This paper provides evidence that stock traders focus on round numbers as cognitive reference points for value. Using a random sample of more than 100 million stock transactions, we find excess buying (selling) by liquidity demanders at all price points one penny below (above) round numbers. Further, the size of the buy–sell imbalance is monotonic in the roundness of the adjacent round number (i.e., largest adjacent to integers, second-largest adjacent to half-dollars, etc.). Conditioning on the price path, we find much stronger excess buying (selling) by liquidity demanders when the ask falls (bid rises) to reach the integer than when it crosses the integer. We discuss and test three explanations for these results. Finally, 24-hour returns also vary by price point, and buy–sell imbalances are a major determinant of that variation across price points. Buying (selling) by liquidity demanders below (above) round numbers yield losses approaching $1 billion per year.

Key words: cognitive reference points; round numbers; left-digit effect; nine-ending prices; trading strategies

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1. Introduction

In an ideal world, liquidity demanders would be equally likely to buy or sell at any given price point. In the real world, they often focus on round number thresholds as cognitive reference points for value. If security traders do focus on round numbers as reference points for value, a security price path that reaches or crosses a round number threshold may generate waves of buying or selling.

This paper examines three different kinds of round number effects. First, we consider the left-digit effect, which claims that a change in the leftmost digit of a price dramatically affects the perception of the magnitude. To illustrate, a price drop from $7.00 to $6.99 is only a one-cent decline, but a quick approximation based only on the leftmost digit suggests a one-dollar drop. In other words, when assessing the drop from $7.00 to $6.99 is only a one-cent decline, but a quick approximation based only on the leftmost digit suggests a one-dollar drop. In other words, when assessing the drop from $7.00 to $6.99, people anchor on the leftmost digit changing from 6 to 7, and believe it is a $1 drop. They do not round $6.99 up to $7.00, because this is mentally costly. The second round number effect we analyze is based on round number thresholds for action, which we call the threshold trigger effect. The idea is that investors have a preference for round numbers, where the hierarchy of roundness from the most round to the least round is whole dollars, half-dollars, quarters, dimes, nickels, and pennies. Therefore, in the example above, when the price reaches the round number $7.00 or crosses below it to $6.99, this drop triggers trades.

Both the left-digit effect and the threshold trigger effect depend on the actions of value traders, who are traders that buy underpriced stocks and sell overpriced stocks relative to their valuations. The trader’s valuation is derived from earnings, dividends, book assets, or other measures of fundamental value. For example, suppose that a value trader engages in fundamental analysis and determines that a particular stock is worth $7.52. If the stock price drops below that level and no new information causes the investor to change his valuation, then the stock will be considered underpriced, and this will generate a buy trade at some point. Theoretically, a buy trade could be triggered by any price below $7.52. However, the left-digit effect causes a great discontinuity in the perceived market price because it crosses a round number threshold, and so a change from $7.00 to $6.99 triggers more buys than a change from, say, $7.08 to $7.07. Similarly, under the threshold trigger effect, some value traders may have selected $7.00 as a target for buying. Thus, if the price falls to $7.00 or goes below it, there is excess buying by value traders. Conversely, with respect to overpriced stocks, both effects predict that if the price rises to $8.00 or above it, there is excess selling by value traders. Note that the left-digit effect, unlike the threshold trigger effect, does
not predict excess buying when prices fall exactly to a round number.

The third round number effect we examine is based on a combination of limit order clustering and undercutting. Limit order clustering occurs when limit order prices are more frequently on round numbers. For example, Chiao and Wang (2009) find that limit order prices are clustered on integers, dimes, nickels, and multiples of two of the tick size on the Taiwan Stock Exchange. Bourghelle and Cellier (2009) document the same phenomenon in Euronext. Undercutting occurs when a new limit sell (buy) is submitted at a penny lower (higher) than the existing ask (bid). The cluster undercutting effect is a combination of both limit order clustering and undercutting. Because of limit order clustering, it is relatively common that existing limit sell orders set the current ask at a round number, say, $7.00. Then a new limit sell undercut at $6.99 and sets a new ask price. Then a market buy hits the new ask price. Thus, a buy trade is frequently recorded below a round number. Conversely, because of limit order clustering, it is relatively common that existing limit buy orders set the current bid at a round number, say, $5.00. Then a new limit buy undercut at $5.01 and sets the new bid price. Then, a market sell hits the new bid price. Thus, a sell trade is frequently recorded above a round number. Hence, the cluster undercutting effect predicts excess buying below round numbers and excess selling above round numbers. Note that unlike the left-digit effect or the threshold trigger effect. To distinguish between these two possibilities, we now turn to a conditional analysis of buy–sell ratios when the price rises or falls around an integer. We conduct four main analyses: ask falls below integer, ask falls to integer, bid rises to integer, and bid rises above integer. We also perform two supplementary analyses as robustness checks: ask rises below integer, ask falls to integer, bid rises above integer, and bid falls while staying above integer. Each of these six tests have the following respective controls: ask rises below integer, ask falls to integer, bid rises above integer, and bid falls while staying above integer.

Under all three buy–sell ratios, we find strong excess buying when the “ask falls to integer” and strong excess selling when the “bid rises to integer.” There is also some excess buying when the “ask falls below integer” and some excess selling when the “bid rises above integer.” However, the excess trading when the price reaches the integer is an order of magnitude larger than the excess trading when the price crosses the integer. This conditional evidence supports that the left-digit effect or the threshold trigger effect takes place on integers.

Very little of the excess buying below round numbers and excess selling above round numbers is because of excess trading after crossing thresholds based on the left-digit effect or the threshold trigger.
effect. Thus, we conclude that the excess buying below round numbers and excess selling above round numbers observed in the unconditional tests must be predominantly due to the cluster undercutting effect.

To summarize, our unconditional tests and our conditional tests provide evidence of all the three effects based on the unifying hypothesis that stock traders focus on round numbers as cognitive reference points for value. A number of further tests, discussed later, confirm this conclusion.

Next, we examine unconditional 24-hour returns. We compute both the trade price returns and the midpoint returns that result from buying whenever buy trades are observed at a .XX price point and the position is closed 24 hours later. Similarly, we compute both the trade price returns and the midpoint returns that result from (short) selling whenever sell trades are observed at a .XX price point and the position is closed 24 hours later. We find a systematic pattern in returns around all round number thresholds: integers, half-dollars, quarters, dimes, and nickels. Specifically, we find that that liquidity demanders who buy (sell) below the threshold have lower (higher) returns, and liquidity demanders who sell (buy) above the threshold have lower (higher) returns.

Given these findings, we next try to determine whether there is a connection between the return pattern surrounding thresholds mentioned above and the buy–sell ratios surrounding thresholds discussed earlier. Our regression tests reveal that buy–sell imbalances are a major determinant of the variation by price point of average 24-hour returns. A higher buy–sell ratio yields a more negative difference in median 24-hour returns (median return to buying minus median return to selling).

We also compute 24-hour returns conditional on reaching (“ask falls to integer” buys and “bid rises to integer” sells) or crossing (“ask falls below integer” buys and “bid rises above integer” sells) integer thresholds. These returns are compared to the analogous 24-hour returns conditional on reaching or crossing nickel thresholds. The conditional returns for reaching (crossing) integer thresholds yield positive (mixed) abnormal 24-hour returns.

