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Prospect Theory or Skill Signaling?*

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This version: September 2006

Abstract

Failure is embarrassing. In gambles involving both skill and chance, we show that a strategic desire to avoid appearing unskilled generates behavioral anomalies that are typically explained by prospect theory's concepts of loss aversion, probability weighting, and framing effects. Loss aversion arises because losing any gamble, even a friendly bet with little or no money at stake, reflects poorly on the decision maker's skill. Probability weighting emerges because winning a gamble with a low probability of success is a strong signal of skill, while losing a gamble with a high probability of success is a strong signal of incompetence. Framing matters when there are multiple equilibria and the framing of a gamble affects beliefs, e.g., when someone takes a "dare" rather than admit a lack of skill. The analysis is based on models from the career concerns literature and is closely related to early social psychology models of risk taking. The results provide an alternative perspective on the existence of prospect theory behavior in economic, financial, and managerial decisions where both skill and chance are important. We identify specific situations where skill signaling makes opposite predictions than prospect theory, allowing for tests between the strategic and behavioral approaches to understanding risk.

JEL Classification Categories: D81; D82; C92; G11

Key Words: Prospect theory; career concerns; probability weighting; loss aversion; framing effects; dare taking; embarrassment aversion

*I thank David Bell, Tom Borcherding, Bill Harbaugh, Tatiana Kornienko, David Laibson, Ricky Lam, Harold Mulherin, Jack Ochs, Al Roth, Lan Zhang, and seminar participants at the Claremont Colleges, Indiana University, IUPUI, the Behavioral Research Council, the Econometric Society Winter Meetings, the Stony Brook Game Theory Conference and the Midwest Theory Meetings. This paper has also circulated under the title, "Skill reputation, prospect theory, and regret theory."

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1 Introduction

Most risky decisions involve both skill and chance. Success is therefore doubly fortunate in that it brings both material gain and an enhanced reputation for skill, while failure is doubly unfortunate. Often the reputational effects are more important than the direct material gain or loss. For instance, the manager of a successful project wins the confidence of superiors to oversee more projects, while the manager of a failed project is viewed as incompetent and loses future opportunities. In other cases the reputational effects are less important but still of some concern. For instance, an investor who picks a successful stock enjoys the esteem of friends and family, while an investor who chooses poorly looks like a foolish loser.

The idea that failure is embarrassing and that decision makers might choose between risky actions to limit embarrassment is emphasized in the early social psychology literature on achievement motivation (Atkinson, 1957), and similar ideas appear in the literatures on self-esteem (James, 1890) and self-handicapping (Jones and Berglas, 1978). More formally, the literature on the career concerns of managers analyzes how the interaction between skill and chance affects Bayesian updating of a manager's skill (Holmstrom, 1982/1999). This literature finds that the incentive to avoid looking unskilled can explain a wide range of seemingly irrational behaviors by managers.

In this paper we use the formal approach of the career concerns literature to reexamine the role of embarrassment and loss of self-image in standard problems involving decision under risk. We find results that are consistent with the principle insights of the early social psychology literatures, and that extend these insights in novel ways. Moreover, we show a close connection between these approaches which are consistent with expected utility maximization, and the non-expected utility approach of prospect theory (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992). We concentrate on identifying what violations of expected utility will appear to arise if a rational decision maker is concerned with appearing skilled, but is instead modeled as only caring about immediate monetary payoffs.

We show that skill signaling leads to a set of behaviors that, depending on the information environment, largely overlap with prospect theory's main concepts of loss aversion, probability weighting, and framing effects (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992). Loss aversion refers to the idea that the utility function is kinked at the status quo wealth level so that utility falls more steeply in losses than it rises in gains (Kahneman and Tversky, 1979). Gambles with a roughly equal chance of winning or losing can therefore have substantial risk premia, even for small stakes where a standard smooth utility function would predict near risk neutrality (Pratt, 1964; Rabin, 2000; Rabin and Thaler, 2001). Probability weighting refers to the idea that decision makers violate expected utility theory by overweighting low probabilities. It can explain the simultaneous purchase of lottery tickets and insurance (Friedman and Savage, 1948), the Allais paradox (Allais, 1953), and the preference for long shots in horse races and other gambling environments (Thaler and Ziemba, 1988). Finally, framing effects arise when different choices result from a simple change in the presentation of choices (Tversky and Kahneman, 1981).

To analyze the role of skill signaling, we follow the career concerns literature in investigating two types of skill. First, with "performance skill" some decision makers face better odds of success than other less

skilled decision makers. For instance, a project might be more likely to succeed under a skilled manager. Performance skill has been used to understand “rat race” career incentives (Holmstrom, 1982/1999), excessive risk-taking (Holmstrom and Costa, 1986), and corporate conformism (Zwiebel, 1995). Second, with “evaluation skill” some decision makers are better at identifying the exact odds of a gamble than their less skilled counterparts. For instance, a skilled manager might be better at choosing promising projects, or a skilled broker might have a talent for identifying profitable companies. Evaluation skill has been used to understand distorted investment decisions (Holmstrom, 1982/1999), herding (Scharfstein and Stein, 1990), anti-herding (Avery and Chevalier, 1999), the sunk cost fallacy (Kanodia, Bushman, and Dickhaut, 1989), conservatism and overconfidence (Prendergast and Stole, 1996), and political correctness (Morris, 2001).

