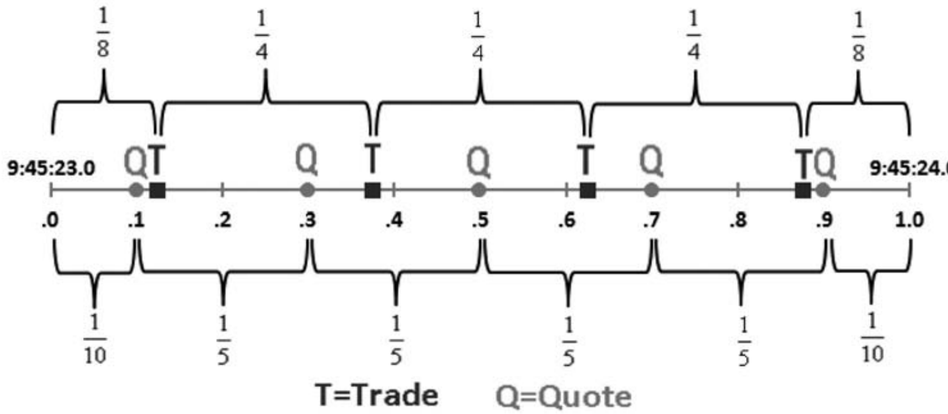

Figure 3. Timeline showing how Interpolated Time assigns times within a second. Suppose there are four trades and five quotes in the second $s = 9{:}45{:}23$. Based on equation (16), the four trades are assigned interpolated trade times of $9{:}45{:}23\frac{1}{8}$, $9{:}45{:}23\frac{3}{8}$, $9{:}45{:}23\frac{5}{8}$, and $9{:}45{:}23\frac{7}{8}$. Based on equation (17), the five quotes are assigned interpolated quote times of $9{:}45{:}23\frac{1}{10}$, $9{:}45{:}23\frac{3}{10}$, $9{:}45{:}23\frac{5}{10}$, $9{:}45{:}23\frac{7}{10}$, and $9{:}45{:}23\frac{9}{10}$. The third trade is presumed to have occurred at the interpolated trade time of $9{:}45{:}23\frac{5}{8} = 9{:}45{:}23.625$ and is matched to the NBBO presumed to be in-force one millisecond earlier at 9:45:23.624. The time 9:45:23.624 is after the third quote's interpolated quote time of $9{:}45{:}23\frac{5}{10} = 9{:}45{:}23.500$, but before the fourth quote's interpolated quote time of $9{:}45{:}23\frac{7}{10} = 9{:}45{:}23.700$. Thus, the third trade is matched to the NBBO presumed to be in-force at 9:45:23.624 based on the currently active quote at that moment by all exchanges and market makers, which is accumulated from the third quote by exchange X and all earlier quotes by exchanges Y, Z, . . . , but *excluding* the fourth and fifth quotes.

that moment by all exchanges and market makers, which is accumulated from the third quote by exchange X and all earlier quotes by exchanges Y, Z, . . . , but *excluding* the fourth and fifth quotes.[29]

Finally, we also consider excluding observations where the calculated NBBO is crossed or locked. To be clear, many researchers throw away observations where the bid of one exchange or market maker is greater than or equal to the ask of the *same* exchange or market maker (e.g., when NYSE bid ≥ NYSE ask). But our clean-up technique goes a step further and throws away observations when the National Best Bid from *any* exchange or market maker is greater than or equal to the National Best Offer from *any* exchange or market maker (e.g., when the National Best Bid quoted by the CHX is greater than or equal to the National Best Offer quoted by the National Stock Exchange). A small

[29] Admittedly, a downside of Interpolated Time is that it uses future information, extending as much as one second into the future. However, exactly the same point applies to the established Same Second technique, which also uses future information extending as much as one second into the future. In both cases, a trade at the beginning of the second (s.000) may be matched with NBBO quotes based on information updated as late as the end of the second (s.999).

minority of researchers exclude NBBO crossed and/or locked markets.[30] The rationale for excluding NBBO locked and crossed markets is that, according to market microstructure theory, the offer price must always be greater than the bid price. As described in Section I, we conjecture that NBBO computations that yield locked or crossed markets are often caused by quotes that are canceled but no revision has been made to the quotes database.[31]

VIII. Empirical Results for Alternative MTAQ Solutions

Table III reports liquidity measure differences for alternative MTAQ solutions. With withdrawn quotes already accounted for, we analyze MTAQ using three quote timing rules (Prior Second, Same Second, and Interpolated Time) and also add the exclusion of NBBO crossed and locked cases to Interpolated Time. DTAQ results are also reported for comparison.

A. Trade Location

In Panel A, *Outside the NBBO* is highest using Prior Second (17.6%), declines using Same Second (15.4%), declines further using Interpolated Time (13.7%), and declines even further when also Excluding NBBO Crossed and Locked (5.6%). The latter is the closest that researchers can get to DTAQ (with a 3.3% *Outside the NBBO*) using MTAQ, but the remaining difference is still significant. Also, in Panel A, *Crossed NBBO* stays the same using Prior Second and Same Second (9.9%), increases slightly using Interpolated Time (10.2%), and drops to 0.0% by definition when also Excluding NBBO Crossed and Locked. The latter is the closest that researchers can get to DTAQ (with a 0.5% *Crossed NBBO*) using MTAQ, but again the remaining difference is still significant.

B. Quoted and Effective Spreads

In Panel B, *Percent Quoted Spread* is nearly the same using the three quote timing rules, ranging from 0.285% to 0.287%. All of these values are significantly *lower* than DTAQ (0.399%). Intuitively, these large differences are related to the frequency of negative quoted spreads under crossed markets and zero quoted spreads under locked markets. Excluding NBBO Crossed and Locked (0.401%) gets researchers very close to DTAQ and the difference is insignificant.

The *Percent Effective Spread* is close using the three quote timing rules, ranging from 0.450% to 0.506%. All of these values are significantly *higher* than

[30] In the JF-JFE-RFS survey described earlier, 10 articles out of the 28 that use MTAQ data explicitly mention that they drop all observations where the *same* exchange is crossed and/or locked. Only two articles explicitly state that they delete NBBO crossed and/or locked observations.

[31] In the Internet Appendix, we analyze an additional possible solution called Duration Limit Control (DLC). This technique, which is proposed by Jain, Upson, and Wood (2008), drops all quotes older than a one-minute duration when computing the NBBO. We find that DLC does better than nothing, but worse than the alternatives. Thus, we conclude that DLC should not be used.