To determine the economic significance of these 24-hour returns, we make a rough estimate of the wealth transfer implied by both the conditional and unconditional returns. We find that the negative abnormal returns for unconditional buys below (sells above) round numbers yield an aggregate wealth transfer of $813 million per year. The positive abnormal returns for conditional buys (sells) when the ask falls (bid rises) to reach an integer yield an aggregate wealth transfer of $40 million per year.

2. Psychological Foundations and Related Findings in Other Fields


One type of heuristic is identified by Rosch (1975), who found that people make judgments based on cognitive reference points. Cognitive reference points are defined as standard benchmarks against which other stimuli are judged. Specifically with regard to numbers, she found that multiples of 10 were cognitive reference points for integer numbers in a decimal number system. More generally, all round numbers (integers, especially multiples of 10, and midpoints between them in a decimal number system) are cognitive reference points. Schindler and Kirby (1997) show that it is easier to remember round numbers. In the context of financial markets, Goodhart and Curcio (1991) and Aitken et al. (1996) argue that investors have an “attraction” to round-numbered prices.

The left-digit effect is present when a change in the left digit of a price leads people to jump from one cognitive reference point to another (e.g., from $7.00 to $6.00 if the price changes from $7.00 to $6.99). Brenner and Brenner (1982) theorize that people economize on their limited mental memory in storing the price of thousands of goods. They note that the economic value of remembering the first digit is much greater than the economic value of remembering the second digit, which in turn is much greater than the economic value of remembering the third digit, and so on. Thomas and Morwitz (2005), in a series of five experiments, provide a cognitive account of when and why the left-digit effect manifests itself. They summarize that:

The effect of a left-digit change on price magnitude perceptions seems to be a consequence of the way the human mind converts numerical symbols to analog magnitudes on an internal mental scale. . . . Since this symbol to analog conversion is an automatic process, the left digit effect seems to be occurring automatically, that is, without consumers’ awareness . . . encoding the magnitude of a multi-digit number begins even before we finish reading all the digits. . . . Since we read numbers from left to right, while evaluating “2.99,” the magnitude encoding process starts as soon as our eyes encounter the digit “2.” Consequently, the encoded magnitude of $2.99 gets anchored on the left most.
digit (i.e., $2) and becomes significantly lower than the encoded magnitude of $3.00. (Thomas and Morwitz 2005, pp. 54–55)

Kahn et al. (2002) develop an interesting application of the left-digit effect in the context of banking. They construct a model in which a fraction of potential bank depositors truncate deposit yields to just the left digit (e.g., truncate 6.27% to 6.00%). They determine the optimal bank policy for setting deposit rates, and find empirical support for their predictions. In accounting, Carlsaw (1988), Thomas (1989), Niskanen and Keloharju (2000), and Van Caneghem (2002) find that company managers manage earnings to change the left digit of reported earnings. Specifically, managers use discretionary accruals in the knife edge cases to report, say, $7 billion in earnings this period, rather than $6.99 billion. Bader and Weinland (1932), Knauth (1949), Gabor and Granger (1964), and Gabor (1977) pioneer the study of the left-digit effect in the realm of marketing. They find that retailers exploit the left-digit effect by setting nine-ending prices (i.e., $6.99) on a wide variety of goods to make them appear less expensive (based on the “underestimation hypothesis”). Nine-ending prices are popular based on surveys of retailers’ pricing practices (Schindler and Kirby 1997) and based on Universal Product Code retail scanning data (Siving and Winer 1997). Nine-ending prices are found to significantly increase retailers’ profits (Anderson and Simester 2003, Blattberg and Neslin 1990, Monroe 2003, and Siving and Winer 1997).

In market microstructure, an extensive literature exists regarding trade price clustering on round numbers. Harris (1991) shows that during the $1/8th tick-size era, the frequency of trade prices was highest on integers, second-highest on half-dollars, third-highest on quarters, and lowest on odd-eighths. Ikenberry and Weston (2007) show that during the decimal era, the frequency of trade prices from highest to lowest is integers, half-dollars, quarters, dimes, nickels, and pennies.3 To explain these patterns, Ball et al. (1985) offer the price resolution hypothesis that uncertain valuations lead to price clustering to reduce search costs. Harris (1991) offers the negotiation hypothesis that price clustering reduces the cost of negotiating between traders and dealers. Ikenberry and Weston (2007) hypothesize that investors have a psychological preference for round numbers. They find that price clustering during the decimal era far exceeds what can be explained by the rational price resolution or negotiation hypotheses. They conclude that a psychological preference for round numbers is a major cause of price clustering. None of the above evidence directly relates to waves of buying or selling because the trades are unsigned. That is, the frequency of trades by price point does not distinguish between the buys and sells of liquidity demanders.

Recent papers by Bagnoli et al. (2006) and Johnson et al. (2007) are the closest to our paper. Using a sample of end-of-day prices, both of these studies show that if the end-of-day price is just below an integer (just above an integer), the overnight or next-day return is lower (higher). However, our paper offers three important distinctions. First, the two aforementioned papers examine overnight or next-day returns starting from closing prices only, whereas we analyze all transactions throughout the day using a high-frequency, intraday data set. Second, unlike the previous two studies, we identify buys and sells of liquidity demanders. Third, because we can identify buys and sells of liquidity demanders, we can directly test three possible explanations for buy–sell imbalances. Johnson et al. (2007) test a number of different hypotheses that may explain their findings—the left-digit effect is not one of their hypotheses—and they come to no definite conclusion. Bagnoli et al. (2006) only observe returns and then infer next-day buying/selling behavior from the returns. Specifically, they observe that closing prices ending in 9 (1) yield negative (positive) overnight returns. They infer that closing prices ending in 9 (1) predict future net selling (buying) the following day. Hence, they conclude that zero-ending round numbers represent a “psychological barrier or hurdle that is difficult to break through” (p. 16). We examine direct evidence of buys and sells rather than inferring buying and selling patterns.

3. Hypotheses and Research Design
We will now formally state our research hypotheses.

Hypothesis 1 (H1). Buys should outnumber sells at trade prices immediately below a round number, and sells should outnumber buys at trade prices immediately above a round number.

The above test checks buys and sells by trade price. It is an unconditional test that does not check whether this particular transaction price was reached after a rise or drop in price. This unconditional test checks for all three effects, but cannot distinguish between them. We thus design conditional tests that offer the ability to distinguish between effects. If asks fall (bids rise) to reach or cross an integer, then both the left-digit effect and the threshold trigger effect predict that value traders who are demanding liquidity are motivated to buy (sell), but the cluster undercutting effect predicts imbalances only for the “crosses” but not

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3 Additional evidence of price clustering can be found in Osborne (1962), Neiderhoffer (1965, 1966), Christie and Schultz (1994), Kavajecz (1999), Chakravarty et al. (2001), Simaan et al. (2003), Kavajecz and Odders-White (2004), and Ahn et al. (2005).
for the “reaches.” Because we do not know whether value traders trigger their trades at the threshold and/or after crossing the threshold, we have three alternative versions of our next hypothesis.

