Comparative Cheap Talk

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Cheap talk about private information

- Seller knows something about quality of a product
- Professor knows something about prospects of a student
- Analyst knows something about value of a stock
- Auditor knows something about viability of a company
- Ebay knows something about quality of sellers

Interval cheap talk (Crawford and Sobel, 1982)

- Biased Sender S knows realization of r.v. θ distributed uniformly on [0,1]
- Receiver R takes action a

$U^S = -(a-(\theta+b))^2$
$U^R = -(a-\theta)^2$

- Informed R chooses $a = \theta$
- But S ideal is $a = \theta+b$

- For any realization θ that S reports, R will believe true θ is lower
- So S must exaggerate even more and R compensates even more
- So communication breaks down and R just chooses $a=E[\theta]$

- S receives expected payoff of only $E[U^S] = -Var[\theta]-b^2$
- Which is less than if S told truth! $E[U^S] = -b^2$

But some communication is still possible for small b

- Consider b=1/10
- S states θ in [0,3/10] or θ in (3/10,1]
- S ideal action for θ=3/10 is a=4/10

- E[θ| θ in [0,3/10]]= 3/20
- E[θ| θ in (3/10,1]]=13/20
- So S is indifferent at θ=3/10 and prefers truth on either side

- Smaller b, more partitions
  b>1/4 no communication
  b<1/4 two partitions max
  b<1/12 three partitions max
  etc.

If interests are too far apart such cheap talk does not work

- Uncertainty over quality of a good - seller tells buyer it’s great
- Uncertainty over prospects for a stock – analyst says it’s a sure bet
- Uncertainty over whether a spending program is worthwhile – administrator says it’s essential
- Uncertainty over quality of a job applicant – recommender says he’s great

Cheap talk is not credible if there is some action the sender always wants the receiver to take (e.g. the maximal action)

What if there is uncertainty along multiple dimensions?

- Seller has multiple goods – they're all great!
- Analyst recommends multiple stocks – buy them all!
- Lobbyist favors multiple spending programs – they're all necessary!
- Professor has multiple students – they will all win Nobel prizes!

Can comparative statements be credible?

- Good A is better than B
- Stock A is better than B
- Proposal A is better than B
- Student A is better than B

- Comparative statements are positive along one dimension and negative along another dimension at the same time
- Can’t exaggerate!

- But still might have an incentive to invert the ordering

The symmetric model

- $\theta = (\theta_1, \ldots, \theta_N)$
  (density strictly positive on $[0,1]^N$ and different θ_k i.i.d.)
- $a = (a_1, \ldots, a_N)$
  (each action in [0,1], can choose actions independently)
- Payoffs additive across dimensions:
  $U^R(a,\theta) = \Sigma_k u^R(\theta_k, a_k)$
  $U^S(a,\theta) = \Sigma_k u^S(\theta_k, a_k)$

- Complete ordering: rank variables from worst to best
- Partial ordering: categorize variables into groups from worst to best (and don't differentiate within a group)

Recommendation game

- Professor (S) has N students to recommend to employer (R). Each student hired ($a_k = 1$) or not ($a_k = 0$).

$$u^S = (\theta_k - T^S)a_k$$
$$u^R = (\theta_k - T^R)a_k$$

- Employer hires a student if expected quality θ_k above T^R
- Professor wants a student to be hired if θ_k above T^S
- "CS" equilibrium in one dimension if $E[\theta_k|\theta_k<T^S]<T^R<E[\theta_k|\theta_k>T^S]$

Recommendation game – comparative cheap talk

- Two students k and k', where $\theta_k > \theta_{k'}$
- $E[\theta_{j:N}]=j/(N+1)$
- Assume $T^R=3/5$ so top student hired, bottom student not hired

- Sender payoff from correct ordering: $(\theta_k-T^S)1+(\theta_{k'}-T^S)0$
- Sender payoff from inverted ordering: $(\theta_k-T^S)0+(\theta_{k'}-T^S)1$
- So gain from deviation: $\theta_{k'} - \theta_k<0$

- True even if $T^S=0$ so professor always wants a student to be hired so CS interval cheap talk impossible

- Both employer and professor are happier if a better student is hired than if a worse student is hired – both utility functions supermodular

Same idea can be applied to standard uniform-quadratic C-S game

For all b, sender and receiver payoffs are supermodular ("sorting condition"), so any ordering is an equilibrium.