We differ from most of the career concerns literature in that we do not explicitly model the details of the career environment. Instead we derive general results for situations where individuals are “embarrassment averse” in the same pattern as is normally assumed for risk aversion regarding wealth. That is, their utility is increasing in their expected skill, and they particularly dislike being thought of as unskilled. Such a pattern could reflect a simple desire to avoid embarrassment or maintain one’s own self-image. Or, from a career concerns perspective, the pattern arises if future income is a linear function of estimated skill and people are risk averse with respect to wealth. It also arises if future income is a concave function of estimated skill because, for instance, the probability of maintaining employment is a concave function of performance (Chevalier and Ellison, 1999). The value of our approach is that the results can be applied to any environment that generates future income based on success or failure in a pattern consistent with the general conditions of embarrassment aversion.

Prospect theory’s concepts of loss aversion, probability weighting, and framing effects were originally identified in laboratory experiments, but have since become widespread tools for analyzing economic, financial, and managerial behavior in environments where the career concerns literature has also shown a strong role for skill signaling.¹ Our results indicate that the behavioral approach of prospect theory and the strategic approach of skill signaling provide very similar predictions in these environments. This overlap can be seen as mutually reinforcing – the theoretical results of skill signaling provide an underlying strategic foundation for the behavioral predictions of prospect theory, and the empirical results of prospect theory indicate that decision makers are capable of understanding and even internalizing the logic of skill signaling. However, there are important situations in which the predictions of prospect theory and skill signaling diverge, so determining whether behavioral or strategic effects are driving behavior in particular situations is often important. We discuss this issue more in Section 4 and in the conclusion.

To see how skill signaling leads to similar predictions as prospect theory, first consider loss aversion. When there is a performance skill component to a gamble, losing implies that there is a good chance that the decision maker bungled the gamble, and when there is an evaluation skill component, losing implies that the decision maker might have unwisely taken a gamble that had worse than expected odds. In either

¹See Camerer (2000) and Barberis and Thaler (2001) for some applications of prospect theory in these areas. One indicator of how widely prospect theory is applied is the fact that Kahneman and Tversky (1979) is typically ranked as the first or second most cited paper in economics.

case, losing reflects poorly on the decision maker’s skill, so if the decision maker is risk averse with respect to skill estimates, then she is more averse to gambling than pure risk aversion regarding monetary payoffs would predict. Since losing even a “friendly bet” with no money at stake is embarrassing, this effect does not disappear as the stakes of the gamble become smaller,² so the utility function in wealth will appear to be kinked at the status quo, i.e., the decision maker will appear to be loss averse.

Regarding probability weighting, prospect theory argues that decision-makers exhibit a “four-fold pattern” of behavior in which they tend to favor long-shots but also avoid near sure-things, and to buy insurance to protect against unlikely losses even as they will take risky chances to win back large losses.³ To capture this observed pattern, probability weighting as developed most fully in “cumulative prospect theory” (Tversky and Kahneman, 1992; Prelec, 1998) assumes that people violate expected utility maximization by overweighting small probability gains (such as taking a 10% chance of winning \$100 over \$10 for sure) and underweighting high probability gains (such as taking \$90 for sure over a 90% chance of winning \$100), and by overweighting low probability losses (such as paying \$10 for sure rather than risking a 10% chance of losing \$100), and underweighting high probability losses (such as risking a 90% chance of losing \$100 rather than paying \$90 for sure).

From the perspective of skill signaling, the four-fold pattern in gains and losses can be interpreted more simply as overweighting of small probabilities of success (e.g., favoring long shots and taking chances to win back large losses) and underweighting of high probabilities of success (e.g., avoiding near sure things and buying insurance). For either performance skill or evaluation skill, we find that losing a gamble that is known to have a low probability of success is less embarrassing than losing a gamble where success is expected, so lower probability gambles are favored. When success is unlikely, failure is common but only slightly reduces the perceived skillfulness of the decision maker because both skilled and unskilled decision makers are expected to fail. But when success is likely, failure is rare but far more embarrassing because a person who fails is probably unskilled. Embarrassment averse decision makers are therefore more willing to take a chance on gambles that observers recognize are long shots, and reluctant to take gambles where success is expected.

For gambles involving performance skill, the preference for long shots is strengthened by the presence of private information held by the decision maker about her own skill. Failure to take a gamble can then be seen as an admission that the decision maker lacks confidence in her own skill, so the decision maker faces pressure to risk failure rather than directly admit her incompetence by refusing to gamble. For gambles that are known to be long shots there is little embarrassment in losing, so the decision maker is better off taking the gamble if the expected monetary return is not too unfavorable. In contrast, for gambles that

²This is consistent with Schlaifer’s (1969, p.161) suggestion that in some cases “nonmonetary consequences” of losing may explain high risk premia for small gambles.

³“Original prospect theory” (Kahneman and Tversky, 1979) assumes that the utility function is convex in losses and concave in gains, implying the simpler pattern that decision makers are risk loving in gambles that involve potential losses and risk averse in gambles that involve potential gains. Applications of prospect theory often allow for interactions between both patterns, but in this paper we concentrate on the four-fold pattern. For an analysis of how concern for status can generate a convex-concave utility function, see Harbaugh and Kornienko (2001).

are recognized to be near sure things, taking the gamble involves substantial risk of embarrassment, so the decision maker is often better off refusing the gamble.