**Hypothesis 2A (H2A)** (Reach Only). Liquidity demanders’ buys should outnumber their sells after ask prices fall to reach an integer, and their sells should outnumber their buys after bid prices rise to reach an integer.

**Hypothesis 2B (H2B)** (Cross Only). Liquidity demanders’ buys should outnumber their sells after ask prices fall to cross an integer, and their sells should outnumber their buys after bid prices rise to cross an integer.

**Hypothesis 2C (H2C)** (Reach and Cross). Liquidity demanders’ buys should outnumber their sells after ask prices fall to reach an integer and to cross an integer, and their sells should outnumber their buys after bid prices rise to reach an integer and to cross an integer.

As a robustness check, we also consider the cases in which the “ask rises while staying below integer” and the “bid falls while staying above integer.” As prices do not reach or cross integers in these cases, the three round number effects have no predictions in these cases.

We devise two additional tests of round number effects. In the decimal era, as we move from a price of $11 to $99 in dollar increments, the first left digit changes around the two-digit integers 20, 30, 40, 50, 60, 70, 80, and 90. The second left digit changes around other two-digit integers 11, 12, …, 19, 21, 22, …, 99. If the left-digit effect exists, a first left-digit change should be more dramatic than a second left-digit change. In other words, the change from $20.00 to $19.99 should have a greater effect than the change from $9.00 to $8.99. This is because if the human brain focuses only on the leftmost digit that is changing, the former is a change of $10, whereas the latter is a change of $1. In addition to the left-digit effect, the other two effects—the threshold trigger effect and the cluster undercutting effect—also yield similar predictions because integers such as 20, 30, 40, 50, 60, 70, 80, and 90 are “more round” than integers such as 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10. This leads us to our final test:

**Hypothesis 3 (H3).** When ask prices fall to hit an integer or fall below an integer, liquidity demanders’ buys should outnumber their sells more around 20, 30, 40, 50, 60, 70, 80, and 90 than around 11, 12, …, 19, 21, 22, …, 99. In addition, when bid prices rise to hit an integer or rise above an integer, liquidity demanders’ sells should outnumber their buys more around 20, 30, 40, 50, 60, 70, 80, and 90 than around 11, 12, …, 19, 21, 22, …, 99.

The next test checks whether the effect of the first left-digit change is greater around certain two-digit integers than around one-digit integers. In other words, the change from $20.00 to $19.99 should have a greater effect than the change from $9.00 to $8.99. This is because if the human brain focuses only on the first left digit, the former is a change of $10, whereas the latter is a change of $1. In addition to the left-digit effect, the other two effects—the threshold trigger effect and the cluster undercutting effect—also yield similar predictions because integers such as 20, 30, 40, 50, 60, 70, 80, and 90 are “more round” than integers such as 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10. This leads us to our final test:

**Hypothesis 4 (H4).** When ask prices fall to hit an integer or fall below an integer, liquidity demanders’ buys should outnumber their sells more around certain two-digit integers (20, 30, 40, 50, 60, 70, 80, and 90) than around one-digit integers (1, 2, 3, 4, 5, 6, 7, 8, 9, and 10). In addition, when bid prices rise to hit an integer or rise above an integer, sells should outnumber buys more around certain two-digit integers (20, 30, 40, 50, 60, 70, 80, and 90) than around one-digit integers (1, 2, 3, 4, 5, 6, 7, 8, 9, and 10).

### 4. Data and Methodology

The intraday data used in this study come from the New York Stock Exchange (NYSE) Trade and Quote (TAQ) data set from 2001 to 2006. Because using the full data set would involve massive computations, we select a random sample of traded stocks. Following the methodology of Hasbrouck (2009), a selected stock must meet five criteria to be eligible: (1) it must be a common stock; (2) it must be present on the first and last TAQ master file for the year; (3) it must have a primary listing on the NYSE, American Stock Exchange, or National Association of Securities Dealers Automated Quotations (NASDAQ); (4) it cannot change primary exchange, ticker symbol, or its Committee on Uniform Security Identification Procedures (CUSIP) code during the course of a year; and (5) it must be listed in the Center for Research in Security Prices (CRSP) database.

Starting with eligible firms in 2001, we divide them into five quintiles based on price, and then randomly select 20 firms from each quintile. We next roll forward to 2002. If firms that were selected in 2001 are eligible in 2002, then they remain in the sample; otherwise, they are replaced by new
firms that are randomly selected from all eligible 2002 firms. This process is repeated for each year through 2006. Thus, in each year we have all trade and quote data of a random sample of 100 traded stocks. The body of our paper analyzes 137,335,376 trades from the decimal era. In the online appendix to this paper, available at http://www.kelley.jhu.edu/cholden/RoundNumbers.pdf, we extend our analysis to include 7,347,675 trades during the $1/8th tick-size era and 15,992,073 trades during the $1/16th tick-size era.3

We then apply the following screens to the trade and quote data. Only quotes/trades during normal market hours (between 9:30 A.M. and 4:00 P.M.) are considered. Cases in which the bid or ask price or bid or ask size is 0 are deleted. In addition, we delete cases in which the bid price was greater than the ask price, or the ask price was twice as big as the bid price. We also remove all prices equal to or greater than $100 and less than $2. The quote condition must be normal, which excludes cases in which trading has been halted. We calculate the National Best Bid and Offer (NBBO) across all nine exchanges and across all market makers for any given second. Each trade is then matched to the NBBO in the prior second, as recommended in Henker and Wang (2006). The market capitalization and the share volume of each stock are obtained from CRSP. From CDA Spectrum we obtain institutional ownership data on each firm.

The “ask falls below integer” sample is constructed as follows. We include it in the data set if (i) the previous best ask is one integer higher than the current best ask, (ii) the digits after the decimal point of the previous best ask are in [.00, .10], and (iii) the digits after the decimal point of the current best ask are in [.90, .99]. If all three conditions are met, we then collect all trades that occur while the ask quote remains in [.90, .99]. An example of the “ask falls below integer” sample would be all trades occurring after the ask quote falls from $10.01 to $9.99. The corresponding control sample is “ask falls below nickel,” which includes all trades after the ask drops from [.01, .10] to [.00] until the ask leaves [.00]. The corresponding control sample is “ask falls below nickel,” which includes all trades after the ask rises from [.90, .99] above an integer threshold until the ask leaves [.90, .99]. The “bid rises above nickel” includes all trades after the bid rises from [.N + .01, .N + .10] to nickel threshold N until the ask leaves [N]. The “bid rises above integer” includes all trades after the bid rises from [.N + .01, .N + .10] to nickel threshold N until the ask leaves [N]. The “bid rises to integer” includes all trades after the bid rises from [.90, .99] above an integer threshold until the bid leaves [90, 99]. The corresponding control sample is “bid rises above nickel,” which includes all trades after the bid rises from [.N − .01, .N − .10] above a nickel threshold N until the bid leaves [N]. The “ask rises while staying above nickel” includes all trades after the ask rises from [.N − .01, .N − .10] to nickel threshold N until the bid leaves [N]. The “ask rises while staying below integer” includes all trades after the ask drops from [.N − .20, .N − .11] to [.N − .10, .N − .01] which is below the nickel threshold N, until the ask leaves [.N − .10, .N − .01]. The “bid falls while staying above nickel” includes all trades after the bid falls from [.11, .20] to [.01, .10] until the bid leaves [.01, .10]. The corresponding control sample is “bid falls while staying above nickel,” and includes all trades after the bid falls from [.N + .01, .N + .10], which is above the nickel threshold N, until the bid leaves [N + .01, .N + .10].