Table III

Alternative MTAQ Solutions: Quote Timing Rules and Excluding NBBO Crossed and Locked

In this table, we present an analysis of how alternative MTAQ solutions affect liquidity measures. Trade locations, cost of trading measures, and depths are shown under alternative treatments. In the base case, withdrawn quotes are already accounted for. Three quote timing rules are compared: Prior Second, Same Second, and Interpolated Time. Exclusion of NBBO Crossed and Locked is also considered. Everything is compared to the DTAQ data set with NBBO file with millisecond time stamps using the prior millisecond quote timing. The sample spans April to June 2008 inclusive and consists of 100 randomly selected stocks, resulting in 33,754,779 trades. Liquidity measures are calculated using firm-day averages. * indicates statistically different from the last column (DTAQ) at the 1% level. Standard errors are robust to clustering along both the firm and the day dimensions.

	MTAQ in Seconds with Withdrawn Quotes Accounted for				
	Prior Second	Same Second	Interpolated Time	Interpolated Time, Exclude NBBO Crossed and Locked	DTAQ in Milliseconds, with NBBO File
Panel A: Trade Location					
At the NBBO	67.8%*	51.4%*	59.5%*	63.9%*	71.0%
Inside the NBBO	14.6%*	33.3%*	26.7%*	30.5%*	25.7%
Outside the NBBO	17.6%*	15.4%*	13.7%*	5.6%*	3.3%
Locked NBBO	1.7%	2.1%*	2.5%*	0.0%*	1.7%
Crossed NBBO	9.9%*	9.9%*	10.2%*	0.0%*	0.5%
Panel B: Quoted and Effective Spreads					
Percent Quoted Spread	0.287%*	0.287%*	0.285%*	0.401%	0.399%
Dollar Quoted Spread	4.76¢*	4.76¢*	4.56¢*	7.56¢*	7.47¢
Percent Effective Spread	0.506%*	0.450%*	0.467%*	0.396%*	0.377%
Dollar Effective Spread	9.62¢*	8.66¢*	9.07¢*	6.86¢*	6.46¢
Time%:% Eff Spd >% Quo Spd	52.1%*	37.6%*	38.4%*	37.0%*	34.5%
Panel C: Realized Spread and Permanent Price Impact					
Percent Realized Spread: LR	0.142%*	0.238%*	0.197%*	0.196%*	0.154%
Percent Realized Spread: EMO	0.089%	0.135%*	0.113%*	0.101%*	0.083%
Percent Realized Spread: CLNV	0.132%*	0.200%*	0.172%*	0.166%*	0.140%
Percent Price Impact: LR	0.365%*	0.212%	0.271%*	0.205%*	0.223%
Percent Price Impact: EMO	0.351%*	0.277%*	0.302%*	0.238%	0.235%
Percent Price Impact: CLNV	0.357%*	0.241%*	0.285%*	0.221%	0.224%
Panel D: Depth Measures					
Dollar Ask Depth (000's)	$14.1	$14.1	$14.0*	$14.0	$14.1
Dollar Bid Depth (000's)	$13.5*	$13.5*	$13.4*	$13.8*	$14.0
Share Ask Depth	561	561	560	558	561
Share Bid Depth	535*	535*	531*	551*	567
# of Trades (Millions)	33.8	33.8	33.8	29.1	33.8
# of Trades Dropped (Mil.)	0.0	0.0	0.0	4.7	0.0

DTAQ (0.377%). Intuitively, this is due to the higher variability of the quote midpoint due to NBBO errors in MTAQ. Excluding NBBO Crossed and Locked (0.396%) gets researchers relatively close to DTAQ, yet the difference is still significant. The *Dollar Quoted Spread* and *Dollar Effective Spread* results are qualitatively similar to their *Percent* counterparts. The percentage of the time that *Percent Effective Spread* is greater than *Percent Quoted Spread* is highest using Prior Second (52.1%), declines using Same Second (37.6%), increases slightly with Interpolated Time (38.4%), and declines when also Excluding NBBO Crossed and Locked (37.0%). The latter is the closest to DTAQ (34.5%) using MTAQ, but the remaining difference is still significant.

C. Realized Spread and Price Impact

Panel C reports *Percent Realized Spread* and *Percent Price Impact* using three trade-typing conventions: (1) Lee and Ready (1991, LR), (2) Ellis, Michaely, and O'Hara (2000, EMO), and (3) Chakrabarty et al. (2006, CLNV). All three versions of *Percent Realized Spread* are largest using Same Second and decline using Interpolated Time. When NBBO crossed and locked trades are excluded, *Percent Realized Spread* decreases under all three trade-typing conventions. All three versions of *Percent Realized Spread* are the lowest using Prior Second, but Prior Second is still significantly different from DTAQ for two of the three trade-typing conventions. All three versions of *Percent Price Impact* are the largest using Prior Second, decline using Same Second (with LR being insignificantly different), increase using Interpolated Time, and decline to the lowest when also Excluding NBBO Crossed and Locked (with two versions being insignificantly different). Intuitively, the differences in *Percent Realized Spread* and *Percent Price Impact* across alternative MTAQ solutions are primarily due to differences in trade typing as they impact the buy/sell indicator D_k for the k^{th} trade. Overall, none of the MTAQ alternatives does very well for *Percent Realized Spread*, but Interpolated Time and Excluding NBBO Crossed and Locked does the best for *Percent Price Impact*.

D. Depth

In Panel D, *Dollar Ask Depth* (in thousands) is nearly constant across all MTAQ alternatives and insignificantly different from DTAQ in all but one case. The *Dollar Bid Depth* (in thousands) is close using the three quote timing rules ($13.4 to $13.5) and significantly different from DTAQ. It is higher when Excluding NBBO Crossed and Locked ($13.8) and, though statistically different, it is close in economic value to DTAQ ($14.0). The *Share Ask Depth* and *Share Bid Depth* results are qualitatively similar to their *Dollar* counterparts. Overall, the depth differences are relatively minimal.

In summary of Table III, Interpolated Time and Exclude NBBO Crossed and Locked is the best MTAQ solution with regard to *Outside the NBBO*, *Crossed NBBO*, *Percent (Dollar) Quoted Spread*, and *Percent (Dollar) Effective Spread*, percent of the time *Percent Effective Spread* is greater than *Percent*

Quoted Spread and *Dollar (Share) Bid Depth*, and is equivalent to the other alternatives for *Dollar (Share) Ask Depth*. However, this method fares less well on *Percent Realized Spread*, where Prior Second is the best, and *Percent Price Impact*, where Interpolated Time alone is the best.

Overall, if a researcher is financially constrained to using only MTAQ data, then the best MTAQ solution is to account for withdrawn quotes, use Interpolated Time, and Exclude NBBO Crossed and Locked.[32] However, this is still a second-best solution, not a first-best. It is significantly different from DTAQ with regard to *Outside the NBBO*, *Crossed NBBO*, and *Percent (Dollar) Effective Spread*, and percent of the time *Percent Effective Spread* is greater than *Percent Quoted Spread*, *Percent Realized Spread*, *Percent Price Impact*, and *Dollar (Share) Bid Depth*.

