Good, Better, Best: Comparative Price Signaling

Michael R. Baye and Rick Harbaugh*

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Abstract

Existing work on price signaling finds that, in the absence of reputational or other effects, a firm typically must distort its price above the full-information monopoly price in order to credibly signal the quality of its product. We reexamine this classic problem for the case of a multi-product monopolist. We show that, owing to the multi-product nature of the firm's pricing problem, relative prices can credibly signal qualities with little or none of the distortion required for a single-product firm. Thus, while factors such as reputation, dynamics, and learning are clearly important in many signaling environments, they are not necessary for a multi-product firm to efficiently signal quality through its prices. We apply our results to versioning and crimping environments where the multi-product firm uses differential prices to screen buyers based on their preferences for quality, and buyers rationally use prices to infer which version is the higher quality. We conclude with a discussion of the implications of our results for cheap talk and provide a novel rationale for resale price maintenance.

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1 Introduction

When can a high price signal high quality? As long recognized, consumers often infer that high-priced products are high quality (Monroe, 1973), but then why doesn't a seller of a low quality product fool buyers by also setting a high price? If the price of a high quality product is set so high that there is very little demand at that price, then a seller of a low quality product will not want to mimic the high price. However, at such a high price, a seller of a low quality product must earn strictly higher profits than the seller of a high quality product, which provides no incentive to generate high quality products in the first place. This presents somewhat of a paradox: price signaling seems pervasive, but theory suggests that it only works in limited circumstances.

A number of approaches have been taken to resolve this problem that credible price signaling appears to require a highly distortionary price. Wolinsky (1983) assumes that some buyers are informed of product quality while others are not. The higher the fraction of informed buyers, the less the distortion that is needed. Milgrom and Roberts (1986) consider a two-period game where buyers become informed of quality in the second period. This gives low quality producers less incentive to mimic high quality producers since their deception will be uncovered. Bagwell and Riordan (1991) combine these approaches in a model where the fraction of informed consumers increases over time so the necessary price distortion decreases with time. Judd and Riordan (1994) assume that consumers have their own noisy signals of product quality which are combined with the price signal. Choi (1998) shows that a firm that can leverage its reputation in one market into another market can introduce a new experience good with less price distortion. Daughety and Reinganum (2008) show how the availability of verifiable information on product quality can also reduce the price distortion required by price signaling.

We present a simple one-period model that does not rely on factors such as reputation or the presence of informed consumers to facilitate price signaling. Our model allows us to address price signaling in a variety of different settings. For example, suppose a consumer walks into an appliance store to purchase a new refrigerator, and observes three different models selling at three different prices. If the seller knows one model is good, one is better, and one is best, can the store signal quality through prices to obtain first-best (full information) profits? Would the answer differ if the models were sold by three different retailers?

As another example, consider the impact that the internet has had on the ability of consumers to observe prices. Prior to the internet, if a clothing manufacturer sold different grades of shirts though different types of retailers (low-grade products through outlet stores and high-grade products through high-street stores), it was difficult for consumers to directly observe the prices at both stores before making a purchase. Today, however, consumers can observe prices at both stores. Does the ability to compare prices at different stores make it more difficult for retailers to pawn off low-grade products as being of a higher grade? Does it enhance retailer's abilities to obtain first-best (full information) profits even when quality is not directly observable?

A related example concerns the placement of products in a store. Wine, for example, comes in a range of qualities. If a consumer walks into a store and observes two bottles of wine next to each other, one with a higher price, can the consumer infer that the higher priced bottle is higher quality? Would price signals be less credible if the consumer did not have the different prices to compare? Does the seller earn higher profits by placing the products in locations that facilitate price comparisons?

More generally, in the absence of economies of scale, scope, and cross-price demand effects, does a firm selling multiple products have an advantage over single product firms?

We show that the answer to all of these questions is yes. Unlike other one-period models which assume that a firm sells only one product, we assume that a single firm derives profits from sales of products of different qualities.[1] These qualities are known by the firm but not by consumers.

2 A Simple Model

A monopolist produces two vertically differentiated products, $i = A, B$. One may view these products as versions of a similar product (e.g., a "good" refrigerator and "best" refrigerator), although this interpretation is not essential in the sequel. The monopolist knows the quality of each product (and thus whether the "best" refrigerator is really better than the "good" one), but consumers do not. From the perspective of a consumer, one of the products is of higher quality and the other of lower quality, but she does not know which is which and thus views each possibility as equally likely. Let $\theta_i \in \{\theta^L, \theta^H\}$ denote the true quality of product i with $\theta^H > \theta^L$, and let c^H and c^L denote the corresponding unit costs with $c^H > c^L$. We assume that under perfect information exchange of both products would be feasible: $\theta^H > c^H$ and $\theta^L > c^L$.

A consumer's realized utility from buying product i is $V_i = \theta_i + \varepsilon_i$, where ε_i is the consumer's idiosyncratic preference parameter for product i. The utility from not buying is normalized to zero. These preference parameters are the private information of the consumer and each ε_i is an

[1] Daughety and Reinganum (2005) and Janssen and Roy (2010) examine competition between multiple single-product firms. Our analysis requires that the products are sold by the same firm.

independent draw from the distribution function F on $[0,1]$. Let μ_i represent the consumer's belief that product i is in fact high quality (i.e., that $\theta_i = \theta^H$), and let $E_{\mu_i}[\theta_i] \equiv \mu_i\theta^H + (1-\mu_i)\,\theta^L$ denote the corresponding expected quality of product i.