For gambles involving evaluation skill, the preference for long shots is strengthened if the outcome of a gamble is observable even when the decision maker turns it down. In this case the decision maker cannot simply refuse to take the gamble and prevent the observer from learning about her skill. If the gamble is refused, a good outcome is a strong indication that the decision maker failed to recognize that the gamble had better than expected odds. For low probability gambles this possibility is more embarrassing than taking the gamble and losing, so the decision maker will take the gamble if the expected monetary return is not too negative. The opposite situation arises with high probability gambles. If such a gamble is refused, little about the decision maker's skill is revealed from the outcome because the good outcome is likely regardless of whether the gamble's true odds were slightly better or worse than expected. The danger from taking the gamble and losing is greater, so high probability gambles are less desirable than pure monetary considerations would imply.

Regarding framing effects, multiple equilibria typically exist depending on whether the observer expects the decision maker to gamble or not, and depending on what the observer believes about the decision maker's skill if she unexpectedly takes a different choice. If refusal to take a gamble is interpreted as an admission of being unskilled, then even an unskilled decision maker might be dared into gambling. Since there are multiple equilibria, contextual cues can help indicate the player's intentions (Schelling, 1960), and in particular the framing of the question is likely to be an important source of information about the observer's expectations. Prospect theory finds that decision makers tend to be risk averse when the framing of the gamble portrays losing rather than winning as the status quo reference point, and risk loving in the opposite case. From a skill signaling perspective, if losing is portrayed as the reference point then taking a fixed sum instead of the gamble is as an improvement over the reference point, so refusing the gamble is unlikely to be viewed negatively. But if winning is portrayed as the reference point, then taking a fixed sum instead of the gamble is worse than the reference point, so the decision maker has reason to expect that refusing the gamble will be viewed as an admission of being unskilled. These beliefs imply that gambling is less likely in the former case than the latter case, which is consistent with prospect theory.

The existence of multiple equilibria also implies a role for cultural factors in determining when gambling is more or less likely. If observers expect some groups to try to prove their skill and others not to, or expect different groups to prove their skill in different environments, behavior can result which confirms the beliefs. For instance, it is documented that men take riskier investments than women do (Jianakoplos and Bernasek, 1998), invest as if they are overconfident (Barber and Odean, 2001), and generally appear to be less risk averse (Eckel and Grossman, 2003; Croson and Gneezy, 2004). Rather than being solely or primarily due to underlying differences in risk attitudes or confidence, this pattern might arise from different equilibrium beliefs among observers about how members of each group try to prove their abilities. That is, if observers expect men but not women to take risky actions, then the negative inference from not taking a gamble is smaller for the latter group, so the beliefs can be self-fulfilling.

These results show a close connection between the predictions of prospect theory and skill signaling,

and also indicate that skill signaling may be related to other risk-taking behavior not captured by current models. In the following section we provide an introductory example, and then in Section 3 we develop a more formal model which considers the existence of multiple equilibria and provides results in terms of risk premia. Section 3 also includes two natural extensions of the model. Section 4 relates the results in more detail to prospect theory, and also shows how the results relate to other models, including achievement motivation, self-esteem, self-handicapping, disappoint aversion, and regret theory. Section 5 concludes the paper.

2 Introductory Example

Consider a gamble with two outcomes, “win” or “lose”, taken by a decision maker whose is either skilled “s” or unskilled “u”. For this example the decision maker does not have any private information about her own skill so that the act of taking a gamble is not itself informative of skill. The probability of being skilled conditional on winning is therefore

$$\begin{aligned}
\Pr[s|win] &= \frac{\Pr[win, s]}{\Pr[win]} = \Pr[s] + \frac{\Pr[win, s] - \Pr[s] \Pr[win]}{\Pr[win]} \\
&= \Pr[s] + \frac{\Pr[win, s] - \Pr[s] \Pr[win, s] - \Pr[s] \Pr[win, u]}{\Pr[win]} \\
&= \Pr[s] + \frac{\Pr[u] \Pr[win, s] - \Pr[s] \Pr[win, u]}{\Pr[win]} \\
&= \Pr[s] + \frac{\Pr[win|s] - \Pr[win|u]}{\Pr[win]} \Pr[s] \Pr[u]. \tag{1}
\end{aligned}$$

Similarly, the probability of being skilled conditional on losing is

$$\Pr[s|lose] = \Pr[s] - \frac{\Pr[win|s] - \Pr[win|u]}{\Pr[lose]} \Pr[s] \Pr[u]. \tag{2}$$

Assuming that the “skill gap” $\Pr[win|s] - \Pr[win|u]$ is positive, the prior skill estimate $\Pr[s]$ is updated favorably when the decision maker wins and unfavorably when the decision maker loses, and the updating is stronger the larger is the skill gap and the weaker is the the prior skill estimate, i.e., the closer is $\Pr[s]$ to 1/2.

To see how such updating can affect behavior, suppose that the decision maker’s utility is a function of both wealth and of her estimated skill by an observer, who could be the decision maker herself if self-esteem is important.⁴ Assuming that the two components of utility are additively separable and letting v represent the skill estimate component, if v is concave then the decision maker prefers that the observer maintains the prior skill estimate $\Pr[s]$ rather than risk the lower estimate $\Pr[s|lose]$. In particular, since $\Pr[win] \Pr[s|win] + \Pr[lose] \Pr[s|lose] = \Pr[s]$, for $v'' < 0$,

$$v(\Pr[s]) > \Pr[win]v(\Pr[s|win]) + \Pr[lose]v(\Pr[s|lose])$$

⁴As we discuss in Section 4, the issue of why self-esteem might affect behavior is addressed formally by Benabou and Tirole (2002).