If all three conditions are met, we then collect all trades that occur while the ask quote remains in [N − .10, N − .01]. An example for the nickel threshold N = .15 would be all trades occurring after the best ask falls from $10.16 to $10.13.

The other samples and corresponding control samples are constructed in an analogous manner. In the “ask falls below nickel,” “ask falls to nickel,” “bid rises above nickel,” and “bid rises to nickel” control samples, N is .15, .25, .35, .45, .55, .65, .75, and .85. In the “ask rises while staying below nickel” and “bid falls while staying above nickel” control samples, N is .25, .35, .45, .55, .65, and .75. All of these nickel thresholds are chosen to avoid any overlap between an integer threshold sample and the corresponding nickel threshold control sample.

5. Buy–Sell Imbalances of Liquidity Demanders

5.1. Unconditional Buy–Sell Imbalances

For each .XX price point, we aggregate all buys and sells for each firm in each year (e.g., trades at $1.99, $2.99, $3.99, etc. are aggregated at the .99 price point). The buy–sell ratio is then computed for each firm-year. This ratio is computed in three different ways: number of buys/number of sells, shares bought/shares sold, and dollars bought/dollars sold. The median of these three ratios over all firm-years is then computed for each price point from .00 to .99.

Figure 1 shows the median number of buys/number of sells by .XX price point, Figure 2 shows the median shares bought/shares sold by .XX price point, Figure 3 shows the median dollars bought/dollars sold by .XX price point. The “ask falls below integer” sample is constructed as follows. We include it in the data set if (i) the previous best ask is one integer higher than the current best ask, (ii) the digits after the decimal point of the previous best ask are in [.00, .10], and (iii) the digits after the decimal point of the current best ask are in [.90, .99]. If all three conditions are met, we then collect all trades that occur while the ask quote remains in [.90, .99]. An example of the “ask falls below integer” sample would be all trades occurring after the ask quote falls from $10.01 to $9.99. The corresponding control sample is “ask falls below nickel,” which includes all trades after the ask drops from [.01, .10] to [.00] until the ask leaves [.00]. The corresponding control sample is “ask falls below nickel,” which includes all trades after the ask drops from [.01, .10] to [.00] until the ask leaves [.00]. The corresponding control sample is “ask falls below nickel,” which includes all trades after the ask drops from [.01, .10] to [.00] until the ask leaves [.00]. The corresponding control sample is “ask falls below nickel,” which includes all trades after the ask drops from [.01, .10] to [.00] until the ask leaves [.00]. The corresponding control sample is “ask falls below nickel,” which includes all trades after the ask drops from [.01, .10] to [.00] until the ask leaves [.00]. The corresponding control sample is “ask falls below nickel,” which includes all trades after the ask drops from [.01, .10] to [.00] until the ask leaves [.00]. The corresponding control sample is “ask falls below nickel,” which includes all trades after the ask drops from [.01, .10] to [.00] until the ask leaves [.00]. The corresponding control sample is “ask falls below nickel,” which includes all trades after the ask drops from [.01, .10] to [.00] until the ask leaves [.00].


5 The TAQ data begins January 4, 1993. The $1/8 tick-size era ends June 23, 1997, for NYSE; May 6, 1997, for AMEX; and June 1, 1997, for NASDAQ. The $1/16 tick-size era ends January 28, 2001, for NYSE and AMEX and March 31, 2001, for NASDAQ.
Figure 1  Median (Number of Buys/Number of Sells) by Liquidity Demanders at .XX Price Points

Figure 2  Median (Shares Bought/Shares Sold) by Liquidity Demanders at .XX Price Points

point, and Figure 3 shows the median dollars bought/dollars sold by .XX price point. All three figures resemble waves. The wave peaks, which represent a high ratio of buys to sells by liquidity demanders, occur at trade prices immediately below dollars, half-dollars, quarters, dimes, and nickels (i.e., .04, .09, .14, .19, etc.). The wave valleys, which represent a low ratio of buys to sells by liquidity demanders, occur at trade prices ending in .99, and the lowest ratio of buys to sells by liquidity demanders occur at trade prices ending in .01. The second-highest ratio occurs at .49 and the second-lowest ratio occurs at .51. In all three figures,
the buy–sell ratio at both .24 and .74 are higher than any of the other .X4 price points, and the buy–sell ratio at both .26 and .76 are lower than any of the other .X6 price points. In other words, the largest imbalances occur at the price points surrounding the whole dollar, the second-largest imbalances surround the half-dollar, and the third-largest imbalances surround quarters.

Further investigation of Figures 1–3 also reveals a regular pattern every 10 cents. Figure 4 explores this further by showing the median buy–sell ratios of liquidity demanders by penny-ending price points: .X0, .X1, .X9. Interestingly, the pattern of buy–sell ratios by penny-ending price points is nearly identical for all three buy–sell ratio measures. Specifically, we notice that the highest buy–sell ratios are at prices ending in .X9 and the lowest buy–sell ratios are at prices ending in .X1, surrounding dimes. Similarly, the second-highest ratios are at prices ending in .X4 and the second-lowest ratios are at prices ending in .X6, surrounding nickels.

Table 1 formalizes these observations by regressing the buy–sell ratios of liquidity demanders for each firm-year on dummy variables for price points that are above or below round numbers. The three regressions are based on three versions of the buy–sell ratio. For all three regressions, the coefficients for Below Integers, Below Half-Dollars, Below Quarters, Below Dimes, and Below Nickels are all positive and statistically significant at the 1% level, indicating significant excess buying below round numbers. Similarly, the coefficients for Above Integers, Above Half-Dollars, Above Quarters, Above Dimes, and Above Nickels are all negative and statistically significant at the 1% level, indicating significant excess selling above round numbers.

Looking at the absolute value of the coefficients in all three regressions, they are monotonically ordered from most round to least round. Specifically, the coefficients for the “below” thresholds adhere to the following pattern of inequalities: Below Integers > Below Half-Dollars > Below Quarters > Below Dimes > Below Nickels. Similarly, the absolute value of the “above” coefficients adhere to the following pattern of inequalities: |Above Integers| > |Above Half-Dollars| > |Above Quarters| > |Above Dimes| > |Above Nickels|. 
5.2. Conditional Buy–Sell Imbalance Tests

Panels A, C, D, and E in Table 2 offer the main conditional results. Panel A in Table 2 shows the difference in median (mean) buy–sell ratio of liquidity demanders between the “ask falls below integer” sample and the “ask falls below nickel” benchmark. The three columns show the results for the three buy–sell ratio measures: number of buys/number of sells, shares bought/shares sold, and dollars bought/dollars sold. All six differences (mean and median for each of the three buy–sell measures) are positive values and are statistically significant at the 1% level. This is evidence of excess buying. Panel C contains the difference in median (mean) buy–sell ratio of liquidity demanders between the “ask falls to integer” sample and the “ask falls to nickel” benchmark. All six differences are large positive values and are statistically significant at the 1% level. This is strong evidence of a huge amount of excess buying. Panel D contains the difference in median (mean) buy–sell ratio of liquidity demanders between the “bid rises to integer” sample and the “bid rises to nickel” benchmark. All six differences are large negative values and four are statistically significant at the 1% level. This is evidence of excess selling.