Given these assumptions, the consumer buys product i if the expected surplus from doing so exceeds the price,

$$E_{\mu_i}[\theta_i] + \varepsilon_i \geq p_i. \tag{1}$$

Since the monopolist knows the true quality of each product (θ_i) but not consumer preferences (ε_i), from its perspective a given consumer purchases product i with probability

$$\Pr[E_{\mu_i}[\theta_i] + \varepsilon_i \geq p_i] = \Pr[\varepsilon_i \geq p_i - E_{\mu_i}[\theta_i]] = 1 - F(p_i - E_{\mu_i}[\theta_i]). \tag{2}$$

Thus the demand for product i given consumer beliefs is

$$q_i(E_{\mu_i}[\theta_i], p_i) = 1 - F(p_i - E_{\mu_i}[\theta_i]). \tag{3}$$

For simplicity, throughout the remainder of this section we assume F is uniform so that demand for product i is

$$\begin{aligned} q_i &= E_{\mu_i}[\theta_i] - p_i \\ &= \mu_i\theta^H + (1-\mu_i)\,\theta^L - p_i. \end{aligned} \tag{4}$$

Our assumptions imply that the demand and cost for each product are independent of the other. This is by design; it allows us to show that, in a multiproduct monopoly setting, such independent demands are linked by consumer inferences about product quality from price signaling.

2.1 Full Information Benchmark

As a benchmark, first consider the case where consumers have complete information about the quality of each product. In this case the demand function is just $q_i = \theta_i - p_i$ and by standard calculations profit maximizing full information prices, quantities, and profits are, for $\theta_i = \theta^L, \theta^H$:

$$\begin{aligned} p^L &= \frac{\theta^L + c^L}{2},\ p^H = \frac{\theta^H + c^H}{2} \\ q^L &= \frac{\theta^L - c^L}{2},\ q^H = \frac{\theta^H - c^H}{2} \\ \pi^L &= \frac{\left(\theta^L - c^L\right)^2}{4},\ \pi^H = \frac{\left(\theta^H - c^H\right)^2}{4} \end{aligned} \tag{5}$$

so

$$\pi_A + \pi_B = \left(p^H - c^H\right)\left(\theta^H - p^H\right) + \left(p^L - c^L\right)\left(\theta^L - p^L\right). \tag{6}$$

There are two factors that affect prices and quantities in this linear demand model, the quality parameters and the unit costs. The higher quality product has a higher demand curve because $\theta^H > \theta^L$, and also has higher unit costs because $c^H > c^L$. Both these factors push up the full information price for the higher quality product,

$$p^H - p^L = \frac{1}{2}\left(\left(\theta^H - \theta^L\right) + \left(c^H - c^L\right)\right). \tag{7}$$

However, for quantities, these factors work in the opposite direction since higher quality pushes up the demand curve, but higher prices driven by higher costs push down quantity demanded,

$$q^H - q^L = \frac{1}{2}\left(\left(\theta^H - \theta^L\right) - \left(c^H - c^L\right)\right). \tag{8}$$

We will show that the key to nondistortionary price signaling is that higher costs for the higher quality product push down the full information quantity demanded more than high quality pushes up quantity demanded, i.e., $q^H \leq q^L$.

2.2 Multiproduct Firm Without Observability

We are interested in how a consumer can infer quality by observing the prices of the two different products. As a comparison for our results on such comparative price signaling, first consider the case where consumers can only see the price of one product.

Proposition 1 *Suppose a multiproduct firm sells high and low quality products in distinct markets, where consumers in each market have identical (but state-dependent) demands. If consumers in each market observe only the price in their market, a necessary condition for the existence of a price signaling equilibrium with full information profits is*

$$q^L - q^H \geq \left(c^H - c^L\right)\frac{q^H}{q^L + q^H}. \tag{9}$$

Proof. We will show that the existence of a price signaling equilibrium with full information profits implies the above inequality. Note that in such an equilibrium, in the market where the firm sells the high quality product, it charges p^H and sell q^H units to consumers in that market, and these consumers correctly infer that the product is of high quality. In the market where it sells the low-quality product it charges charges p^L and sells q^L units, and consumers in this market correctly infer that the product is of low quality. Recall that these full information prices and quantities are defined in equation (5).

Consider the market in which the firm sells the low-quality product. If the firm conforms it earns profits of

$$(p^L - c^L)q^L + (p^H - c^H)q^H. \tag{10}$$

If the firm deviates in this market and sets a high price p^H, consumers incorrectly infer that the product is high quality and the firm earns deviation profits of

$$(p^H - c^L)q^H + (p^H - c^H)q^H. \tag{11}$$

Thus a price signaling equilibrium with full information prices implies that such a deviation is not profitable:

$$(p^H - c^L)q^H \leq (p^L - c^L)q^L. \tag{12}$$

Substituting the full information prices p^H and p^L, this implies

$$\begin{aligned} \left(\frac{\theta^H + c^H}{2} - c^L\right) q^H &\leq \left(\frac{\theta^L + c^L}{2} - c^L\right) q^L, \text{ or} \\ \left(\frac{\theta^H + c^H - 2c^L}{2}\right) q^H &\leq q^L q^L. \end{aligned} \tag{13}$$

Adding and subtracting c^H from the left-hand-side permits us to write this inequality as

$$\begin{aligned} \left(\frac{\theta^H - c^H + c^H + c^H - 2c^L}{2}\right) q^H &\leq q^L q^L, \text{ or} \\ \left(\frac{\theta^H - c^H}{2} + c^H - c^L\right) q^H &\leq q^L q^L \end{aligned} \tag{14}$$

Using the definition of q^H and then factoring gives us

$$\begin{aligned} \left(q^H + c^H - c^L\right) q^H &\leq q^L q^L, \text{ or} \\ \left(c^H - c^L\right) q^H &\leq \left(q^L - q^H\right)\left(q^L + q^H\right) \end{aligned} \tag{15}$$

which implies

$$\left(c^H - c^L\right) \frac{q^H}{q^L + q^H} \leq q^L - q^H \tag{16}$$

as required. ■

Notice that in the absence of observability, there is no qualitative difference in this model between price signaling by a single two-product firm and price signaling by two independent single-