E. By Trade Frequency

Table IV examines the robustness of the results in Table III by breaking out trade frequency quintiles based on the number of trades per day, where quintile 1 is the lowest and quintile 5 is the highest. Panels A and B break out *Outside the NBBO* and *Crossed NBBO* by trade frequency quintiles. Within each column, *Outside the NBBO* is relatively similar across trade frequency quintiles, whereas *Crossed NBBO* varies modestly in a hump-shaped pattern that peaks at quintile 4. Across columns, Interpolated Time and Excluding NBBO Crossed and Locked yields the lowest values for *all quintiles* among the MTAQ alternatives, but they are still significantly different from the DTAQ values (with one exception).

Panels C and D break out *Percent Quoted Spread* and *Percent Effective Spread* by trade frequency quintiles. Within each column, they decrease nearly monotonically. In the first four columns, *Percent Quoted Spread* crosses over into negative values in the higher quintiles. Across columns, Interpolated Time and Excluding NBBO Crossed and Locked yields values closest to DTAQ for *all quintiles* among the MTAQ alternatives and all values are strictly positive. The values for *Percent Quoted Spread* in the two lowest quintiles are not statistically different from DTAQ values; the values for *Percent Effective Spread* are not statistically different from DTAQ values in all but one trade quintile.

Panel E breaks out *Dollar Ask Depth* by trade frequency quintiles. Within each column, depth increases monotonically. Across columns, there is very little difference among the MTAQ alternatives and compared to DTAQ.

To summarize, in the first four panels, Interpolated Time and Excluding NBBO Crossed and Locked yields values closest to DTAQ for *all quintiles* among the MTAQ alternatives and eliminates negative values, but the values are still significantly different from DTAQ in most cases. In the fifth panel, depth is largely the same among the MTAQ alternatives and DTAQ.

[32] The Internet Appendix considers excluding regional exchanges. We find that doing so helps reduce liquidity measure differences relative to DTAQ, but excluding NBBO crossed and locked trades does better.

Table IV

Alternative MTAQ Solutions: Results by Trade Frequency Quintiles

In this table, we present an analysis of how alternative MTAQ solutions affect liquidity measures by trade frequency quintiles. In the base case, there are no adjustments. Then, withdrawn quotes are accounted for. Next, three quote timing rules are compared: Prior Second, Same Second, and Interpolated Time. Finally, Exclusion of NBBO Crossed and Locked is also considered. Everything is compared to DTAQ in millisecond time stamps with the NBBO file. The sample spans April to June 2008 inclusive and consists of 100 randomly selected stocks, resulting in 33,754,779 trades. Liquidity measures are calculated using firm-day averages. * indicates statistically different from the last column (DTAQ) at the 1% level. Standard errors are robust to clustering along both the firm and the day dimensions.

		MTAQ in Seconds with Withdrawn Quotes Accounted for				
	MTAQ in Seconds, with No Adjustments	Prior Second	Same Second	Interpolated Time	Interpolated Time, Exclude NBBO Crossed and Locked	DTAQ in Milliseconds, with NBBO File
Panel A: Outside the NBBO						
# of Trades 1 (Low)	26.9%*	17.6%*	15.8%*	14.8%*	9.3%*	2.7%
# of Trades 2	24.7%*	18.3%*	16.6%*	15.1%*	5.6%*	2.9%
# of Trades 3	25.0%*	15.5%*	13.8%*	12.4%*	3.9%*	2.8%
# of Trades 4	28.9%*	19.1%*	16.6%*	14.9%*	4.4%*	3.5%
# of Trades 5 (High)	26.2%*	17.5%*	14.1%*	11.4%*	4.7%	4.8%
Panel B: Crossed NBBO						
# of Trades 1 (Low)	11.0%*	7.2%*	7.3%*	7.3%*	0.0%*	0.2%
# of Trades 2	16.9%*	11.9%*	11.7%*	11.8%*	0.0%*	0.4%
# of Trades 3	20.0%*	10.4%*	10.3%*	10.6%*	0.0%*	0.5%
# of Trades 4	24.2%*	12.5%*	12.5%*	13.1%*	0.0%*	0.5%
# of Trades 5 (High)	19.2%*	7.4%*	7.5%*	8.1%*	0.0%*	0.7%
Panel C: Percent Quoted Spread						
# of Trades 1 (Low)	1.141%*	1.217%*	1.217%*	1.219%*	1.345%	1.363%
# of Trades 2	0.125%*	0.164%*	0.164%*	0.169%*	0.315%	0.301%
# of Trades 3	0.000%*	0.066%*	0.066%*	0.064%*	0.189%*	0.180%
# of Trades 4	−0.107%*	−0.011%*	−0.011%*	−0.018%*	0.105%*	0.101%
# of Trades 5 (High)	−0.090%*	0.006%*	0.006%*	−0.002%*	0.057%*	0.055%
Panel D: Percent Effective Spread						
# of Trades 1 (Low)	1.472%*	1.376%*	1.151%	1.181%*	1.277%	1.225%
# of Trades 2	0.529%*	0.484%*	0.367%*	0.362%*	0.331%	0.312%
# of Trades 3	0.384%*	0.320%*	0.250%*	0.252%*	0.200%	0.189%
# of Trades 4	0.306%*	0.224%*	0.186%*	0.193%*	0.106%*	0.098%
# of Trades 5 (High)	0.216%*	0.132%*	0.099%*	0.105%*	0.070%	0.067%
Panel E: Dollar Ask Depth (000's)						
# of Trades 1 (Low)	$4.3*	$4.7	$4.7	$4.7	$4.8*	$4.7
# of Trades 2	$6.8	$6.5	$6.5	$6.5	$6.5	$6.5
# of Trades 3	$8.3*	$8.8	$8.8	$8.8*	$8.7	$8.8
# of Trades 4	$14.2	$14.4	$14.4	$14.4*	$14.3	$14.5
# of Trades 5 (High)	$33.8	$35.8	$35.8	$35.7*	$35.8	$35.8

Thus, the conclusion that we draw from Table III, that the best MTAQ solution is to account for withdrawn quotes, use Interpolated Time, and exclude NBBO crossed and locked, is found in Table IV to be robust by trade frequency.

F. Trade Classification

Table V reports additional liquidity measure differences for alternative MTAQ solutions. Panel A compares MTAQ and DTAQ trade classifications using the three trade-typing conventions: LR, EMO, and CLNV.[33] For example, in the first three rows, each trade is classified as a buy or a sell using MTAQ under the LR convention and as a buy or sell using DTAQ under the LR convention. When the two classifications match, the trade is reported as "*DTAQ and MTAQ agree—LR*." When DTAQ says it is a buy (sell) and MTAQ says it is a sell (buy), then the trade is reported as "*DTAQ Buy vs. MTAQ Sell—LR*" ("*DTAQ Sell vs. MTAQ Buy—LR*"). Using MTAQ with no adjustments, the percentage of trades where DTAQ and MTAQ agree is 88.4% using LR, 91.7% using EMO, and 90.2% using CLNV, and all three are significantly less than 100.0%. Interestingly, the two types of disagreement are close to evenly balanced using all three techniques. For example, using LR, 5.7% of trades are "*MTAQ Buy vs. DTAQ Sell*" and 5.9% are "*DTAQ Sell vs. MTAQ Buy.*" The same close balance holds using the other two trade-typing conventions.