Panels B and F contain the results when prices do not reach or cross an integer. The left-digit effect and the round number effect have no predictions here. Panel B gives the difference in median (mean) buy–sell ratio of liquidity demanders between the “ask rises while staying below integer” sample and the “ask rises while staying below nickel” benchmark. All six differences are positive values, but only three of the differences are statistically significant at the 1% level, considerably weaker evidence than was seen in panel A. Thus, we see that excess buying when prices fall to cross the integer (panel A) is slightly stronger than when prices rise but do not cross the integer (panel B). Panel F gives the difference in median (mean) buy–sell ratio of liquidity demanders between the “bid falls while staying above integer” sample and the “bid falls while staying above nickel” benchmark. Five of the six coefficients are negative, but only two are statistically significant. The magnitude of the median coefficients in panel E is about 2.5 times larger than the median coefficients in panel F. Thus, we see that excess selling when prices rise to cross the integer (panel E) is much stronger than the excess selling when prices fall but do not cross the integer (panel F).

Table 2

<table>
<thead>
<tr>
<th>Number of buys/number of sells</th>
<th>p-value</th>
<th>Shares bought/shares sold</th>
<th>p-value</th>
<th>Dollars bought/dollars sold</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>1.175*</td>
<td>&lt;0.0001</td>
<td>1.338*</td>
<td>&lt;0.0001</td>
<td>1.348*</td>
</tr>
<tr>
<td>Below Integers (.99)</td>
<td>1.493*</td>
<td>&lt;0.0001</td>
<td>2.548*</td>
<td>&lt;0.0001</td>
<td>2.626*</td>
</tr>
<tr>
<td>Above Integers (.01)</td>
<td>−0.367*</td>
<td>&lt;0.0001</td>
<td>−0.449*</td>
<td>0.0002</td>
<td>−0.458*</td>
</tr>
<tr>
<td>Below Half-Dollars (.49)</td>
<td>0.904*</td>
<td>&lt;0.0001</td>
<td>1.225*</td>
<td>&lt;0.0001</td>
<td>1.240*</td>
</tr>
<tr>
<td>Above Half-Dollars (.51)</td>
<td>−0.356*</td>
<td>&lt;0.0001</td>
<td>−0.366*</td>
<td>0.0025</td>
<td>−0.385*</td>
</tr>
<tr>
<td>Below Quarters (24, .74)</td>
<td>0.626*</td>
<td>&lt;0.0001</td>
<td>0.859*</td>
<td>&lt;0.0001</td>
<td>0.856*</td>
</tr>
<tr>
<td>Above Quarters (26, .76)</td>
<td>−0.268*</td>
<td>&lt;0.0001</td>
<td>−0.297*</td>
<td>0.0006</td>
<td>−0.309*</td>
</tr>
<tr>
<td>Below Dimes (.09, .19, .29, .39, .59, .69, .79, .89)</td>
<td>0.483*</td>
<td>&lt;0.0001</td>
<td>0.647*</td>
<td>&lt;0.0001</td>
<td>0.642*</td>
</tr>
<tr>
<td>Above Dimes (.11, .21, .31, .41, .51, .61, .71, .81, .91)</td>
<td>−0.246*</td>
<td>&lt;0.0001</td>
<td>−0.276*</td>
<td>&lt;0.0001</td>
<td>−0.277*</td>
</tr>
<tr>
<td>Below Nickels (.04, .14, .34, .44, .54, .64, .84, .94)</td>
<td>0.270*</td>
<td>&lt;0.0001</td>
<td>0.370*</td>
<td>&lt;0.0001</td>
<td>0.381*</td>
</tr>
<tr>
<td>Above Nickels (.06, .16, .36, .46, .56, .66, .86, .96)</td>
<td>−0.177*</td>
<td>&lt;0.0001</td>
<td>−0.222*</td>
<td>&lt;0.0001</td>
<td>−0.227*</td>
</tr>
<tr>
<td>N</td>
<td>55,503</td>
<td></td>
<td>55,503</td>
<td></td>
<td>55,503</td>
</tr>
</tbody>
</table>

Notes: The buy–sell ratio of liquidity demanders for each firm-year is regressed on dummy variables for price points that are below or above round numbers. Three definitions of the buy–sell ratio are provided: number of buys/number of sells, shares bought/shares sold, and dollars bought/dollars sold. The sample spans 2001–2006 in the decimal era and consists of 100 randomly selected stocks with annual replacement of stocks that do not survive.

*Means statistically significant at the 1% level.
trade by a liquidity demander. The table reports the difference in regression coefficients between each of the six integer cases and their corresponding nickel benchmarks. Although the coefficients are not shown, each regression includes the following controls: trade size dummies, price level dummies, firm size dummies, institutional ownership dummies, share volume dummies, penny-ending dummies (e.g., .00 – .99), exchange dummies, and year dummies.

Table 3 confirms the univariate results found in Table 2 and shows that they are robust to controlling for firm-specific, trade-specific, exchange-specific, and year-specific effects. Under each of the three buy–sell specifications, the regression coefficients for the “ask falls below integer,” the “ask falls to integer,” the “bid rises to integer,” and the “bid rises above integer” samples, less their corresponding nickel benchmarks, are of the predicted sign and are statistically significant.

Interestingly, the magnitude of the coefficients of the “ask falls to integer” case is seven or more times larger than the “ask falls below integer” case. Similarly, the magnitude of the coefficients of the “bid rises to integer” case is three or more times larger than the “bid rises above integer” case. This is strong evidence in favor of H2A—the reach-only version—which states that excess trades are predominantly determined by prices reaching the integer. This also represents strong evidence against H2B and H2C—the cross-only case and the reach-and-cross case, respec-
Ask Rises While Staying Below Integer

Table 4 shows results from testing H3 in a multivariate setting. The table reports the difference in coefficients between first left-digit changes and second left-digit changes for each price path. We find that the first left-digit change is stronger around two-digit integers than second left-digit changes around two-digit integers. Although the difference in coefficients correctly predicts the sign in 11 of 12 cases, it is statistically significant at the 1% level in only 3 of the 12 tests. This provides only modest support for H3 after controlling for other influences. Table 5 shows results from testing H4 in a multivariate setting. The table reports the difference in coefficients between the first left-digit change around two-digit integers and the first left-digit change around one-digit integers. We find that the first left-digit change is stronger around two-digit integers than around one-digit integers in 9 of 12 cases. In all 3 of the cases in which the sign is not correctly predicted, the result is not statistically significant. Of the 9 cases in which the sign is correctly predicted, statistical significance exists for 8 of them. On balance, this supports H4 after controlling for other influences.