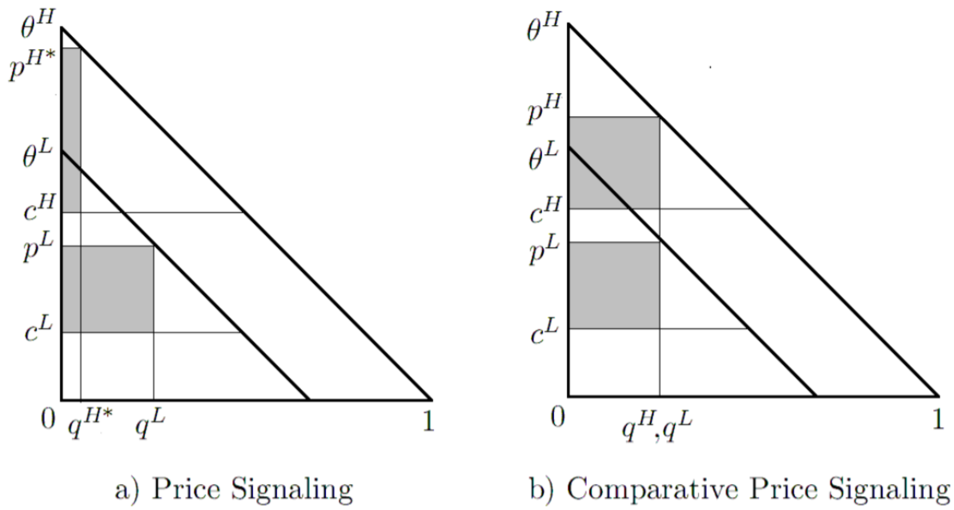

Figure 1: Equilibrium prices, quantities, and profits.

product firms where prices are not observable across markets. Hence the necessary condition from Proposition 1 also applies to price signaling by a one-product firm.[2]

Figure 1(a) depicts the case where $\theta^H = 1$, $\theta^L = 2/3$, $c^H = 1/2$, and $c^L = 1/6$. These parameters do not satisfy (9) so a price signaling equilibrium with full information profits does not exist. Instead a price signaling equilibrium requires a distorted price p^{H*} and corresponding quantity $q^{H*} < q^H$. This price is high enough that the firm with a low quality product (and hence low unit cost) does not benefit by deviating to it from a low price p^L and corresponding high quantity q^L, i.e., $\left(p^{H*} - c_L\right) q^{H*} \leq (p^L - c^L)q^L$. Because the high quality firm has higher unit costs, and the low quality firm can choose to mimic the same high price, in any equilibrium the high quality firm's profits must be less than the low quality firm's profits as seen by the shaded areas in the figure.

With this understanding of the difficulty of sustaining a price signaling equilibrium in the standard model, we now focus on our topic of how the observability of prices of multiple products sold by the same firm can facilitate the use of prices as signals.

[2]As a step toward understanding reputation effects, Choi (1998) characterizes price signaling equilibria for a one-product firm. Note that to make the observable and non-observable cases quantitatively equivalent, demand in the non-observable case needs to be doubled.

2.3 Comparative Price Signaling

We define a *comparative price signaling equilibrium* (CPS) as a perfect Bayesian equilibrium where consumers infer product quality by observing both prices. This is distinct from the price signaling environment above where consumers wishing to buy product i base beliefs only on p_i rather than the pair (p_A, p_B).

Proposition 2 *For a multiproduct firm facing independent linear demands, a comparative price signaling equilibrium with full information profits (i) exists if and only if $q^L \geq q^H$ (or, equivalently in terms of demand and cost parameters, $c^H - c^L \geq \theta^H - \theta^L$), and (ii) when such an equilibrium exists it survives the Divinity refinement, and is the unique separating perfect Bayesian equilibrium.*

Proof. (i) (Necessity) For the equilibrium to have full information profits, the price of the high quality product must be p^H and the price of the low quality product must be p^L, where $p^H > p^L$ from (5), and the consumers must believe the higher priced product is the higher quality product, and equilibrium profits must be

$$\frac{\left(\theta^H - c^H\right)^2}{4} + \frac{\left(\theta^L - c^L\right)^2}{4}. \tag{17}$$

Since these are profits in a CPS there cannot be a profitable deviation. We show that $q^L \geq q^H$ follows from the no deviation property. To see this, consider a deviation where the firm charges p^H for the lower quality product and p^L for the higher quality product. Since this deviation is on the equilibrium path, when consumers observe these prices they believe the low quality product is high quality and vice-versa. Therefore, the firm's profit from this deviation is

$$\begin{aligned} & (p^L - c^H)(\theta^L - p^L) + (p^H - c^L)(\theta^H - p^H) \\ = & \left(\frac{\theta^L + c^L}{2} - c^H\right)\left(\theta^L - \frac{\theta^L + c^L}{2}\right) + \left(\frac{\theta^H + c^H}{2} - c^L\right)\left(\theta^H - \frac{\theta^H + c^H}{2}\right) \end{aligned} \tag{18}$$

so for this deviation to not be profitable, the gain to the deviation must be negative:

$$\begin{aligned} \Delta = & \left(\frac{\theta^L + c^L}{2} - c^H\right)\left(\theta^L - \frac{\theta^L + c^L}{2}\right) + \left(\frac{\theta^H + c^H}{2} - c^L\right)\left(\theta^H - \frac{\theta^H + c^H}{2}\right) \\ & - \left(\left(\frac{\theta^H + c^H}{2} - c^H\right)\left(\theta^H - \frac{\theta^H + c^H}{2}\right) + \left(\frac{\theta^L + c^L}{2} - c^L\right)\left(\theta^L - \frac{\theta^L + c^L}{2}\right)\right) \\ = & \frac{1}{2}\left(c^H - c^L\right)\left(\left(\theta^H - c^H\right) - \left(\theta^L - c^L\right)\right) \\ = & \left(c^H - c^L\right)\left(q^H - q^L\right). \end{aligned} \tag{19}$$

Since $c^H > c^L$, $\Delta < 0$ implies $q^L \geq q^H$. This proves necessity.