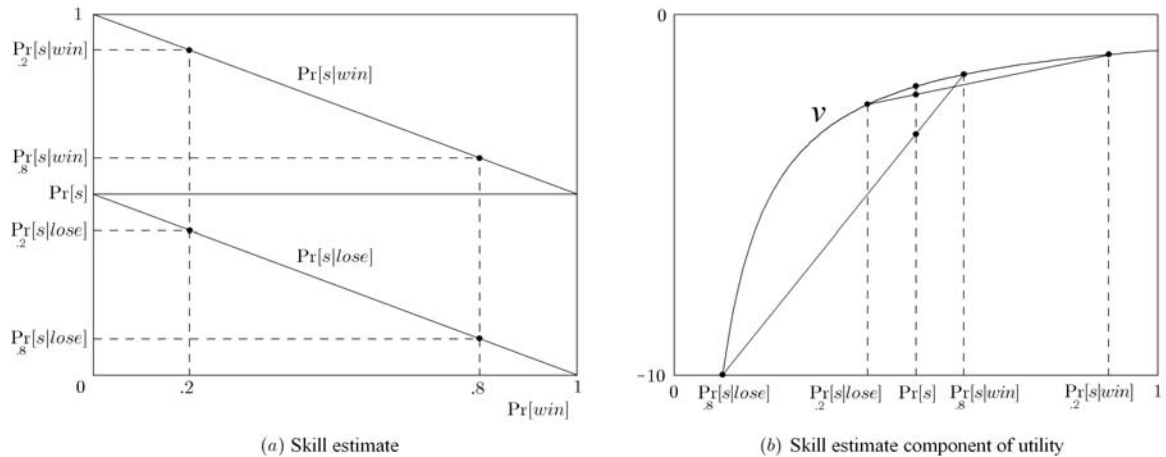


Figure 1: Impact of winning and losing on estimated skill and expected utility

so concern for appearing skilled leads a decision maker to be more wary of gambles than pure monetary considerations would suggest. As the monetary size of the gamble becomes smaller, any risk aversion with respect to the monetary component of utility should asymptotically disappear for a smooth utility function, but the fear of looking unskilled remains even for a “friendly bet” with no money at stake.⁵ Therefore, embarrassment aversion can provide a basis for prospect theory’s idea of loss aversion that is consistent with a standard smooth utility function.⁶

Now consider how the decision maker’s attitude toward a gamble is affected by the odds of the gamble. From (1) and (2), for a given skill gap $\Pr[\text{win}|s] - \Pr[\text{win}|u]$, both $\Pr[s|\text{win}]$ and $\Pr[s|\text{lose}]$ are decreasing in $\Pr[\text{win}]$, so the higher is $\Pr[\text{win}]$, the weaker is the favorable updating and the stronger is the unfavorable updating. Therefore, when a gamble is a “long shot” ($\Pr[\text{win}]$ is low) the decision maker has little to fear from losing and a lot to gain from winning. And when a gamble is a “near sure thing” ($\Pr[\text{win}]$ is high) the decision maker has little to gain and a lot to lose. More generally, if the skill gap depends on $\Pr[\text{win}]$, as is necessary to keep $\Pr[\text{win}|s]$ and $\Pr[\text{win}|u]$ bounded in $[0,1]$ as $\Pr[\text{win}]$ approaches 0 or 1, then these updating patterns hold as long as the skill gap does not change too rapidly as $\Pr[\text{win}]$ rises.⁷

To see these updating patterns, consider Figure 1(a) which shows the probability of being skilled when the prior is $\Pr[s] = 1/2$ and the skill gap for any gamble takes the form $\Pr[\text{win}|s] - \Pr[\text{win}|u] =$

⁵In practice the skill gap itself is likely to shrink as the size of the gamble shrinks because the decision maker cares less about the outcome. This idea appears in career concerns models where decision makers devote more resources to evaluating or performing well at larger gambles (Dewatripont, Jewitt and Tirole, 1999; Milbourn, Shockley, and Thakor, 2001). However, given a concern for appearing skilled, there is no reason to assume that the skill gap should go to zero.

⁶The issue of reputational risk appears in various contexts in the career concerns literature starting with Holmstrom (1982/1999), but the connection with loss aversion does not appear to have been noted previously.

⁷From (1), winning leads to weaker favorable updating as $\Pr[\text{win}]$ increases if $(\Pr[\text{win}|s] - \Pr[\text{win}|u]) / \Pr[\text{win}]$ is decreasing in $\Pr[\text{win}]$, and from (2), losing leads to stronger unfavorable updating as $\Pr[\text{win}]$ increases if $(\Pr[\text{win}|s] - \Pr[\text{win}|u]) / (1 - \Pr[\text{win}])$ is increasing in $\Pr[\text{win}]$.