6. 24-Hour Returns

6.1. Unconditional Returns

We begin this section with unconditional returns. For each XX price point, we compute 24-hour returns in two different ways. First, we compute 24-hour trade...
price returns. For every buy trade observation, we compute the return to buying at the actual trade price and then selling at the bid price 24 hours later to close the position.\textsuperscript{7} Similarly, for every sell trade observation, we compute the return to (short) selling at the actual trade price and then buying at the ask price 24 hours later to close the position. Second, we compute 24-hour midpoint returns. For every buy trade observation, we compute the return to buying at the contemporaneous quote midpoint price and then selling at the quote midpoint price 24 hours later to close the position. For every sell trade observation, we compute the return to (short) selling at the contemporaneous quote midpoint price and then buying at the quote midpoint price 24 hours later to close the position. Thus, for each .XX price point, we end up with four return categories: (1) the 24-hour trade price return to buying, (2) the 24-hour midpoint return to buying, (3) the 24-hour trade price return to selling, and (4) the 24-hour midpoint return to selling.

Figure 5 plots the buy–sell ratios of all 100 price points on the left $y$-axis and the difference in median 24-hour trade price returns (median return to selling minus median return to buying) at all 100 price points on the right $y$-axis. The solid curve is the buy–sell ratio. The dashed curve is the difference in median 24-hour trade price returns. Clearly they are related! As before, the solid curve of the buy–sell ratio oscillates in a smooth wave reaching a peak at one penny below each round number and reaching a valley at one penny above each round number. The dashed curve of the difference in median 24-hour trade price returns almost always reaches a peak at one penny below each round number and reaches a valley at one penny above each round number. The two curves are very similar and the correlation between the two variables is 0.58.

Table 6 reports the regression of the difference in median 24-hour trade price or midpoint returns (i.e., median return to buying minus median return to selling)\textsuperscript{8} for each firm-year on dummy variables for the price points that are immediately above and below round numbers. We see a clear pattern of how liquidity demanders who buy below a round number threshold have lower returns than those who sell below that round number threshold. Likewise, those

\begin{table}
\centering
\caption{Multivariate Regressions: First vs. Second Digit Changes}
\begin{tabular}{lcccc}
\hline
 & \multicolumn{2}{c}{(1)} & \multicolumn{2}{c}{(2)} \\
 & Logistic: Probability of a buy trade & p-value & OLS: + shares bought for a buy or − shares sold for a sell & p-value \\
\hline
(Ask Falls Below Integer) × (First Left-Digit Change) & 0.009\textsuperscript{*} & 0.0029 & 5.494 & 0.3673 & −220.682 & 0.1522 \\
− (Ask Falls Below Integer) × (Second Left-Digit Change) & 0.014 & 0.0819 & 29.876 & 0.0679 & 830.711 & 0.0748 \\
(Ask Falls to Integer) × (First Left-Digit Threshold) & −0.044\textsuperscript{*} & <0.0001 & −42.337 & 0.0125 & −1,166.614 & 0.0157 \\
− (Ask Falls to Integer) × (Second Left-Digit Threshold) & −0.055\textsuperscript{*} & <0.0001 & −21.014 & 0.2307 & −755.117 & 0.1306 \\
\hline
Trade size dummies & YES & YES & YES & YES & YES & YES \\
Price level dummies & YES & YES & YES & YES & YES & YES \\
Firm size dummies & YES & YES & YES & YES & YES & YES \\
Institutional ownership level dummies & YES & YES & YES & YES & YES & YES \\
Share volume level dummies & YES & YES & YES & YES & YES & YES \\
Exchange dummies & YES & YES & YES & YES & YES & YES \\
Year dummies & YES & YES & YES & YES & YES & YES \\
Penny-ending dummies & YES & YES & YES & YES & YES & YES \\
\hline
$N$ & 74,819,798 & 74,819,798 & 74,819,798 & 74,819,798 & 74,819,798 & 74,819,798 \\
\hline
\end{tabular}
\textit{Notes.} Column (1) is a logistic regression in which the dependent variable takes a value of 1 if the trade is a buy or a 0 if it is a sell. Column (2) is an OLS regression where the dependent variable is + shares bought for a buy or − shares sold for a sell. Column (3) is an OLS regression where the dependent variable is + dollars bought for a buy or − dollars sold for a sell. Controls for trade size, price, firm size, institutional holdings, volume, exchange, year, and penny-ending are included in each regression. Interaction terms select cases where reaching or crossing the threshold causes a first left-digit change (e.g., ask price falls from $30.01$ to $29.99$) versus causing a second left-digit change (e.g., ask price falls from $21.01$ to $20.99$). The sample spans 2001–2006 in the decimal era and consists of 100 randomly selected stocks with annual replacement of stocks that do not survive.

\textsuperscript{*}Means statistically significant at the 1% level.
\end{table}

\textsuperscript{7}For example, if there is a buy at 11:00 a.m. on day $t$, then a return is computed from buying at the trade price to selling at the bid price at 11:00 a.m. on day $t + 1$. Twenty-four-hour returns are slightly cleaner than returns until the end of the day, because they avoid the end-of-day pricing anomaly documented in Harris (1989).

\textsuperscript{8}In Tables 6 and 7 the dependent variable is the median return to buying minus the median return to selling, which is easier to interpret, but it is the opposite convention to that used in Figures 5 and 6.
who sell above a round number threshold have lower returns than those who buy above that round number threshold. Specifically, in the first column, which reports the coefficients for the differential trade price returns, we find that .99 has a negative coefficient (a lower differential return between buying and selling than the other price points) and .01 has a positive coefficient (a higher differential return between buying and selling than the other price points). Similarly, below half-dollars is negative and above half-dollars is positive. Below quarters is negative and above quarters is positive, and so on. Although the signs alternate, the coefficients are sometimes not statistically significant. In the second column, which reports the coefficients for the differential midpoint returns, the same positive/negative pattern is true, though with diminished magnitude.

Figure 6 plots the buy–sell ratios for penny-ending price points (X0, X1, X2, . . . , X9) on the left y-axis and the difference in median 24-hour trade price returns (median return to selling minus median return to buying) for penny-ending price points on the right y-axis. The buy–sell ratios and the difference in returns both form W-shaped figures that almost perfectly overlap. Clearly, there is a strong relationship between buy–sell ratios and the difference in returns. The correlation between the two variables is 0.87.