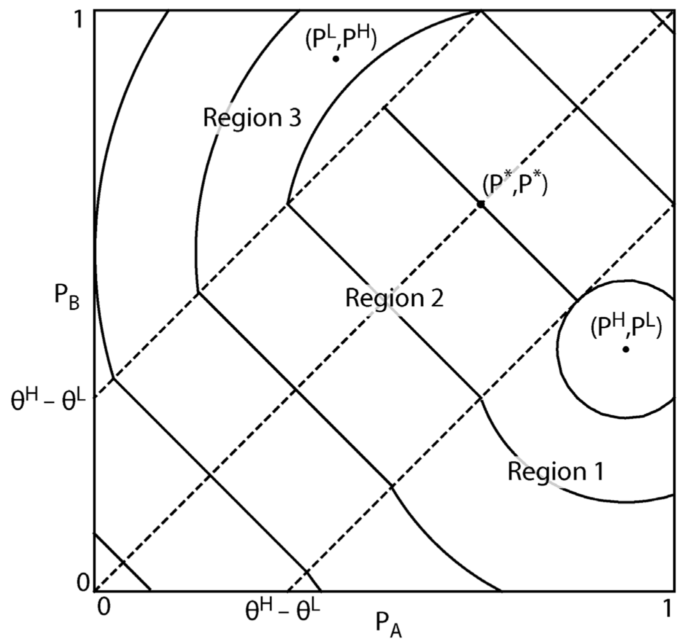

Figure 2: Independent linear demand: Isoprofit curves for type A given equilibrium beliefs

(Sufficiency) Suppose $q^L \geq q^H$ and consider a candidate CPS where the firm charges full information prices (p^L for the low quality product and p^H for the high quality product) and consumers correctly believe the higher priced product is the higher quality product. The only on-the-path deviation is where the firm flips the prices to trick consumers into believing that the low-quality product is of high quality. But, as in our proof of necessity, this deviation is not profitable to the firm when $q^L \geq q^H$. Thus, consider off-the-equilibrium-path deviations, $(p_A, p_B) \notin \{(p^L, p^H), (p^H, p^L)\}$.

Off the equilibrium path, we are free to choose beliefs so let

$$\mu_A = \begin{cases} 1 & \text{if} \quad p_A - p_B \geq \theta^H - \theta^L \quad \text{(Region 1)} \\ \frac{1}{2} + \frac{1}{2}\frac{p_A - p_B}{\theta^H - \theta^L} & \text{if} \quad |p_A - p_B| < |\theta^H - \theta^L| \quad \text{(Region 2)} \\ 0 & \text{if} \quad p_B - p_A \geq \theta^H - \theta^L \quad \text{(Region 3)} \end{cases} \tag{20}$$

and $\mu_B = 1 - \mu_A$. Notice that these beliefs have the intuitive property that when prices are equal consumers believe each product is equally likely to be high quality. And the greater the difference in prices, the stronger the belief that the higher priced product is higher quality. Without loss of generality, suppose the firm knows A is the high quality product.

First consider a deviation to prices in Region 1 where $p_A - p_B \geq \theta^H - \theta^L$. For these prices, consumers beliefs are $\mu_A = 1$. Given such beliefs, profits are maximized at $(p_A, p_B) = (p^H, p^L)$ by the definition of p^L, p^H so there is not a profitable deviation in Region 1.

Next consider a deviation to prices $(p_A, p_B) = (p', p'')$ in Region 3. Since these prices are in Region 3, $p'' - p' \geq \theta^H - \theta^L$ and consumers erroneously believe that product B is high quality ($\mu_B = 1$). In this case the firm sells the high-cost product based on the low quality demand function and vice versa, so the profits from such a deviation are

$$\left(p_A - c^H\right)\left(\theta^L - p_A\right) + \left(p_B - c^L\right)\left(\theta^H - p_B\right) \equiv \left(p' - c^H\right)\left(\theta^L - p'\right) + \left(p'' - c^L\right)\left(\theta^H - p''\right). \quad (21)$$

Compare this with an alternative deviation that flips these prices to $(p_A, p_B) = (p'', p')$. Since these flipped prices are in Region 1, consumers correctly believe that product A is high quality ($\mu_A = 1$). Profits from this alternative deviation are thus

$$\left(p_A - c^H\right)\left(\theta^H - p_A\right) + \left(p_B - c^L\right)\left(\theta^L - p_B\right) \equiv \left(p'' - c^H\right)\left(\theta^H - p''\right) + \left(p' - c^L\right)\left(\theta^L - p'\right). \quad (22)$$

Subtracting (21) from (22) yields $\left(c^H - c^L\right)\left((p'' - p') - \left(\theta^H - \theta^L\right)\right)$, which is non-negative since $p'' - p' \geq \theta^H - \theta^L$ by hypothesis. This implies that when the firm knows A is of high quality, any deviation to prices in Region 3 is dominated by a deviation in Region 1. We have already shown that any deviation to prices in Region 1 yields profits below those in the candidate equilibrium, so we conclude that it does not pay to deviate to prices in Region 3.