$2 \Pr[\text{win}] \Pr[\text{lose}]$. This simple formulation, which ensures that $\Pr[\text{win}|s] < 1$ and $\Pr[\text{lose}|u] > 0$ as $\Pr[\text{win}]$ approaches 1 or 0, is symmetric in that the skill gap is the same for a gamble with probability $\Pr[\text{win}] = p$ and a gamble with probability $\Pr[\text{lose}] = p$.⁸ As we will see in Section 4.2, this formulation is implicitly assumed by the achievement motivation literature. When $\Pr[\text{win}]$ is low, winning has a large impact on estimated skill as seen from the divergence of the top line $\Pr[s|\text{win}]$ from the center line representing expected skill, $\Pr[s|\text{win}] \Pr[\text{win}] + \Pr[s|\text{bad}] \Pr[\text{bad}] = \Pr[s] = 1/2$, while losing has only a small impact as seen from the closeness of the bottom line $\Pr[s|\text{lose}]$ to expected skill. Low probability gambles therefore present a chance of standing out with little downside risk. Conversely, when $\Pr[\text{win}]$ is high, winning has only a small impact on estimated skill whereas losing has a large impact. Such gambles offer little opportunity to prove the sender's skill but carry substantial danger of embarrassment. In the figure a gamble with $\Pr[\text{win}] = .2$ generates expected skill from winning of $\Pr_{.2}[s|\text{win}] = 1/2 + ((2(.2)(.8)) / (.2)) (1/2)(1/2) = .9$ and expected skill from losing of $\Pr_{.2}[s|\text{lose}] = 1/2 - ((2(.2)(.8)) / (.8)) (1/2)(1/2) = .4$, while a gamble with $\Pr[\text{win}] = .8$ generates, by similar calculations, expected skill from winning of $\Pr_{.8}[s|\text{win}] = .6$ and expected skill from losing of $\Pr_{.8}[s|\text{lose}] = .1$.

The prospect theory literature and other literatures find that decision makers tend to be more wary of near sure things relative to long shots, which is consistent with this pattern that losing is more embarrassing for high probability gambles. However, even though losing at a near sure thing implies severe embarrassment, losing occurs only rarely, so it is not immediately clear whether decision makers will be more wary of such gambles. As we show in the next section, a near sure thing has more “downside risk” (Whitmore 1970; Menezes, Geiss, and Tressler, 1980), so a sufficient condition for the decision maker to prefer the long shot is that she is downside risk averse with respect to the observer's estimate of her skill, $v' > 0$, $v'' < 0$, and $v''' > 0$, a standard assumption for risk aversion with respect to monetary outcomes.⁹ As we discuss in Section 4, this may provide an expected utility basis for the probability weighting phenomenon identified in the prospect theory literature and formalizes a basic insight behind the achievement motivation and self-handicapping literatures.

The effect of downside risk on expected utility is seen in Figure 1(b) for a constant relative risk aversion function $v(x) = -1/x$.¹⁰ Skill estimates from winning and losing for the $\Pr[\text{win}] = .2$ gamble (flatter line) and the $\Pr[\text{win}] = .8$ gamble (steeper line) are given. In each case estimated skill is $\Pr[s] = 1/2$ on average, but the downside risk is clearly much higher for the higher probability gamble. If we make the simplifying assumption that the utility function is linear in wealth, the risk premium for a gamble is just the difference

⁸As long as the skill gap has the form $\Pr[\text{win}|s] - \Pr[\text{win}|u] = \alpha \Pr[\text{win}] \Pr[\text{lose}]$ for $\alpha \in [0, 2]$ the posterior skill estimates are linear in $\Pr[\text{win}]$ as seen in Figure 1(a). The case $\alpha = 2$ represents the largest skill gap consistent with $\Pr[\text{win}|s]$ and $\Pr[\text{lose}|u]$ remaining bounded in $[0, 1]$.

⁹Note that absolute risk aversion, $-v''/v'$, is decreasing if $((v'')^2 - v'v''') / (v')^2 < 0$, which, if $v' > 0$ and $v'' < 0$, requires $v''' > 0$. Decreasing absolute risk aversion implies that the demand for risky assets increases with wealth (Pratt, 1964) and that consumers engage in precautionary savings (Kimball, 1990). It is satisfied by most standard utility functions.

¹⁰For a CRRA function $v(x) = x^{1-a}/(1-a)$ this corresponds to a relative risk aversion parameter of $a = 2$, whereas a log utility function corresponds to $a = 1$ and a square root utility function to $a = 1/2$. Typical estimates of risk aversion with respect to wealth, which may be confounded by embarrassment aversion, range from $a = 2$ upwards. Note that $v' > 0$, $v'' < 0$, and $v''' > 0$ for all $a > 0$.

between $v(1/2)$ and the expected value of v from the gamble. As seen in the figure, the risk premium is over six times larger for the $\Pr[\textit{win}] = .8$ gamble than for the $\Pr[\textit{win}] = .2$ gamble.

In the following section we examine a more formal model where we allow the decision maker to have some private information about her own ability (performance skill) and/or about the gamble itself (evaluation skill). We show that the tendency to favor low probability gambles continues to hold, and is strengthened by two effects. First, when the decision maker has some private information about her ability, refusing to take a gamble can be seen as acknowledging one’s own weakness. Therefore, for a sufficiently low probability gamble where the embarrassment from taking the gamble and losing is small, the risk premium is negative rather than positive. Second, when the decision maker has some private information about the gamble, rejecting a gamble that ultimately succeeds reflects unfavorably on the decision maker’s judgement. This is more dangerous than gambling and losing for a low probability gamble, so the risk premium is negative for a sufficiently low probability gamble.