Across columns, Interpolated Time and Excluding NBBO Crossed and Locked together yield the highest amount of agreement using all three conventions (90.4% using LR, 93.2% using EMO, and 92.1% using CLNV), and maintain a close balance between the two types of disagreement using all three conventions. However, the agreement values are still significantly less than 100%.

G. Absolute Order Imbalance

Panel B reports on *Absolute Order Imbalance*, which is an increasingly popular measure of the PIN (as discussed in Section II). The *Absolute Order Imbalance* using MTAQ with no adjustments is relatively close to DTAQ for all three trade-typing conventions. Similarly, *Absolute Order Imbalance* using all of the alternative MTAQ solutions is relatively close to DTAQ for all three trade-typing conventions. It appears that the even balance between the two types of disagreement reported in Panel A leads to roughly offsetting errors and thus we find very little difference in *Absolute Order Imbalance*. Overall, the *Absolute Order Imbalance* differences are relatively minimal.

[33] Other studies of trade classification include Odders-White (2000) and Chakrabarty, Moulton, and Shkilko (2012).

Table V

Alternative MTAQ Solutions: Trade Classification and Absolute Order Imbalance Results

In this table, we present trade classification differences between DTAQ and MTAQ and Absolute Order Imbalance under alternative MTAQ solutions and DTAQ. In the base case, there are no adjustments. Then, withdrawn quotes are accounted for. Next, three quote timing rules are compared: Prior Second, Same Second, and Interpolated Time. Finally, Exclusion of NBBO Crossed and Locked is also considered. Everything is compared to DTAQ in milliseconds with the NBBO file. The sample spans April to June 2008 inclusive and consists of 100 randomly selected stocks, resulting in 33,754,779 trades. All measures are calculated using firm-day averages. In Panel A, * for "DTAQ and MTAQ agree" indicates statistically different from 100% and * for "DTAQ Buy vs. MTAQ Sell" and "DTAQ Sell vs. MTAQ Buy" indicates statistically different from 0%. In Panel B, * indicates statistically different from the last column (DTAQ) at the 1% level. Standard errors are robust to clustering along both the firm and the day dimensions.

	MTAQ in Seconds, with No Adjustments	MTAQ in Seconds with Withdrawn Quotes Accounted for				DTAQ in Milliseconds, with NBBO File
		Prior Second	Same Second	Interpolated Time	Interpolated Time, Exclude NBBO Crossed and Locked	
Panel A: Trade Classification Differences between DTAQ and MTAQ						
DTAQ and MTAQ agree—LR	88.4%*	89.4%*	83.5%*	89.5%*	90.4%*	
DTAQ Buy vs. MTAQ Sell—LR	5.7%*	5.5%*	8.9%*	5.8%*	5.3%*	
DTAQ Sell vs. MTAQ Buy—LR	5.9%*	5.1%*	7.6%*	4.7%*	4.2%*	
DTAQ and MTAQ agree—EMO	91.7%*	92.2%*	88.6%*	92.7%*	93.2%*	
DTAQ Buy vs. MTAQ Sell—EMO	4.1%*	4.1%*	6.0%*	3.9%*	3.6%*	
DTAQ Sell vs. MTAQ Buy—EMO	4.2%*	3.7%*	5.4%*	3.4%*	3.2%*	
DTAQ and MTAQ agree—CLNV	90.2%*	91.1%*	86.8%*	91.4%*	92.1%*	
DTAQ Buy vs. MTAQ Sell—CLNV	4.8%*	4.6%*	7.0%*	4.6%*	4.2%*	
DTAQ Sell vs. MTAQ Buy—CLNV	4.9%*	4.3%*	6.2%*	4.0%*	3.7%*	
Panel B: Absolute Order Imbalance						
Absolute Order Imbalance: LR	12.6%	13.0%*	13.7%*	12.8%	13.9%*	12.6%
Absolute Order Imbalance: EMO	11.2%	11.4%*	10.7%	10.7%*	11.4%*	11.0%
Absolute Order Imbalance: CLNV	12.0%*	12.4%	12.2%	12.0%*	12.7%*	12.2%
# of Trades (Millions)	33.8	33.8	33.8	33.8	29.1	33.8
# of Trades Dropped (Millions)	0.0	0.0	0.0	0.0	4.7	0.0

IX. Does Methodology Affect Research Inference? Case 1: Hybrid Market Reform

This section examines whether the three different methods we consider (MTAQ with no adjustments,[34] DTAQ, and MTAQ with our recommended adjustments) yield different research inferences. In particular, we reexamine the analysis of HM, who conduct an event study around NYSE's Hybrid Market reform, which significantly increased the exchange's automation and speed.

We select the HM study because implementation of the Hybrid Market is when the NYSE converted a significant fraction of its floor trading to electronic trading. HM find that floor trading declined from 17% of NYSE trading pre-Hybrid to 9% post-Hybrid and that the average time to execute a market order declined from 10 seconds pre-Hybrid to one second post-Hybrid.

Hybrid Market implementation was rolled out stock-by-stock over October 2006 to January 2007. HM analyze a sample of 400 stocks listed on the NYSE before and after each individual stock's switch to Hybrid. They also compare these NYSE stocks to 400 stocks listed on NASDAQ that are matched by size and price. This allows them to compute a difference-in-difference of *Percent Effective Spread*, which controls for any time trends unrelated to Hybrid Market implementation. HM generously make their list of NYSE stocks, the Hybrid event date for each individual stock, and their list of matching NASDAQ stocks available online.[35]

HM match trades *to NYSE quotes only*, not to NBBO quotes. By focusing on NYSE-only trades and quotes, HM may well have sidestepped the most egregious withdrawn/canceled quote problems, if they were a bigger problem on other exchanges. It was sufficient for HM to have used NYSE quotes only because, during this period, NYSE still held a dominant position,[36] even though it had already started its downward slide.

We replicate the HM study in three ways, all of which are different from what HM did: (i) using the "conventional" MTAQ methods (NBBO across all markets, no adjustments for withdrawn quotes, etc.); (ii) using the first-best DTAQ approach; and (iii) using the second-best adjusted MTAQ approach. Table VI reports results for these three methods and reproduces the original HM results. Panel A reports *Percent Effective Spread* of the NYSE stocks over the 20 days preceding Hybrid activation (pre-Hybrid) and the 20 days immediately following Hybrid activation (post-Hybrid). The change in *Percent Effective Spread* using MTAQ with no adjustments is 0.7 basis points, but this change is *insignificant* (p-value = 0.1971).[37] By contrast, using both the DTAQ first-best

[34] In other words, the standard way MTAQ is used in the literature. This includes all of the standard screens described in Section III.