Finally, consider a deviation to prices in Region 2. Suppose first that the firm deviates by setting the same price for each product, $p_A = p_B = p$. From (20), consumers believe each product is equally likely to be high quality, i.e., $\mu_A = \mu_B = 1/2$, so $E_{\mu_A}[\theta_A] = E_{\mu_B}[\theta_B] = \overline{\theta}$ where $\overline{\theta} \equiv \left(\theta^H + \theta^L\right)/2$. Letting $\overline{c} = \left(c^H + c^L\right)/2$, the firm's profit from such a deviation are

$$\begin{aligned} & \left(p_A - c^H\right)\left(\overline{\theta} - p_A\right) + \left(p_B - c^L\right)\left(\overline{\theta} - p_B\right) \\ = \; & \left(2p - c^H - c^L\right)\left(\overline{\theta} - p\right) \\ = \; & 2\left(p - \overline{c}\right)\left(\overline{\theta} - p\right). \end{aligned} \quad (23)$$

This expression is maximized at a price $p^* = \left(\overline{\theta} + \overline{c}\right)/2$, and results in profits of

$$2\left(p^* - \overline{c}\right)\left(\overline{\theta} - p^*\right) = 2\frac{\left(\overline{\theta} - \overline{c}\right)^2}{4} = \frac{\left(\theta^H + \theta^L - c^H - c^L\right)^2}{8}. \quad (24)$$

The maximum gain from such a deviation is thus the difference in expressions (24) and (17):

$$\begin{aligned} \Delta &= \frac{\left(\theta^H + \theta^L - c^H - c^L\right)^2}{8} - \left(\frac{\left(\theta^H - c^H\right)^2}{4} + \frac{\left(\theta^L - c^L\right)^2}{4}\right) \\ &= -\frac{\left(\theta^H - \theta^L - c^H + c^L\right)^2}{8} \\ &= -\frac{\left(q^H - q^L\right)^2}{4} \leq 0 \end{aligned} \quad (25)$$

so there is no gain from deviating.

The final step is to consider a deviation to prices $p_A \neq p_B$ in Region 2. From equation (20), these prices induce beliefs that satisfy

$$\mu_A \theta^H + (1-\mu_A)\,\theta^L - p_A = (1-\mu_A)\,\theta^H + \mu_A \theta^L - p_B \tag{26}$$

so using equation (4) this implies that demands for the two products are equal:

$$E_{\mu_A}[\theta_A] - p_A = E_{\mu_B}[\theta_B] - p_B. \tag{27}$$

The payoff from such a deviation by type A is

$$\begin{aligned} \pi_A^D &\equiv \left(p_A - c^H\right)\left(E_{\mu_A}[\theta_A] - p_A\right) + \left(p_B - c^L\right)\left(E_{\mu_B}[\theta_B] - p_B\right) && (28)\\ &= \left(p_A - c^H\right)\left(\mu_A \theta^H + (1-\mu_A)\,\theta^L - p_A\right) + \left(p_B - c^L\right)\left((1-\mu_A)\,\theta^H + \mu_A \theta^L - p_B\right) && (29) \end{aligned}$$

Using equation (27),

$$\begin{aligned} \pi_A^D &= \left(p_A - c^H\right)\left(E_{\mu_A}[\theta_A] - p_A\right) + \left(p_B - c^L\right)\left(E_{\mu_A}[\theta_A] - p_A\right)\\ &= \left(p_A - c^H + p_B - c^L\right)\left(E_{\mu_A}[\theta_A] - p_A\right)\\ &= \left(p_A - c^H + p_B - c^L\right)\left(\mu_A \theta^H + (1-\mu_A)\,\theta^L - p_A\right). \end{aligned} \tag{30}$$

Substituting in for μ_A using equation (20), we obtain

$$\begin{aligned} \pi_A^D &= \left(p_A - c^H + p_B - c^L\right)\left(\frac{p_A - p_B + \theta^H - \theta^L}{2\left(\theta^H - \theta^L\right)}\theta^H + \left(1 - \frac{p_A - p_B + \theta^H - \theta^L}{2\left(\theta^H - \theta^L\right)}\right)\theta^L - p_A\right)\\ &= \left(p_A - c^H + p_B - c^L\right)\left(\frac{1}{2}\theta^H + \frac{1}{2}\theta^L - \frac{1}{2}p_A - \frac{1}{2}p_B\right)\\ &= 2\left(\overline{p} - \overline{c}\right)\left(\overline{\theta} - \overline{p}\right) \end{aligned} \tag{31}$$

where $\overline{p} = (p_A + p_B)/2$. Thus, the payoff to a deviation in Region 2 cannot exceed the payoffs from setting optimal equal prices,

$$p^* = \arg\max_p \left\{2\left(p - \overline{c}\right)\left(\overline{\theta} - p\right)\right\} \tag{32}$$

where $p^* = \left(\overline{\theta} + \overline{c}\right)/2$ and resulting profits are as in equation (24). But equation (25) implies these profits are less than those in the candidate equilibrium, so we conclude there is no profitable deviation.

(ii) (Survives Divinity) We now check whether the above oep beliefs satisfy Divinity. With two types, the Divinity refinement puts strictly more weight on whichever type will deviate for a superset of rationalizable best responses by the consumer. This is equivalent in our environment to putting more weight on whichever type will deviate for a superset of beliefs. Note that the equilibrium payoffs to each type are the same. From (28) the payoff to deviating is linear in μ_i. Comparing the deviation payoffs for each type evaluated at the endpoints,

$$\begin{aligned} \pi_A^D(\mu_A &= 1) - \pi_B^D(\mu_B = 1) = \left(\left(p_A - c^H\right)(\theta_H - p_A) + \left(p_B - c^L\right)(\theta_L - p_B)\right) \\ &\quad - \left(\left(p_A - c^L\right)(\theta_L - p_A) + \left(p_B - c^H\right)(\theta_H - p_B)\right) \\ &= (p_A - p_B)\left(\theta_H - \theta_L + c^H - c^L\right) < 0 \text{ for } p_A < p_B \end{aligned} \tag{33}$$

$$\begin{aligned} \pi_A^D(\mu_A &= 0) - \pi_B^D(\mu_B = 0) = \left(\left(p_A - c^H\right)(\theta_L - p_A) + \left(p_B - c^L\right)(\theta_H - p_B)\right) \\ &\quad - \left(\left(p_A - c^L\right)(\theta_H - p_A) + \left(p_B - c^H\right)(\theta_L - p_B)\right) \\ &= -(p_A - p_B)\left(\theta_H - \theta_L - c^H + c^L\right) < 0 \text{ for } p_A < p_B \end{aligned} \tag{34}$$

so for $p_A < p_B$ type B will deviate for a wider range of beliefs. By symmetry for $p_A > p_B$ type A will deviate for a wider range of beliefs. Hence the beliefs (20) that put more weight on B and A in each case respectively survive the Divinity refinement.