3 The Model

We consider a generalization of the above example that allows for both performance skill and evaluation skill. A decision maker faces a gamble with payoff $x \in \{\textit{lose}, \textit{win}\} \subset \mathbb{R}$ where *lose* (“losing” or “failure”) is strictly less than *win* (“winning” or “success”). The decision maker is of skill $q \in \{u, s\} \subset \mathbb{R}$ where u (“unskilled”) is strictly less than s (“skilled”). The decision maker does not know her skill q but has an unverifiable private signal $\theta \in \{b, g\} \subset \mathbb{R}$ where the probability of winning is higher given a “good” signal g than a “bad” signal b , $\Pr[\textit{win}|g] > \Pr[\textit{win}|b]$. The probability of winning is increasing in skill, $\Pr[\textit{win}|s] \geq \Pr[\textit{win}|u]$, and when it is strictly increasing we say the gamble has “performance skill”. The informativeness of the signal θ regarding the probability of winning is also increasing in skill, $\Pr[\textit{win}|s, g] - \Pr[\textit{win}|u, g] \geq \Pr[\textit{win}|s, b] - \Pr[\textit{win}|u, b]$, and when it is strictly increasing we say the gamble has “evaluation skill”. We assume $\Pr[\textit{win}|s, g] - \Pr[\textit{win}|u, g] > 0$ but do not restrict $\Pr[\textit{win}|s, b] - \Pr[\textit{win}|u, b]$, which can be negative if performance skill is sufficiently strong. Performance (evaluation) skill is “pure” if there is no evaluation (performance) skill and $\Pr[s|g] > (=) \Pr[s|b]$. The joint distribution $F(x, q, \theta)$ has full support on $\{\textit{lose}, \textit{win}\} \times \{u, s\} \times \{b, g\}$.¹¹ The decision maker’s strategy is to accept or reject the gamble at a given price $z \in \mathbb{R}$. Her wealth is normalized to zero.

After the decision maker accepts or rejects the gamble, an observer observes the decision and the outcome of the gamble if it is accepted. The observer’s only role is to estimate the decision maker’s skill q given all available information Ω . The observer could be an employer, the market more generally, a potential mate, or even the decision maker herself if self-esteem is important. To simplify the presentation we normalize type $u = 0$ and $s = 1$ so that $E[q|\Omega] = \Pr[s|\Omega]$. The decision maker’s utility is a quasilinear function of wealth Y and her estimated skill by the observer, $U = Y + v(\Pr[s|\Omega])$ where v is thrice-differentiable on $(0, 1)$. We are particularly interested in cases where a better skill estimate is preferred, $v' > 0$, where the decision maker is risk averse with respect to skill estimates, $v'' < 0$, and where this

¹¹Therefore neither the outcome x nor signal θ is fully revealing of skill q .

aversion is stronger at lower estimates, $v''' > 0$. The utility function is “reduced form” in that we do not explicitly model why the decision maker is concerned with appearing skilled. The quasilinearity assumption simplifies the analysis by allowing us to isolate the effect of embarrassment aversion on risk premia. In practice risk premia are likely to be higher due to regular risk aversion regarding wealth.

Our equilibrium concept is perfect Bayesian equilibrium with D1-proof beliefs (Cho and Kreps, 1987). Therefore we assume that the expected skill $\Pr[s|\Omega]$ by the observer is based on Bayes’ Rule for play on the equilibrium path, and that the observer believes any deviation from the equilibrium path is by the type who benefits from such a deviation for the largest range of responses by the observer. The D1 refinement picks a unique type in our context and therefore simplifies the presentation while also highlighting the fact that the results do not depend on arbitrary choices of observer beliefs. In particular, the D1 refinement implies that if a decision maker is not expected to gamble and unexpectedly gambles then the observer believes that she has good news ($\theta = g$), and that if a decision maker is expected to gamble and does not then the observer believes that she has bad news ($\theta = b$).¹²

We restrict our attention to pure strategy equilibria. First consider a separating equilibrium in which a decision maker with good news gambles and a decision maker with bad news does not. Generalizing (1) and (2), if a gamble is accepted the expected skill conditional on x and the equilibrium belief that $\theta = g$ is

$$\Pr[s|x, g] = \Pr[s|g] + \frac{\Pr[x|s, g] - \Pr[x|u, g]}{\Pr[x|g]} \Pr[s|g] \Pr[u|g] \quad (3)$$

and if a gamble is rejected the expected skill given equilibrium the belief that $\theta = b$ is just $\Pr[s|b]$. The payoff for type θ from gambling is therefore $E[x|\theta] + E[v(\Pr[s|x, g])|\theta] - z$, where $E[v(\Pr[s|x, g])|\theta] = \Pr[win|\theta]v(\Pr[s|win, g]) + \Pr[lose|\theta]v(\Pr[s|lose, g])$, and the payoff from not gambling is $v(\Pr[s|b])$. For the candidate equilibrium to exist, it must be that for a b type the net financial reward from taking a chance and gambling at price z is less than the net skill estimate benefit from playing it safe and not gambling, and that the opposite is true for a g type. Assuming that an indifferent decision maker always gambles, the condition for type b is

$$E[x|b] - z < v(\Pr[s|b]) - E[v(\Pr[s|x, g])|b] \quad (4)$$

and for type g is

$$E[x|g] - z \geq v(\Pr[s|b]) - E[v(\Pr[s|x, g])|g]. \quad (5)$$

Since $E[x|\theta]$ and $\Pr[win|\theta]$ are increasing in θ and $\Pr[s|win, g] > \Pr[s|lose, g]$, for $v' > 0$ a z always exists satisfying these conditions.