Chordia et al. (2002) show that daily “order imbalance” (number of buy trades minus the number of sell trades),9 is a major determinant of daily stock returns. The buy–sell ratio is a nonlinear transformation of buys minus sells. In the spirit of Chordia

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Table 5  Multivariate Regressions: Two-Digit vs. One-Digit Integers

<table>
<thead>
<tr>
<th></th>
<th>(1) Logistic: Probability of a buy trade</th>
<th>(2) OLS: +shares bought for a buy or −shares sold for a sell</th>
<th>(3) OLS: +dollars bought for a buy or −dollars sold for a sell</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Ask Falls Below Integer) × (First Left-Digit Change in Two-Digit Integers ≥20)</td>
<td>0.0271∗&lt;0.0001</td>
<td>−10.670 0.1261</td>
<td>1,056.33∗&lt;0.0001</td>
</tr>
<tr>
<td>(Ask Falls Below Nickel) × (Nickel Thresholds &gt; 20])</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Ask Falls Below Integer) × (First Left-Digit Change in One-Digit Integers &lt;10)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Ask Falls Below Nickel) × (Nickel Thresholds &lt; 10])</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Bid Rises to Integer) × (First Left-Digit Threshold in Two-Digit Integers ≥20)</td>
<td>−0.0176 0.0695 2.33 0.8687</td>
<td>4,611.49∗&lt;0.0001</td>
<td></td>
</tr>
<tr>
<td>(Ask Falls to Nickel) × (Nickel Thresholds &gt; 20])</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Bid Rises to Integer) × (First Left-Digit Change in Two-Digit Integers &lt;10)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Ask Falls to Nickel) × (Nickel Thresholds &lt; 10])</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Bid Rises Above Integer) × (First Left-Digit Change in Two-Digit Integers ≥20)</td>
<td>−0.0390∗&lt;0.0001</td>
<td>−66.69∗ 0.0005</td>
<td>−5,657.05∗&lt;0.0001</td>
</tr>
<tr>
<td>(Bid Rises to Nickel) × (Nickel Thresholds &gt; 20])</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Bid Rises Above Integer) × (First Left-Digit Change in One-Digit Integers &lt;10)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Bid Rises Above Nickel) × (Nickel Thresholds &lt; 10])</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trade size dummies</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Price level dummies</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Firm size dummies</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Institutional ownership level dummies</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Share volume level dummies</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Exchange dummies</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Year dummies</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Penny-ending dummies</td>
<td>YES</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>N</td>
<td>74,819,798</td>
<td>74,819,798</td>
<td>74,819,798</td>
</tr>
</tbody>
</table>

Notes. Column (1) is a logistic regression in which the dependent variable takes a value of 1 if the trade is a buy or a 0 if it is a sell. Column (2) is an OLS regression where the dependent variable is +shares bought for a buy or −shares sold for a sell. Column (3) is an OLS regression where the dependent variable is +dollars bought for a buy or −dollars sold for a sell. Controls for trade size, price, firm size, institutional holdings, volume, exchange, year, and penny-ending are included in each regression. Interaction terms select cases where reaching or crossing the threshold causes a first left-digit change in two-digit integers ≥20 (e.g., ask price falls from $30.01 to $29.99) versus causing a first left-digit change in one-digit integers <10 (e.g., ask price falls from $9.01 to $8.99). The sample spans 2001–2006 in the decimal era and consists of 100 randomly selected stocks with annual replacement of stocks that do not survive. ∗Means statistically significant at the 1% level.
Figure 5Median Buy–Sell Ratio Compared to the Difference in Median 24-Hour Trade Price Returns by .XX Price Points

Table 6Difference in Median 24-Hour Returns Regressed on Price Point Dummies

<table>
<thead>
<tr>
<th>Price Point Dummies</th>
<th>Difference in median 24-hour trade price returns (median return to buying−median return to selling) (%)</th>
<th>p-value</th>
<th>Difference in median 24-hour midpoint returns (median return to buying−median return to selling) (%)</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Intercept</td>
<td>0.11*</td>
<td>&lt;0.0001</td>
<td>0.12*</td>
<td>&lt;0.0001</td>
</tr>
<tr>
<td>Below Integers (.99)</td>
<td>−0.25</td>
<td>0.0019</td>
<td>−0.18</td>
<td>0.0090</td>
</tr>
<tr>
<td>Above Integers (.01)</td>
<td>0.14</td>
<td>0.0709</td>
<td>0.07</td>
<td>0.3145</td>
</tr>
<tr>
<td>Below Half-Dollars (.49)</td>
<td>−0.12</td>
<td>0.1372</td>
<td>−0.11</td>
<td>0.1360</td>
</tr>
<tr>
<td>Above Half-Dollars (.51)</td>
<td>0.20</td>
<td>0.0104</td>
<td>0.10</td>
<td>0.1520</td>
</tr>
<tr>
<td>Below Quarters (.24,.74)</td>
<td>−0.10</td>
<td>0.0772</td>
<td>−0.07</td>
<td>0.1440</td>
</tr>
<tr>
<td>Above Quarters (.26,.76)</td>
<td>0.11</td>
<td>0.0539</td>
<td>0.05</td>
<td>0.3613</td>
</tr>
<tr>
<td>Below Dimes (.09,.19,.29,.39,.59,.69,.79,.89)</td>
<td>−0.11*</td>
<td>0.0001</td>
<td>−0.10</td>
<td>0.0003</td>
</tr>
<tr>
<td>Above Dimes (.11,.21,.31,.41,.61,.71,.81,.91)</td>
<td>0.06</td>
<td>0.0392</td>
<td>0.04</td>
<td>0.1390</td>
</tr>
<tr>
<td>Below Nickels (.04,.14,.24,.34,.44,.54,.64,.74,.84,.94)</td>
<td>−0.03</td>
<td>0.3924</td>
<td>−0.02</td>
<td>0.4304</td>
</tr>
<tr>
<td>Above Nickels (.06,.16,.26,.36,.46,.56,.66,.76,.86,.96)</td>
<td>0.09*</td>
<td>0.0015</td>
<td>0.06</td>
<td>0.0361</td>
</tr>
</tbody>
</table>

N55,838  55,838

Notes. The difference in median 24-hour trade price (midpoint) returns for each firm-year is regressed on dummy variables for price points that are below or above round numbers. The 24-hour trade price (midpoint) return to buying is the return from buying at the trade price (midpoint) when a buy trade is observed and closing the position 24-hours later at the bid (midpoint) price. The 24-hour trade price (midpoint) return to selling is the return from short selling at the trade price (midpoint) when a sell trade is observed and closing the position 24-hours later at the ask (midpoint) price. The difference in median returns is the median return to buying minus the median return to selling. The sample spans 2001–2006 in the decimal era and consists of 100 randomly selected stocks with annual replacement of stocks that do not survive.

*Means statistically significant at the 1% level.
6.2. Conditional Returns

We now turn to conditional returns, which is a computation of returns conditional on the price path. We compute the 24-hour returns for buying after the ask falls to reach or cross the integer, and for selling after the bid rises to reach or cross the integer. These returns are compared to analogous 24-hour returns relative to benchmark nickel price points.

Table 8 reports the results of multivariate regressions. In panel A, the dependent variable is the 24-hour midpoint return. In panel B, the dependent variable is 24-hour midpoint return. The first four rows of both panels report abnormal 24-hour returns, defined as the difference in regression coefficients between four indicator variables for the “ask falls below integer buys,” “ask falls to integer buys,” “bid rises to integer sells,” and “bid rises above integer sells” benchmarks. Each regression includes controls for price level, firm size, institutional ownership, share volume, penny-ending (e.g., .X0–.X9), exchange, and year. The first column represents the full sample and includes a further control for trade size.