(Unique Separating PBE) Now consider any candidate separating PBE with prices not equal to $\left(p^H, p^L\right)$ or $\left(p^L, p^H\right)$, and consider a deviation by type A to $\left(p^H, p^L\right)$. By the definition of full information prices and the strict concavity of the profit function, when $\mu_A = 1$ type A receives strictly higher profits from $\left(p^H, p^L\right)$ than any other price. Considering $\mu_A < 1$, from (28), the deviation payoff to A is decreasing in μ_A if $p^H - p^L \leq c^H - c^L$. Substituting $p^H - p^L = \frac{1}{2}\left(\left(\theta^H - \theta^L\right) + \left(c^H - c^L\right)\right)$ from (7), this payoff is decreasing if $\frac{1}{2}\left(\left(\theta^H - \theta^L\right) + \left(c^H - c^L\right)\right) \leq c^H - c^L$, or $\theta^H - \theta^L \leq c^H - c^L$, which holds by assumption. Hence the deviation payoff to type A is higher for $\mu_A < 1$ than for $\mu_A = 1$, implying type A will deviate for any beliefs. ■

This result provides a simple necessary and sufficient condition for the firm to achieve full information profits through comparative price signaling; if the quantity demanded is lower for the higher priced product (even after buyers correctly infer it is high quality) then there is no need to distort the price of the high quality product above its optimal (full-information level) price. This condition seems plausible; indeed, market shares of premium-quality products are often smaller than lower-quality, mass-marketed products.

The beliefs used in the proof of Proposition 2 ensure that the quantity demanded for the higher-priced product always remains (weakly) below the quantity demanded for the lower-priced product. The key is that as the price gap increases the beliefs change slowly enough that the positive effect on quantity demanded from higher expected quality does not overwhelm the negative effect from a higher price. So the beliefs have the intutitive property that a relatively higher price for a product leads to a greater belief that the product is high quality, but they also require that small changes in prices do not lead to large changes in beliefs. If this were the case a seller would be tempted to price the high quality good only slightly above the low quality good (Region 2), and then switch the goods so as to sell a large amount of cheap to produce low quality goods at a higher price.

Figure 1(b) shows a comparative price signaling equilibrium for the same parameters as the price signaling equilibrium in Figure 1(a). Since $\theta^H = 1$, $\theta^L = 2/3$, $c^H = 1/2$, and $c^L = 1/6$, the equilibrium condition is just satisfied, $c^H - c^L = \theta^H - \theta^L$, so $q^H = q^L$ and the seller has a weak incentive to not deviate. The full information profits in the shaded areas are therefore attained. If the cost difference is larger, or the quality difference smaller, then $q^H < q^L$. In this case the incentive to follow the equilibrium pricing strategy generating full information profits is strict.[3]

Comparative price signaling is still an equilibrium even if the condition $c^H - c^L \geq \theta^H - \theta^L$ is not met, but there is some upward price distortion for the high quality product. In general this price just has to be high enough such that equilibrium demand for the high quality good is no higher than equilibrium demand for the low quality good. It is straightforward to show that the necessary price is lower than that required for a price signaling equilibrium without observability, so there is less distortion with comparative price signaling.

3 Results for General Demand Functions

The above results provide a simple model of independent linear demand that captures the essential features of the problem. We now consider more general demand structures and find that the quantity condition discussed above is still key to sustaining full information prices through comparative price signaling. For simplicity we maintain our assumption of linear costs.

3.1 Interdependent Demand with Two Products

Suppose demands are interdependent (in the sense that the demand for product i depends on the prices and values of both products, $q_i(\theta_i, \theta_j, p_i, p_j)$. Further assume that demands are symmetric

[3] For this borderline case of $q^H = q^L$ the beliefs used in the proof must be exact to ensure the seller does not deviate. However if $q^H < q^L$ then the beliefs do not have to be exact, and if the gap is large enough they just have to fit the rough pattern that higher prices lead to a greater belief that the product is high quality.

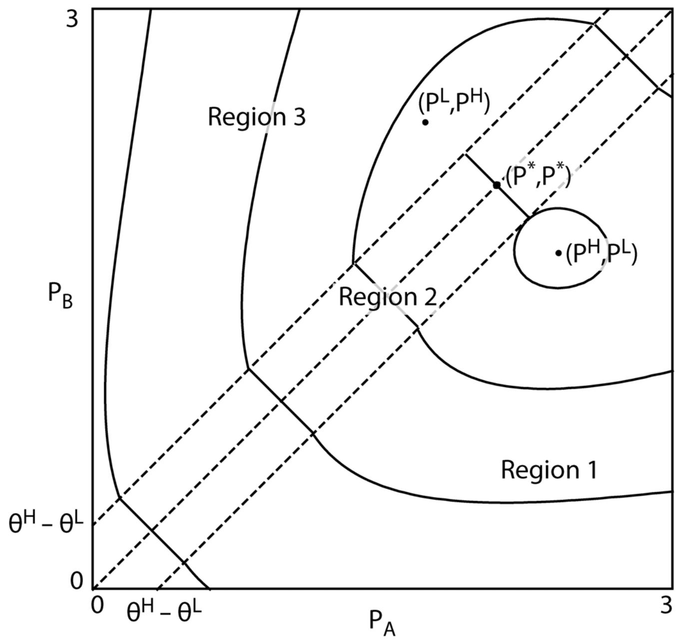

Figure 3: Logit demand: Isoprofit curves for type A given equilibrium beliefs

at the full-information profit-maximizing prices p^H and p^L; that is, $q^H \equiv q_A\left(\theta^H, \theta^L, p^H, p^L\right) = q_B\left(\theta^H, \theta^L, p^H, p^L\right)$ and $q^L \equiv q_A\left(\theta^L, \theta^H, p^L, p^H\right) = q_B\left(\theta^L, \theta^H, p^L, p^H\right)$. Then we have the following result.