[35] See http://www.hotelschool.cornell.edu/research/facultybios/research-papers/p-moulton-research.html.

[36] Specifically, AHS report that, during this period, NYSE's market share was 55% and Wolfe (2010) reports that, during this period, NYSE's information share (i.e., percentage contribution to price discovery) in Dow Jones Industrial Average stocks ranged from 74% to 88%.

[37] We follow HM and calculate equal-weighted effective spreads and t-statistics using nonclustered standard errors.

Table VI

Percent Effective Spread Pre- and Post-Hybrid Implementation

In this table, we present the equal-weighted percent effective spread in basis points of 400 NYSE stocks and net of 400 NASDAQ stocks matched on size and price. The average values are estimated over the 20 days before (pre-Hybrid) and the 20 days after (post-Hybrid) the implementation of the NYSE Hybrid Market, which increased the automation and speed of the NYSE. "Time trend removed" means that the data from [−80 days, −21 days] were regressed on time, predicted values were computed for all days, and residuals were evaluated over the 20 days pre-Hybrid and the 20 days post-Hybrid. ** (*) denotes significance at the 1% (5%) level. Significance tests are calculated using nonclustered standard errors.

	MTAQ with No Adjustments	DTAQ First-Best Solution	MTAQ Second-Best Solution	Original HM: MTAQ with NYSE Quotes Only
Panel A: Percent Effective Spread (NYSE)				
Pre-Hybrid	21.7	5.7	6.0	5.6
Post-Hybrid	22.4	6.1	6.4	5.9
Change	0.7	0.4**	0.4**	0.3**

	MTAQ with No Adjustments	MTAQ with No Adjustments, Time Trend Removed	DTAQ First-Best Solution	MTAQ Second-Best Solution	Original HM: MTAQ with NYSE Quotes Only
Panel B: Percent Effective Spread (NYSE) − Percent Effective Spread (NASDAQ)					
Pre-Hybrid	8.3	−1.2	−1.5	−1.7	−1.4
Post-Hybrid	10.2	−0.4	−0.7	−0.9	−0.9
Change	1.9**	0.8	0.8**	0.8**	0.5**

and MTAQ second-best solutions, the change is smaller at 0.4 basis points, but is *significant* at the 1% level. This is nearly the same as the original HM change of 0.3 basis points that was *significant* at the 1% level. Since 0.4, 0.4, and 0.3 basis points are smaller in magnitude than 0.7 basis points, the reason that the latter three are significant must be smaller standard errors than using MTAQ with no adjustments. This is confirmed in Figure 4, which plots *Percent Effective Spread* from day −80 to day +80. The figure shows clearly that *Percent Effective Spread* using MTAQ with no adjustments is very noisy (creating a large standard error), whereas using both the DTAQ first-best and MTAQ second-best solutions is very smooth (creating small standard errors).

Panel B reports the *Percent Effective Spread* of the NYSE stocks minus *Percent Effective Spread* of the matching NASDAQ stocks. Using MTAQ with no adjustments, the change from pre- to post-Hybrid is 1.9 basis points and significant, but there is reason to doubt this result. Figure 5 shows the difference in *Percent Effective Spread* (NYSE − NASDAQ) by Hybrid event time. MTAQ with no adjustments is in a completely different range of values from the other methods, shows a general upward trend, and is very volatile. Indeed, it is not

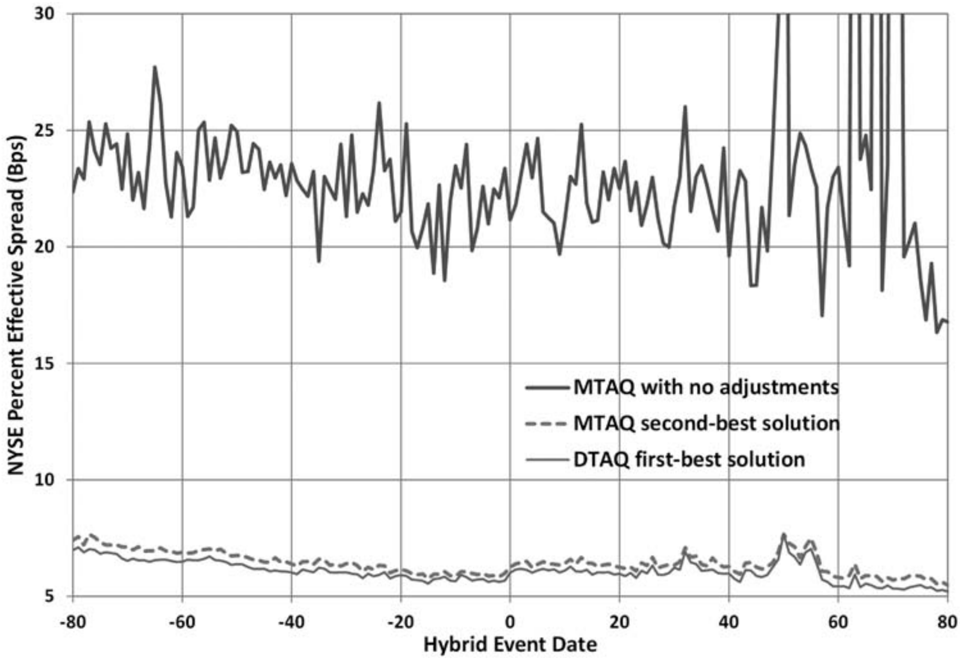

Figure 4. NYSE percent effective spread by hybrid event date.

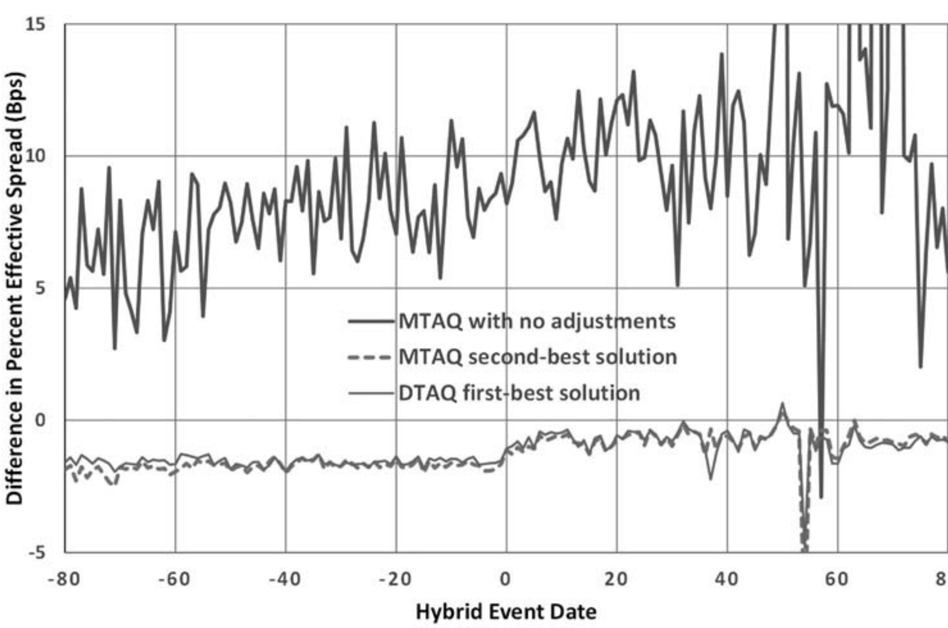

Figure 5. Difference in percent effective spread (NYSE − NASDAQ) by hybrid event date.