$u^R = -(a_k - \theta_k)^2$ $\quad\quad$ $u_{12}^R = 2>0$

$u^S = -(a_k-(\theta_k+b))^2$ $\quad\quad$ $u_{12}^S = 2>0$

$a_{j:N} = E[\theta_{j:N}]=j/(N+1)$

So for any two $j> j'$, and $\theta_k > \theta_{k'}$ sender won't deviate if:

$-(E[\theta_{j':N}] -(\theta'+b))^2 -(E[\theta_{j:N}] -(\theta+b))^2 \geq$

$-(E[\theta_{j':N}] -(\theta+b))^2 -(E[\theta_{j:N}] -(\theta'+b))^2$

or $(E[\theta_{j:N}]-E[\theta_{j':N}])(\theta_k-\theta_{k'}) \geq 0$

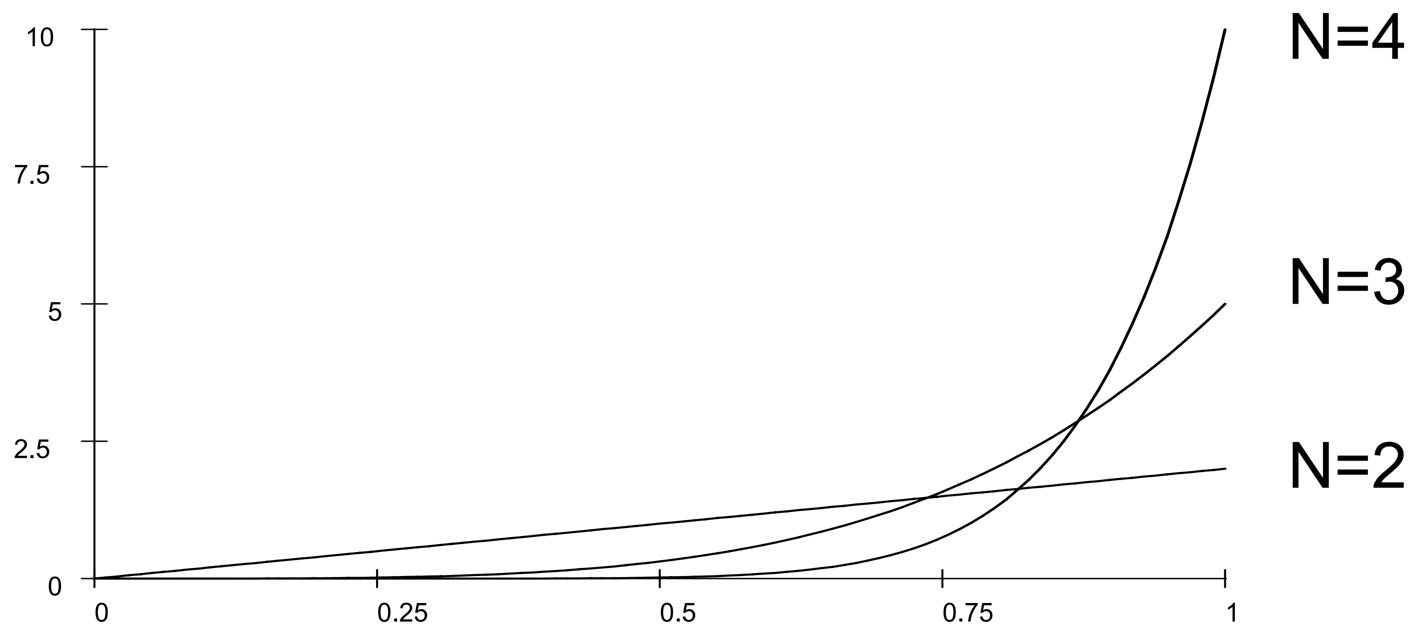
- Theorem 1: If u^R and u^S are supermodular in (θ_k,a_k) then the complete ordering and every partial ordering are equilibrium orderings.