In the separating equilibrium a good type “shows off” her favorable information by gambling, but when evaluation skill is sufficiently important relative to performance skill and when the financial costs to losing

¹²Divinity (Banks and Sobel, 1987) implies the weaker restriction that the observer puts more weight on the decision maker having good (bad) news in the former (latter) case, while the intuitive criterion (Cho and Kreps, 1987) need not restrict beliefs at all in our context. As shown in an earlier version of this paper, we find similar behavioral predictions under these weaker restrictions on beliefs as under D1, though the lack of uniqueness complicates the presentation.

are sufficiently small, a “reverse separating” equilibrium is sometimes possible. In this case a decision maker with a bad rather than good signal is expected to gamble, so that losing rather than winning is a positive sign of having an accurate signal. Since this equilibrium seems comparatively unlikely and does not share the properties of the other equilibria, to simplify the presentation we rule it out by assuming that the financial costs of such behavior outweigh any reputational gains from appearing skilled,¹³

$$E[x|b] + E[v(\Pr[s|x, b])|b] < E[x|g] + E[v(\Pr[s|x, b])|g] \quad (6)$$

or

$$lose - win < v(\Pr[s|win, b]) - v(\Pr[s|lose, b]). \quad (7)$$

Since $lose < win$ by assumption, and since losing is never a good sign for a pure performance skill gamble, $\Pr[s|win, b] > \Pr[s|lose, b]$, this assumption is only relevant if there is an evaluation skill component to the gamble. We emphasize that this assumption is purely to streamline the analysis and that there might be cases where the reverse separating equilibrium is of interest.

For pooling equilibria, the two possibilities are a both-gamble equilibrium and a neither-gamble equilibrium. In the both-gamble equilibrium, the observer cannot condition on θ so expected quality from accepting the gamble is just $\Pr[s|x]$ as defined in (1) and (2). If the gamble is unexpectedly refused then, as we confirm in the proof of Proposition 1, the D1 refinement implies that the observer believes that the decision maker has a b signal, so the expected quality is just $\Pr[s|b]$. Therefore the condition for existence of a both gamble-equilibrium is, for $\theta = b, g$,

$$E[x|\theta] - z \geq v(\Pr[s|b]) - E[v(\Pr[s|x])|\theta] \quad (8)$$

which clearly holds for z small enough.¹⁴

Regarding the neither-gamble equilibrium, the expected quality from not gambling is just $\Pr[s]$ and the expected quality from gambling depends on the observer’s beliefs about who would unexpectedly gamble. As we confirm in the proof of Proposition 1, the D1 refinement implies in this case that the observer believes that the decision maker has a g signal, so the expected quality is $\Pr[s|x, g]$. Therefore the condition for a neither-gamble equilibrium is, for $\theta = b, g$,

$$E[x|\theta] - z < v(\Pr[s]) - E[v(\Pr[s|x, g])|\theta] \quad (9)$$

which clearly holds for z big enough.

These conditions are summarized in the following proposition. The details of the proof regarding the D1 refinement are in the Appendix, as are all subsequent proofs.

Proposition 1 *Suppose $v' > 0$. The separating equilibrium exists iff (4) and (5) hold, the both-gamble equilibrium exists iff (8) holds, and the neither gamble equilibrium exists iff (9) holds.*

¹³As shown in the proof of Proposition 1, this assumption also simplifies the analysis of off-equilibrium-path beliefs in the neither-gamble equilibrium.

¹⁴Note that z can be negative. For instance, if we interpret gambling as not buying insurance, then a very high price of insurance corresponds to a very negative z in which case both types will gamble by not taking insurance.

To gain insight into the properties of these different equilibria, and to better understand when they exist, it is helpful to think in terms of the risk premium for a gamble, i.e., the amount that the decision maker would have to be given to make her indifferent to taking the gamble. The risk premium is always positive in gambles without a skill signaling component when utility is concave in wealth (Pratt, 1964), but here we are assuming that utility is linear in wealth and concentrating on the effects created by skill signaling. Because of the linearity assumption, the risk premium in our model just equals the net loss or gain from the skill estimate component of expected utility. That is, for the separating equilibrium, the risk premium for type b , denoted by π_b , is just the RHS of (4), and the risk premium for type g , denoted by π_g , is just the RHS of (5). Similarly the risk premia for the both-gamble and neither-gamble equilibria are given by the RHS of (8) and (9) respectively. We will also refer to the average risk premium, $\pi = \pi_b \Pr[b] + \pi_g \Pr[g]$.

If the risk premium π_θ is positive (negative) then there is an expected loss (gain) to the skill estimate component of utility from taking the gamble, so the price of the gamble z that would make type θ indifferent is less (greater) than $E[x|\theta]$. Therefore the sign of the risk premium gives a simple indication of whether a “fair gamble” with price $z = E[x|\theta]$ will be accepted or rejected by type θ . In this sense the risk premia provide a summary measure of whether skill signaling leads to more or less risky behavior. They also provide a basis for relating our results to non-expected utility models of probability weighting as we discuss in Section 4.

In the example of the previous section the decision maker has no private information about her skill, so the observer has no reason to infer anything about her skill from the choice to gamble or not. Refusing to gamble therefore offers a safe alternative without loss of reputation so the risk premium is always positive for $v'' < 0$. But when we allow for the possibility of private information, refusal to gamble can be seen as lack of confidence in one’s own ability, as seen by the estimates $\Pr[s|b]$ in the conditions (4), (5), and (8) for the separating and both-gamble equilibria, so the decision maker has an added incentive to gamble, implying that the risk premia can be negative rather than positive. A similar situation arises in the neither-gamble equilibrium where if the decision maker decides to gamble then the observer thinks more favorably of her, as seen by the term $\Pr[s|x, g]$ in condition (9). In each case the decision maker faces a trade-off between revealing a lack of confidence in her own skill by not gambling, and looking better on average from gambling but at the risk of being embarrassed.