obvious whether there is any shift on the event date at all, as opposed to a general time trend that has not adequately been removed by the matching of NASDAQ stocks. By contrast, the DTAQ first-best and MTAQ second-best solutions clearly show an immediate, upward shift on the event date that continues over time. To investigate this issue, we remove the time trend by estimating the time trend from [−80 days, −21 days] and then computing the residual deviations from the forecasted time trend over the 20 days pre-Hybrid and the 20 days post-Hybrid. The next column of Table VI, Panel B, reports that, when the time trend is removed, the change drops to 0.8 basis points and becomes *insignificant*. By contrast, using the first- and second-best solutions, the change is 0.8 basis points and *significant*. The last column reports that the original HM change was 0.5 basis points and *significant* at the 1% level. Again, the different inferences come from the much smaller standard errors in the last three cases compared with the relatively volatile series using MTAQ with no adjustments and the time trend removed.

Using our first- and second-best solutions leads to the finding that Hybrid Market implementation caused a *significant* increase in *Percent Effective Spread*, which confirms HM's original finding based on NYSE quotes only. However, using MTAQ with no adjustments based on the NBBO across all exchanges and market makers leads to the finding that the Hybrid reform had an *insignificant* effect. We conclude that different methods yield different research inferences. Researchers analyzing the relatively competitive markets of recent years (2009 and thereafter) must compute the NBBO across all exchanges and market makers. Hence, it matters that researchers use our first- or second-best solutions, not the prior conventional way of using MTAQ.

X. Does Methodology Affect Research Inference? Case 2: Order Routing Decisions

This section performs a second test of whether different methods yield different research inferences. Suppose that a researcher used different methods to study how traders choose to route an order to minimize trading costs. During our sample period, investors could route their orders to nine stock exchanges: Chicago Board Options Exchange (CBOE), CHX, International Securities Exchange (ISE), National Association of Security Dealers (NASD): Alternative Display Facility (ADF) and Trade Reporting Facility (TRF),[38] National Association of Security Dealers Automatic Quotation (NASDAQ), National Stock Exchange (NSX), NYSE including the American Stock Exchange (AMEX), NYSE Archipelago (NYSE ARCA), and the Philadelphia Stock Exchange (PHLX). Following the methodology of Boehmer, Jennings, and Wei (2007), we compute *Dollar Effective Spread* for each stock-day for both Complete Official NBBO and MTAQ Quotes using alternative methods. Then, for each stock-day, we rank each stock exchange from one to nine. A rank of one is the lowest *Dollar*

[38] This is a catch-all category that in the sample period included the Better Alternative Trading System (BATS), Direct Edge A and X, and dark pool trades.

Effective Spread, which is most likely to attract orders, whereas a rank of nine is the highest *Dollar Effective Spread*, which is least likely to attract orders. We examine what percentage of the stock-days these rankings change based on alternative methods using MTAQ versus DTAQ.[39]

Table VII, Panel A, reports the difference in *Dollar Effective Spread* rankings using MTAQ with no adjustments versus using the DTAQ first-best solution. Looking in the Average column, we see that the rankings agree (the difference is zero) on only 49.1% of stock-days. Conversely, the rankings differ the majority (50.9%) of the time. For example, the difference "1" means that MTAQ with no adjustments yields a one-rank-higher number (i.e., one-rank-worse performance) 15.2% of the time. Looking at the bottom two rows, we see that MTAQ with no adjustments gives the CBOE a lower rank (better performance) 41.7% of the time and a higher rank (worse performance) 16.5% of the time. MTAQ with no adjustments makes some exchanges (e.g., CBOE, CHX, PHLX, etc.) appear to perform better and other exchanges (e.g., ISE, NSX, NYSE ARCA, etc.) appear to perform worse.

Panel B reports the difference in *Dollar Effective Spread* rankings using MTAQ with our second-best solution (accounting for withdrawn quotes, using Interpolated Time, and Excluding NBBO Crossed and Locked) versus using the DTAQ first-best solution. In the Average column, the rankings that agree (the difference is zero) rises to 60.4%. This is an improvement over Panel A, but still leaves 39.6% disagreement. Looking at the bottom two rows, all but two exchanges have a smaller value for lower rank time than in Panel A and all but one exchange have a smaller value for higher rank time than in Panel A. Again, this is an improvement over Panel A, but still leaves potential bias about exchange performance.

In summary, we find that using MTAQ with no adjustments yields different rankings than DTAQ the majority of the time and yields biased conclusions about which exchanges have superior versus inferior performance. Further, we find that using our second-best solution reduces the frequency of different rankings than DTAQ and reduces the bias about exchange performance, but falls short of using our first-best solution.

XI. Impact on Other Research Areas

This section examines whether different methods might impact other research areas. Corporate finance and asset pricing frequently use MTAQ variables to study a wide range of research questions (e.g., Nimalendran, Ritter, and Zhang (2007) and Chen, Goldstein, and Jiang (2007)). We conduct a firm trading costs sort (*Dollar Effective Spread* quintiles) that is common in the corporate finance and asset pricing literature. We then ask, do different methods yield different quintiles?

[39] In the Internet Appendix, we compute dollar effective spread exchange rankings for the stocks and time period (2001 to 2004) used in Boehmer, Jennings, and Wei (2007). We compare alternative MTAQ methods to Rule 605 data.

Table VII
The Difference in Dollar Effective Spread Rankings between MTAQ and DTAQ by Exchange

In this table, we present the difference in dollar effective spread quintiles by stock-day under alternative MTAQ treatments versus DTAQ by exchange. Panel A compares MTAQ with no adjustments. Panel B compares MTAQ with withdrawn quotes accounted for, Interpolated Time, and Excluding NBBO Crossed and Locked. Both panels are compared to DTAQ in milliseconds with the NBBO file. The sample spans April to June 2008 inclusive and consists of 100 randomly selected stocks, resulting in 33,754,779 trades.