- S states ordering, worst to best
- R chooses action $a_{j:N}$ for jth worst issue
- Sender won't lie if for any j>j' and $\theta_k>\theta_{k'}$:

$$u^S(\theta,a_{j:N})+u^S(\theta',a_{j':N}) \geq u^S(\theta',a_{j:N})+u^S(\theta,a_{j':N})$$

- But if $a_{j:N}> a_{j':N}$ this is just supermodularity
- So when does receiver take higher action for higher ranked issue?
- Receiver takes action to maximize utility. If knew θ for certain then supermodularity good enough – but only knows ranking of θ.
- Ranking implies $\theta_{j:N} >_{FOSD} \theta_{j':N}$.
- FOSD plus supermodularity implies that action is higher for higher ranked variables

Rankings become more informative as number of issues increases



Distribution of top-ranked issue

- Theorem 2: Under the complete ordering, expected sender and receiver payoffs asymptotically approach the full information case as the number of issues N increases.
- Application to uniform-quadratic game:
- Sender payoffs are concave:

  $u^R = -(a_k-\theta_k)^2$

  $u^S = -(a_k-(\theta_k+b))^2$

- Babbling per-issue payoff:

  $E[u^R] = -Var[\theta_k]$

  $E[u^S] = -Var[\theta_k]-b^2$

- Complete ordering payoff for issue j:

  $E[u^R] = -Var[\theta_{j:N}]$

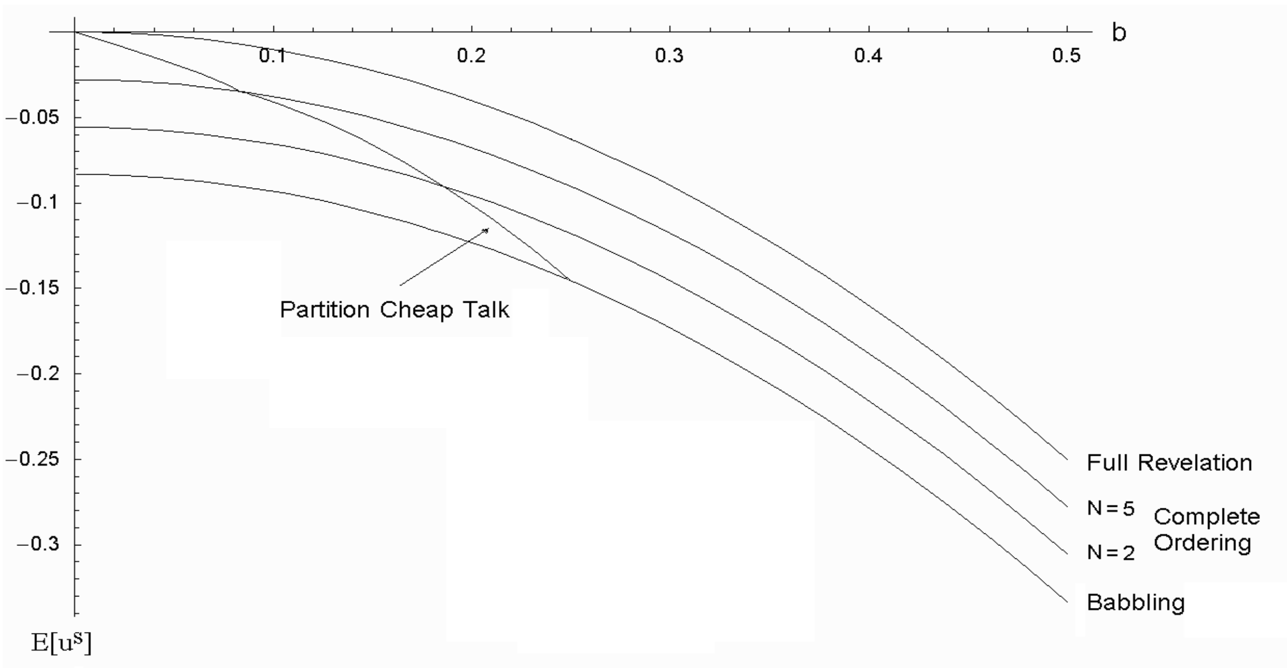
  $E[u^S] = -Var[\theta_{j:N}]-b^2$

- In complete ordering $Var[\theta_{j:N}]$ goes to 0 in limit:

  $E[u^R] = 0$

  $E[u^S] = -b^2$

Effect on expected sender payoffs



For given N, partition cheap talk outperforms ordinal cheap talk for small enough bias b

For given b, ordinal cheap talk outperforms partition cheap talk for large enough N

What if asymmetric preferences, distributions?

- Theorem 3 (roughly): For N=2 comparative cheap talk is generically robust to small asymmetries.
  - Have to take rankings with “a grain of salt” and adjust for sender bias
  - Large enough asymmetries then cheap talk can break down
  - But in some cases still works for arbitrary asymmetries in preferences
  - Information loss from asymmetries

- Theorem 4 (roughly): For large enough N there are always some issues similar enough to be compared.

As before let $a_{j:N}$ be R's best action for issue ranked *j*th from the bottom. And let $a(\theta_k)$ be R's best action when θ_k is known. For each $q \in (0,1)$, by the Glivenko-Cantelli Theorem

$$\lim_{N\to\infty}\theta_{\lceil qN\rceil:N} = F^{-1}(q) \text{ a.s.}$$

where $F(\theta_k)$ is distribution of each θ_k.

Therefore for i=S,R,

$$\lim_{N\to\infty}(1/N)\sum_j E[u^i(a_{j:N},\theta_{j:N})]=E[u^i(a(\theta_k),\theta_k)]$$

So in the limit as the number of issues increases, the sender reveals **all** information and sender and receiver payoffs are equivalent

Is complete order always better than partial order?

Recommendation game as before, $T^R=3/5$, $T^S=0$

Babbling/No Information:

- $E[\theta_k]=1/2$ so no one is hired

Complete ordering:

- N=2: $E[\theta_{1:2}]=1/3$, $E[\theta_{2:2}]=2/3$
- N=3: $E[\theta_{1:3}]=1/4$, $E[\theta_{2:3}]=2/4$, $E[\theta_{3:3}]=3/4$
- N=4: $E[\theta_{1:4}]=1/5$, $E[\theta_{2:4}]=2/5$, $E[\theta_{3:4}]=3/5$, $E[\theta_{4:4}]=4/5$
- N=5: $E[\theta_{1:5}]=1/6$, $E[\theta_{2:5}]=2/6$, $E[\theta_{3:5}]=3/6$, $E[\theta_{4:5}]=4/6$, $E[\theta_{5:5}]=5/6$

Partial ordering:

- N=3: $E[\theta_{1:3}]=1/4$, $E[\theta_{\{2,3\}:3}]=5/8$,
- N=4: $E[\theta_{\{1,2\}:4}]=3/10$, $E[\theta_{\{3,4\}:4}]=7/10$
- N=5: $E[\theta_{\{1,2\}:5}]=1/4$, $E[\theta_{\{3,4,5\}:5}]=2/3$

Conclusion

- Multiple dimensions increase the scope for communication
- Simple rankings are often credible when other forms of cheap talk are not
- These rankings can be surprisingly informative
- Sometimes sender prefers a partial ordering
- And sometimes only a partial ordering is credible
- Asymmetries, private receiver info reduce scope for communication but often still possible

Extensions

- Interdependent actions
  - Can only hire one person
  - Must buy goods in bundle
- Sender and receiver take actions, e.g. bargaining
- Private receiver information, e.g. auctions
- Non-additive payoffs
- Reputation