First consider the two crossing cases in the Full Sample column of panel A. “Ask falls below integer buys” and “bid rises above integer sells” exhibit abnormal 24-hour trade price returns of −0.07% and 0.01%, respectively. The former is significantly negative with a p-value less than 0.0001, and the latter is insignificant. Next consider the two reaching cases. “Ask falls to integer buys” and “bid rises to integer sells” exhibit abnormal 24-hour trade price returns of 0.06% and 0.04%, respectively. Both reaching cases are significantly positive, with p-values below 0.0001.

The abnormal 24-hour midpoint returns are relatively similar in sign and magnitude, but only three of the four midpoint returns are significant at the 1% level. To summarize, the two cross cases yield mixed abnormal returns, whereas the two reach cases yield positive abnormal returns that are significantly positive in three out of four returns.

The next three columns break out the sample by trade size. Small trades are those involving fewer than 500 shares, medium trades involve 500 to 2,000 shares, and large trades are those in excess of 2,000 shares. First consider the two crossing cases. “Ask falls below integer buys” yield abnormal 24-hour trade price returns that are significantly negative for small, medium, and large trades. The magnitude of negative return becomes larger for larger trades, increasing from −7 basis points (bps) for small trades to −9 bps for medium trades to −13 bps for large trades. “Bid rises above integer sells” yield insignificant abnormal 24-hour trade price returns for all trade sizes. The midpoint returns follow the same pattern.

Next consider the two reaching cases. “Ask falls to integer buys” and “bid rises to integer sells” yield abnormal 24-hour trade price returns that are significantly positive for small and medium trades, but insignificant for the large trades. The magnitude of the positive returns decreases as trade size increases. The midpoint returns follow a similar pattern.

In summary, the conditional returns for crossing cases

Note that the abnormal return coefficients do not imply that arbitrage profits can be made net of transaction costs. Rather, they suggest that liquidity demanders who are influenced by round number effects earn lower returns on these trades compared to other benchmark liquidity demanders.

Lee and Radhakrishna (2000) develop a methodology for breaking trades into small, medium, and large sizes that takes into
yield mixed returns, but the conditional returns for reach cases are robustly positive.

To determine the economic significance of threshold trigger effects, we make a very rough estimate of the wealth transfer implied by the unconditional and conditional returns. For the unconditional returns, we examine the size and frequency of buy trades below round numbers and sell trades above round numbers. We also compute the abnormal return to buying below (selling above) round numbers by regressing the median 24-hour trade price return to buying (selling) on dummy variables for below round numbers and for above round numbers. Together this information can be used to determine the aggregate size of the unconditional wealth transfer/year as follows:

\[
\text{Wealth transfer/year from buys below and sells above round numbers} = \left( \frac{\text{Abnormal return to Buying Below Integers}}{\text{Agg. dollar value of Buying Below Integers}} \right) + \cdots + \left( \frac{\text{Abnormal return to Buying Below Nickels}}{\text{Agg. dollar value of Buying Below Nickels}} \right) + \left( \frac{\text{Abnormal return to Selling Above Integers}}{\text{Agg. dollar value of Selling Above Integers}} \right) + \cdots + \left( \frac{\text{Abnormal return to Selling Above Nickels}}{\text{Agg. dollar value of Selling Above Nickels}} \right) \times \left( \frac{3,721 \text{ eligible firms/year on average}}{(100 \text{ firms/year in our sample})} \right) / (6 \text{ years}) = -0.00121 \text{ million/year.}
\]

The last multiplier scales our sample size up to the full size of the TAQ data set during 2001–2006. We assume that our random sample of 100 firms is representative of all firms. The $-1,021.2 million/year figure is based on abnormal 24-hour trade price returns. Repeating the calculation using abnormal 24-hour midpoint returns yields $-605.5 million/year. Averaging the two estimates, we obtain an unconditional yearly wealth transfer above and below round numbers of approximately $-813 million. Clearly, this is a sizable amount of money. It should also be noted that this is a somewhat conservative estimate of the yearly wealth transfer, because we are ignoring ineligible firms, such as those that change their listing exchange, ticker symbol, or CUSIP code.

We make a similar back-of-the-envelope computation for the conditional reach cases:

\[
\text{Wealth transfer/year from buys when the ask falls and sells when the bid rises to an integer} = \left( \frac{\text{Abnormal return to Ask Falls to Integer Buys}}{\text{Agg. dollar value of Ask Falls to Integer Buys}} \right) + \left( \frac{\text{Abnormal return to Bid Rises to Integer Sells}}{\text{Agg. dollar value of Bid Rises to Integer Sells}} \right) \times \left( \frac{3,721 \text{ eligible firms/year on average}}{(100 \text{ firms/year in our sample})} \right) / (6 \text{ years}) = 59.8 \text{ million/year.}
\]

The $59.8 million/year figure is based on abnormal 24-hour trade price returns from panel A in Table 8. Repeating the calculation using abnormal 24-hour midpoint returns from panel B in Table 8 yields $19.4 million/year. Averaging, we obtain a conditional yearly wealth transfer on integers of approximately $40 million.
Conditioning on the price path, we find strong excess buying (selling) by liquidity demanders when the ask falls (bid rises) to \textit{reach} the integer. We find relatively little buy–sell imbalance when the ask falls (bid rises) to \textit{cross} the integer. This evidence supports the left-digit effect and threshold trigger effect. All of these findings hold true under three different measures of the buy–sell ratio, in multivariate regressions with various controls, and in multiple robustness checks.

We find that 24-hour returns vary by price point, and buy–sell imbalances are a major determinant of that variation across price points. This motivates us

### 7. Conclusion

Using a random sample of more than 100 million stock transactions, we find excess buying by liquidity demanders at all price points one penny below round numbers (e.g., .04, .09, .14, .19, etc.) and excess selling by liquidity demanders at all price points one penny above round numbers (e.g., .01, .06, .11, .16, etc.). We find that the size of the buy–sell imbalance is monotonically ordered by the roundness of the adjacent round number (i.e., largest imbalance above and below integers, second-largest above and below half-dollars, etc.). This and further evidence supports the cluster undercutting effect.
to estimate the profits or losses incurred by trading on and around round numbers. We find that unconditional buys below (sells above) round numbers yield negative abnormal returns with an aggregate wealth transfer of $−813 million per year. Conditional buys (sells) when the ask falls (bid rises) to reach an integer yield positive abnormal returns with an aggregate wealth transfer of $40 million per year.

Finally, we consider the wider implications of our study. Liquidity-supplying, limit order submitters might consider fighting their behavioral tendency to cluster on round numbers. It appears that cluster undercutting is a relatively profitable strategy that might be an improvement over clustering. Similarly, liquidity-demanding value traders might consider fighting their behavioral tendency to buy below (sell above) round numbers. This could be done by intentionally switching their trading strategies to non-round price thresholds for action. Researchers might explore whether similar buy–sell imbalances on and around round numbers and similar variations by price point of average 24-hour returns exist in other asset classes, time periods, and countries. Directions for future research might include whether order imbalance trading halts vary by price point and whether arbitrage trading profits vary by price point.

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