MTAQ Quotes Rank − Benchmark Rank	CBOE	CHX	ISE	NASD ADF & TRF	NASDAQ	NSX	NYSE (Including AMEX)	NYSE ARCA	PHLX	Average
Panel A: MTAQ in Seconds with No Adjustments versus DTAQ in Milliseconds with the NBBO File										
−8	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
−7	0.5%	0.2%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	1.6%	0.2%
−6	1.0%	2.1%	0.2%	0.1%	0.0%	0.1%	0.1%	0.0%	1.6%	0.6%
−5	2.9%	2.4%	0.7%	0.9%	0.1%	0.4%	0.4%	0.1%	1.6%	1.0%
−4	4.3%	3.2%	1.5%	1.8%	0.5%	0.9%	0.9%	0.4%	6.3%	2.2%
−3	7.7%	4.3%	2.2%	4.2%	1.7%	2.6%	2.7%	1.2%	1.6%	3.2%
−2	8.6%	6.3%	5.1%	8.9%	5.9%	5.7%	5.0%	4.6%	4.8%	6.1%
−1	16.6%	14.6%	11.9%	20.7%	17.3%	14.1%	14.6%	14.6%	12.7%	15.2%
0	41.8%	47.3%	46.7%	47.8%	49.6%	45.9%	54.5%	43.6%	65.1%	49.1%
1	8.3%	6.5%	14.1%	10.5%	15.9%	15.3%	14.8%	20.8%	1.6%	12.0%
2	3.5%	3.1%	6.7%	3.1%	6.0%	6.6%	4.7%	9.5%	0.0%	4.8%
3	1.3%	2.6%	3.3%	1.2%	2.0%	4.2%	1.6%	3.6%	0.0%	2.2%
4	1.9%	2.9%	3.5%	0.6%	0.8%	2.6%	0.5%	1.3%	0.0%	1.6%
5	0.8%	2.3%	2.9%	0.2%	0.2%	1.3%	0.2%	0.3%	1.6%	1.1%
6	0.3%	2.2%	1.1%	0.0%	0.1%	0.4%	0.1%	0.0%	0.0%	0.4%
7	0.4%	0.2%	0.0%	0.0%	0.0%	0.1%	0.0%	0.0%	0.0%	0.1%
8	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	1.6%	0.2%
Lower Rank Time	41.7%	33.1%	21.7%	36.7%	25.5%	23.7%	23.7%	20.9%	30.2%	
Higher Rank Time	16.5%	19.7%	31.6%	15.5%	24.9%	30.5%	21.8%	35.5%	4.8%	
Panel B: MTAQ in Seconds with Withdrawn Quotes Accounted for, Using Interpolated Time, and Excluding NBBO Crossed and Locked versus DTAQ in Milliseconds with the NBBO File										
−8	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
−7	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
−6	0.3%	0.4%	0.0%	0.0%	0.0%	0.1%	0.0%	0.0%	0.0%	0.1%
−5	1.5%	1.0%	0.4%	0.2%	0.0%	0.1%	0.2%	0.0%	1.8%	0.6%
−4	3.7%	2.0%	0.4%	0.3%	0.2%	0.5%	0.6%	0.2%	0.0%	0.9%
−3	5.7%	2.6%	1.0%	0.9%	1.4%	1.1%	1.9%	1.0%	3.5%	2.1%
−2	8.8%	4.7%	2.9%	3.4%	6.0%	3.7%	5.7%	3.9%	3.5%	4.7%
−1	17.1%	13.9%	9.8%	13.4%	20.0%	12.6%	17.4%	15.1%	8.8%	14.2%
0	48.5%	59.1%	61.8%	62.1%	56.7%	58.2%	60.7%	53.9%	82.5%	60.4%
1	8.5%	9.5%	15.7%	15.6%	12.5%	16.2%	9.9%	20.1%	0.0%	12.0%
2	2.8%	2.9%	4.7%	3.3%	2.5%	5.0%	2.5%	4.7%	0.0%	3.1%
3	1.3%	1.7%	1.8%	0.6%	0.6%	2.0%	0.7%	0.9%	0.0%	1.1%
4	1.1%	0.9%	1.1%	0.1%	0.1%	0.5%	0.4%	0.2%	0.0%	0.5%
5	0.4%	0.9%	0.4%	0.0%	0.0%	0.0%	0.1%	0.0%	0.0%	0.2%
6	0.3%	0.4%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%
7	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
8	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
Lower Rank Time	37.1%	24.7%	14.5%	18.3%	27.6%	18.1%	25.8%	20.2%	17.5%	
Higher Rank Time	14.4%	16.2%	23.7%	19.6%	15.7%	23.8%	13.6%	25.9%	0.0%	

Table VIII
Alternative MTAQ Solutions: Difference in Dollar Effective Spread Quintiles

In this table, we present the difference in dollar effective spread quintiles by stock-week under alternative MTAQ treatments versus DTAQ. In the base case, there are no adjustments. Then, withdrawn quotes are accounted for. Next, three quote timing rules are compared: Prior Second, Same Second, and Interpolated Time. Finally, Exclusion of NBBO Crossed and Locked is also considered. Everything is compared to DTAQ in milliseconds with the NBBO file. The sample spans April to June 2008 inclusive and consists of 100 randomly selected stocks, resulting in 33,754,779 trades.

		MTAQ in Seconds with Withdrawn Quotes Accounted for			
MTAQ Quintile – DTAQ Quintile	MTAQ in Seconds, with No Adjustments	Prior Second	Same Second	Interpolated Time	Interpolated Time, Exclude NBBO Crossed and Locked
–4	0%	0%	0%	0%	0%
–3	0%	0%	0%	0%	0%
–2	4%	1%	1%	1%	0%
–1	30%	26%	26%	27%	10%
0	43%	54%	53%	53%	81%
1	13%	11%	12%	12%	9%
2	7%	6%	6%	6%	0%
3	3%	1%	2%	2%	0%
4	1%	0%	0%	0%	0%
Lower Quintiles	34%	27%	28%	28%	10%
Higher Quintiles	23%	19%	20%	19%	9%

We analyze the impact as follows. First, using each MTAQ alternative and DTAQ, we compute *Dollar Effective Spread* for firm i in day t. Second, we sort all of the firms in day t into *Dollar Effective Spread* quintiles, where quintile 1 has the smallest values and quintile 5 has the largest values. Finally, for each firm-day, we compute the difference between each MTAQ alternative quintile and the DTAQ quintile.

Table VIII reports the difference between the MTAQ alternative quintile and the DTAQ quintile. Using MTAQ with no adjustments, only 43% of the firm-days agree (are in the same quintile) with the DTAQ quintile. Conversely, 57% of firm-days disagree, with 34% placed in lower quintiles and 23% placed in higher quintiles. Accounting for withdrawn quotes raises the agreement to 54% (Prior Second), 53% (Same Second), and 53% (Interpolated Time). Adding the exclusion of NBBO crossed and locked raises the agreement to 81%.