Proposition 3 *For a multiproduct firm facing interdependent demands for two products, a necessary condition for the existence of a comparative price signaling equilibrium with full information profits is* $q^L \geq q^H$*.*

Proof. For the equilibrium to have full information profits, the price of the high quality product must be p^H and the price of the low quality product must be p^L, where $p^H > p^L$ and the consumers must believe the higher priced product is the higher quality product, where $\left(p^H, p^L\right)$ are the prices that maximize profits in the full information case and $\left(q^H, q^L\right)$ are the corresponding quantities demanded. The profits in a full information equilibrium are thus

$$(p^H - c^H)q^H + (p^L - c^L)q^L \tag{35}$$

We show that $q^L - q^H \geq 0$ follows from the no deviation property. To see this, consider a deviation where the firm charges p^H for the lower quality product and p^L for the higher quality product. Since this deviation is on the equilibrium path, when consumers observe these prices they believe the low quality product is high quality and vice-versa. Therefore, the firm's profit from this deviation is

$$(p^L - c^H)(\theta^L - p^L) + (p^H - c^L)(\theta^H - p^H) \tag{36}$$

so for this deviation to not be profitable, the gain to the deviation must be negative:

$$\begin{aligned}\Delta &= \left((p^L - c^H)q^L + (p^H - c^L)q^H\right) - \left((p^H - c^H)q^H + (p^L - c^L)q^L\right) \\ &= \left(c^H - c^L\right)\left(q^H - q^L\right). \end{aligned} \tag{37}$$

Since $c^H > c^L$, a necessary condition for $\Delta < 0$ is $q^L - q^H \geq 0$. ∎

3.2 Interdependent Demand with n Products

Now suppose the multiproduct firm produces n products where demands are again interdependent. The products are all of different qualities, and as before the firm knows the quality of each product but consumers do not. Let the demand for product $i \in \{1, ..., n\}$ be given by $q_i(\theta_i, \theta_{-i}, p_i, p_{-i})$ where θ_{-i} and p_{-i} are vectors of the other product's qualities and prices respectively. Let $\theta_i \in \{\theta^n, \theta^{n-1}, ..., \theta^1\}$ where $\theta^j > \theta^{j-1}$; that is, the product with the highest quality is associated with θ^n and so on. Let c^j denote the corresponding unit costs with $c^j < c^k$ for all $j < k$. We assume that under perfect information exchange of all products is feasible: $\theta^j > c^j$ for all j. Notice that $n = 3$ is the special case where the firm sells "good" (θ^1), "better" (θ^2), and "best" (θ^3) versions of a product. As before we assume that demands are symmetric at $(p^n, p^{n-1}, ..., p^1)$ where p^j is the full-information profit-maximizing price for a product of quality θ^j. For any two products i, k we assume that demands are symmetric at the full-information profit-maximizing prices: $q^j \equiv q_i(\theta^j, \theta^{-j}, p^j, p^{-j}) = q_k(\theta^j, \theta^{-j}, p^j, p^{-j})$ for all j.

The same quantity condition identified in the two-product case arises in the n-product case.

Proposition 4 *For a multiproduct firm facing interdependent demands for* n *products, a necessary condition for the existence of a comparative price signaling equilibrium with full information profits is* $q^1 \geq q^2 \geq \cdots \geq q^n$.

Proof. In a full information equilibrium, for any $j < k$, we can decompose profits into those from products j and k and those from the remaining products,

$$\sum_i^n (p_i - c_i)\, q_i = (p^j - c^j)q^j + (p^k - c^k)q^k + \sum_{m \neq j,k} (p^m - c^m)q^m. \tag{38}$$

Consider a deviation from full information prices where the firm charges p^j for the product of quality θ^k and charges p^k for the product of quality θ^j. Since this deviation is on the equilibrium path, when consumers observe these prices they believe the lower quality product is higher quality and vice-versa. Therefore, the firm's profit from this deviation is

$$(p^j - c^k)(\theta^j - p^j) + (p^k - c^j)(\theta^k - p^k) + \sum_{m \neq j,k} (p^m - c^m) q^m \tag{39}$$

so for this deviation to not be profitable, the gain to the deviation must be negative:

$$\begin{aligned} \Delta &= \left((p^j - c^k) q^j + (p^k - c^j) q^k \right) - \left((p^k - c^k) q^k + (p^j - c^j) q^j \right) \\ &= \left(c^k - c^j \right) \left(q^k - q^j \right). \end{aligned} \tag{40}$$

Since $c^k > c^j$, a necessary condition for $\Delta < 0$ is $q^j \geq q^k$ for all $j < k$. ■

4 Extensions and Applications

4.1 Crimping

Our main results are based on the assumption that the high-quality product is more costly to produce. As noted by Deneckere and McAfee (1996) sometimes the lower quality product is actually more expensive to produce. The next proposition follows from the above results:

Proposition 5 *Suppose a multiproduct firm produces the lower quality product at the higher unit cost, $c^L > c^H$. (i) If demand is independent and linear, a comparative price signaling equilibrium with full information profits exists if and only if $q^L \leq q^H$. (ii) If demands are interdependent and symmetric, a necessary condition for the existence of a comparative price signaling equilibrium with full information profits is $q^L \leq q^H$.*

4.2 Cheap Talk

We have shown how comparative price signaling can credibly rank the qualities of different products. Now consider when it is credible to rank the products through pure cheap talk. For instance, a salesperson tells a consumer which of two products is better for her, or multiple products are literally labelled “good”, “better”, “best”, etc. (Chakraborty and Harbaugh, 2007, 2010). To focus on pure cheap talk, assume that the prices are fixed so that consumers do not infer anything directly from the prices.