We find that, using MTAQ with no adjustments, the majority of *Dollar Effective Spread* quintiles differ from our first-best solution, whereas, using our second-best solution, the vast majority are the same as the first-best solution. We conclude that different methods have a large impact on other research areas.

XII. The Ultimate Breakdown of the NBBO

Hasbrouck and Saar (2010) provide evidence that some traders are responding to market events in milliseconds. Specifically, they find that, after a quote improves or deteriorates, there is a peak in the hazard rate two to three milliseconds later for limit order submission, limit order cancellation, and execution at the updated quoted price. In other words, current trading algorithms are able to observe the market event, process the information, and take action in about two to three milliseconds.

There is every reason to believe that response times will get faster in the decades ahead. Moore (1965) ("Moore's Law") and other similar formulations have found evidence that, over the past half-century, there has been an exponential increase in computing power (as measured by CPU speed per dollar, memory capacity per dollar, hard disk capacity per dollar, etc.) and an exponential increase in network power (as measured by internet backbone bandwidth, wireless network speeds, network latency, etc.). This has fueled a competitive "arms race" by proprietary trading firms to continually reduce network latency and increase processing speed in order to leapfrog the competition (see Aldridge (2009)). Projecting these trends into the future, response times will likely accelerate into microseconds (10^{-6} seconds) in the late 2010s and into nanoseconds (10^{-9} seconds) in the 2020s.

As response times accelerate, we predict that the fundamental legal and economic concept of the NBBO will ultimately break down.[40] In 1687, Sir Isaac Newton published his seminal book on the law of gravity, which is based on a framework of absolute time and space that is independent of any observer. In a Newtonian universe, it would be possible for bid and offer update messages to travel arbitrarily fast so there could always exist a common NBBO for all economic agents in all locations.[41] However, in 1905, Albert Einstein published his special theory of relativity in which time and space differ depending on the relative motion of each observer. Critically, the special theory of relativity yields the fundamental result that no physical entity can travel faster than the speed of light (186,282 miles per second). In an Einsteinian universe, a bid and offer update message could *not* travel faster than the speed of light so high-speed traders face immutable lag times in receiving bid and offer updates from remotely located exchanges.[42]

[40] Macey and O'Hara (1997) note that the broker's fundamental legal obligation of providing "best execution" for the client (the trader) is complex and hard to define, but has generally been interpreted by the courts as requiring that the trader at least get the NBBO.

[41] Indeed, there could always exist a common best bid and offer throughout the entire universe. Stocks crossed-listed on exchanges all over the universe could trade at the *Universal* Best Bid and Offer.

[42] The lag times could be viewed as endogenous. For example, an exchange might strategically decide to move its data center closer to the data centers of other exchanges in order to reduce most lag times.

Consider a trading algorithm colocated[43] with the servers of the CHX and a second trading algorithm colocated with the servers of the NYSE. Suppose, for a particular security for a five millisecond period leading up to 9:47:25.000, both locations observe that the best ask is \$47.10 and the best bid is \$47.00. Then, one millisecond later at 9:47:25.001, the trading algorithm in Chicago submits a limit sell at \$47.09, which improves the CHX ask price. Then, one millisecond later at 9:47:25.002, the trading algorithm in New York submits a market buy to the NYSE. Chicago and New York are 790 miles apart, or equivalently, 4.24 milliseconds apart when traveling at the speed of light. At that moment (9:47:25.002), it is physically impossible for a message about the improved CHX ask price to have traveled the 790 miles in just one millisecond to inform the matching engine on the NYSE, so the market buy is executed at the unimproved NYSE ask price of \$47.10. This example illustrates that the Newtonian-type concept of a single absolute NBBO for all economic agents in all locations breaks down under sufficiently fast trading.

As a replacement for the NBBO, we propose an Einsteinian-type concept of an RBBO that is different for each market center, where a market center is defined as an exchange or market maker. The RBBO for a given market center matching engine is based on local information in real time and remote information with various lag times. Suppose that market center i is one of N market centers in the United States. Let D_{ij} be the distance from the data center for market center j to the data center for market center i. Let τ_{ij} be the theoretical minimum lag time in communicating from the data center for market center j to the data center for market center i as given by

$$\tau_{ij} = D_{ij} c, \tag{18}$$

where c is the speed of light.[44] Let $Bid(j,t)$ and $Offer(j,t)$ be the bid and offer of market center j at time t. We propose that the RBBO for market center i at time t is

$$RBBO(i,t) = \left\{ \begin{array}{l} \text{Max}[Bid(1,t-\tau_{i1}), Bid(2,t-\tau_{i2}), \ldots, Bid(N,t-\tau_{iN})], \\ \text{Min}[Offer(1,t-\tau_{i1}), Offer(2,t-\tau_{i2}), \ldots, Offer(N,t-\tau_{iN})] \end{array} \right\}. \tag{19}$$

By construction, the local lag time τ_{ii} is zero. In the example above, the RBBO on the CHX at 9:47:25.002 is given by *RBBO*(*CHX*, 9:47:25.002) = {\$47.00, \$47.09}, which reflects the improved ask price. However, the RBBO on the NYSE in the

[43] Colocation is where a trading firm's computer is placed in the same room as an exchange's computer to minimize the computer-to-computer communication time. Colocation has grown explosively in recent years.

[44] In practice, proprietary and high-frequency traders are rapidly approaching the theoretical minimum lag time. For example, several trading firms recently invested in line-of-sight microwave towers between Chicago and New York so that they can communicate trading instructions between Chicago and New York in 4.25 milliseconds, rather than use fiber optic cables that take 6.55 milliseconds because they must follow the available communication right-of-ways and contour of the earth (*Wall Street Journal*, May 30, 2012).

same millisecond is given by *RBBO*(*NYSE*, 9:47:25.002) = {$47.00, $47.10}, which is based on the unimproved ask price.

XIII. Conclusion

We investigate whether today's fast, competitive U.S. equity markets yield liquidity measurement problems. We find that the widely used MTAQ database yields a 54% higher *Percent Effective Spread* than the expensive DTAQ database and a *Percent Quoted Spread* that goes negative 37 times more often. We find that these differences are driven by: (1) withdrawn quotes, (2) second (versus millisecond) time stamps, and (3) other causes, including canceled quotes. We test ways to eliminate or mitigate these differences. We find that the expensive solution—using DTAQ—is first-best. For financially constrained researchers, the cheap solution—using MTAQ with our new Interpolated Time technique, adjusting for withdrawn quotes, and throwing away economically nonsensical states—is second-best. We find that both our first-best and second-best solutions yield different inferences from using MTAQ with no adjustments. Looking to the future, we anticipate the ultimate demise of the NBBO and propose that it be replaced with an RBBO that is different for each market center.

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Supporting Information

Additional Supporting Information may be found in the online version of this article at the publisher's website:

Appendix S1: Internet Appendix.