Comparative price signaling is credible when the different equilibrium prices ensure that the quantity demanded for the higher cost product is lower, so that the seller earns more by selling less of the higher cost product at the higher price and more of the lower cost product at the lower price. In particular, even though consumers have higher demand for a higher quality product, the associated price keeps the quantity demanded from being higher. With given prices that do not signal information, there is no assurance that prices will play this role.

The following proposition shows that if prices are fixed and do not signal information, a necessary condition for the existence of a comparative cheap talk equilibrium is that demands (at the given prices) are lower for the higher quality products. For notational convenience and without loss of generality, relabel the products so that p_i is the price associated with product of true quality θ^i. We assume that the firm first communicates the ranking of the products to the consumers (e.g., good, better, best), and then consumers observe the prices.

Proposition 6 *For a multiproduct firm facing interdependent demands for n products and charging fixed prices, a necessary condition for the existence of a fully revealing cheap talk equilibrium is* $q\left(\theta^1, \theta^{-1}, p_1, p_{-1}\right) \geq q\left(\theta^2, \theta^{-2}, p_2, p_{-2}\right) \geq \cdots \geq q\left(\theta^n, \theta^{-n}, p_n, p_{-n}\right)$.

Proof. Consider a fully revealing cheap talk equilibrium where consumers learn the qualities of all products and consider any two products $j < k$. This implies $q\left(\theta^j, \theta^{-j}, p\right) < q\left(\theta^k, \theta^{-k}, p\right)$. Profits in this putative equilibrium are then

$$\begin{aligned} \sum_i^n \left(p_i - c_i\right) q_i &= (p_j - c^j) q\left(\theta^j, \theta^{-j}, p_j, p_{-j}\right) + (p_k - c^k) q\left(\theta^k, \theta^{-k}, p_k, p_{-k}\right) \\ &\quad + \sum_{m \neq j,k} (p_m - c^m) q\left(\theta^m, \theta^{-m}, p_m, p_{-m}\right). \end{aligned} \tag{41}$$

Consider a deviation that flips the messages to trick consumers. Profits from this deviation are

$$(p_j - c^k) q\left(\theta^j, \theta^{-j}, p_j, p_{-j}\right) + (p_k - c^j) q\left(\theta^k, \theta^{-k}, p_k, p_{-k}\right) + \sum_{m \neq j,k} (p - c^m) q\left(\theta^m, \theta^{-m}, p_m, p_{-m}\right). \tag{42}$$

Since this deviation is not profitable by hypothesis, the gain to the deviation must be non-positive:

$$\Delta = \left(c^j - c^k\right)\left(q\left(\theta^j, \theta^{-j}, p_j, p_{-j}\right) - q\left(\theta^k, \theta^{-k}, p_k, p_{-k}\right)\right) \leq 0. \tag{43}$$

By assumption $c^j < c^k$ so Δ is non-positive if $q\left(\theta^j, \theta^{-j}, p_j, p_{-j}\right) \geq q\left(\theta^k, \theta^{-k}, p_k, p_{-k}\right)$ holds for all j, k, which implies the required inequalities. ■

This result has several implications. First, if prices are all identical then a cheap talk ranking of the goods cannot be credible if buyer demand for a product is increasing in its perceived quality and seller costs are increasing in quality.[4] Second, if prices happen to be set such that quantities are ordered to permit cheap talk, then cheap talk provides no additional information to buyers beyond the prices. In this sense if the retailer has set prices consistent with comparative price signaling, a salesperson with incentives proportional to the retailer's profits cannot provide extra information to consumers, but also has no incentive to provide contradictory information to consumers.

Similarly, if a manufacturer were to "dictate" that the retailer charge the retailer's optimal full information prices, then the retailer has no incentive to "mislabel" the good, better, and best products in an attempt to mislead consumers. As discussed below, this is also related to resale price maintenance.

4.3 Resale Price Maintenance

A final application of the above results is to resale price maintenance (RPM)—a situation where a manufacturer imposes a vertical restraint to prevent retailers from charging retail prices that are, from the manufacturer's perspective too high (in the case of maximum RPM) or too low (in the case of Minimum RPM). The standard rationale for maximum RPM is to mitigate the double-marginalizaton problem, while minimum RPM may be imposed to provide retailers sufficient margins to provide point-of-sale service or to mitigate service-related free-rider problems among retailers. Our results provide a potential signaling rationale for RPM, as discussed below.

Consider an environment where a manufacturer produces high-end and low-end versions of a product. If these two products are distributed through two different retailers, the retailer selling the low-end product will, under the usual conditions, have an incentive to mimic the firm selling the high quality product. But if the manufacturer imposes vertical restraints in the form of minimum or maximum resale price maintenance, the informational value of prices is maintained and the manufacture may obtain the full information outcome when this outcome would not occur in the absence of such restraints.

In contrast, if the manufacturer distributes the two products through a single firm that sells both versions–and if consumers can observe both prices of both versions of products (searching the internet, for example), our results imply that the need for RPM is reduced. Indeed, our results show that for configurations in which separate retailers cannot achieve full-information profits, the multiproduct retailer may achieve this outcome. While RPM is not per-se illegal in the United States at the Federal level, it is per-se illegal in many states and, indeed, in many countries around

[4] When prices are strictly increasing in estimated quality the sender's complementarity condition of Theorem 1 of Chakraborty and Harbaugh (2007) is violated.

the word. When RPM is not possible or otherwise not preferable, channeling both versions through a single retailer rather than through independent retailers may still preserve the gains from credible price signaling.

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