The following proposition provides conditions under which this tradeoff between admitting incompetence and risking embarrassment can be signed, so that the risk premia are definitely positive or negative. Part (i) considers the case where the gamble has some performance skill so that $\Pr[s|g] - \Pr[s|b] > 0$. If the probability of winning is small enough then there is very little loss in estimated skill from losing, so the decision maker is better off taking a fair gamble than admitting to having a bad signal. Similarly, if this signal is very informative, $\Pr[s|g] - \Pr[s|b]$ approaches 1, then the amount of bad information provided by choosing to not gamble dominates any risk from gambling even when losing is very embarrassing, so the decision maker is better off gambling. Part (ii) considers the opposite case where $\Pr[s|g] - \Pr[s|b]$ is very small, e.g., $\Pr[s|g] = \Pr[s|b]$ as in a pure evaluation skill gamble. In this case very little information

is revealed by refusing to gamble, so for $v'' < 0$ the decision maker prefers to not gamble rather than risk embarrassment.^{15,16} This is similar to the case in the introductory example.

Proposition 2 (i) Suppose $v' > 0$. In any pure strategy equilibrium the risk premia π_θ are negative (a) for fixed $\Pr[s|g] - \Pr[s|b] > 0$ and sufficiently low $\Pr[\text{win}|g]$ and $\Pr[\text{win}|b]$ or (b) for fixed $\Pr[\text{win}|g]$ and $\Pr[\text{win}|b]$ and sufficiently high $\Pr[s|g] - \Pr[s|b]$. (ii) Suppose $v'' < 0$. In any pure strategy equilibrium the average risk premium π is positive for fixed $\Pr[\text{win}|g]$ and $\Pr[\text{win}|b]$ and sufficiently low $\Pr[s|g] - \Pr[s|b]$.

Since the risk premia can be negative with private information about skill, the model captures the idea that people can be dared into taking a gamble. For instance, teenagers often “overconfidently” take risky actions to prove that they are not afraid. Even if they are not skilled at the risky activity they may prefer to give in to peer pressure and take a chance rather than confirm their inadequacy through refusing the dare.¹⁷ The potential role for dare-taking and related behavior is enhanced by the existence of multiple equilibria. For instance, comparing conditions (8) and (9), the both-gamble and neither-gamble equilibrium can coexist depending on the parameter values, and comparing conditions (5) and (9), the separating and neither-gamble equilibria always coexist for some z . The risk premia in each equilibrium are different so the incentive to gamble or not depends in part on what equilibrium is expected. For instance, not gambling is not penalized in the neither-gamble equilibrium while it is in the separating and both-gamble equilibria, so the risk premium (positive or negative) necessary to induce gambling is higher in the neither-gamble equilibrium.

The existence of multiple equilibria also implies that the “framing” of the gamble can potentially make a difference in determining whether a decision maker gambles or not. In particular, if the framing leads the decision maker to expect the observer to have a negative impression of those who do not gamble, then the decision maker is more likely to gamble, e.g., the both-gamble or separating equilibrium is more likely than the neither-gamble equilibrium. Prospect theory finds that the “reference point” of a gamble can affect behavior, in that gambles are more likely when the fixed alternative to gambling is framed as being below the reference point. In a skill signaling model, manipulating the reference point can be seen as providing information to the decision maker about how gambling or not gambling will be perceived, and a high reference point could indicate that failure to gamble will be interpreted negatively by the observer.

The finding from Proposition 2(i) that sufficiently low probability gambles have a negative risk premium is consistent with the tendency to favor low probability gambles that is found in the prospect theory and related literatures. A related result was found in the example of the previous section where it was shown that a decision maker prefers a fair gamble with a 20% chance of winning to a fair gamble with an 80%

¹⁵Cowen and Glazer (2006) consider many labor market applications where decision makers are likely to be risk averse with respect to estimates of their ability and therefore prefer to keep observers ignorant of their exact ability.

¹⁶In some contexts v might be convex, e.g., when a higher performance evaluation will ensure a promotion but a lower evaluation will not lead to a demotion. Similar issues arise regarding reversal of the standard assumption of risk aversion with respect to wealth, e.g., when higher wealth allows purchase of a large indivisible good.

¹⁷In richer information environments, e.g., when the observer also has private information about the decision maker’s skill, trying too hard to prove one’s skill can itself signal a lack of confidence (Feltovich, Harbaugh, and To, 2002).

chance of winning. To investigate this issue of favoring low probability gambles more generally, we now compare risk premia for pairs of gambles where, as in the example, the probability of winning at one gamble is equal to the probability of losing at the other gamble. The current environment is more complicated in that we must hold constant any differences in private information by the decision maker about her skill and/or the gamble. As stated formally in the following definition, we refer to a pair of gambles as symmetric if the probability of winning at one gamble equals the probability of losing at the other, if the skill gap is the same, and if the probability of being skilled given the decision maker's signal is the same. To simplify the comparison we also assume that the probabilities of a good and bad signal are the same.

Definition 1 *Two gambles F and G are a symmetric pair if $\Pr_F[\text{win}] = \Pr_G[\text{lose}]$, $\Pr_F[\text{win}|s, \theta] - \Pr_F[\text{win}|u, \theta] = \Pr_G[\text{win}|s, \theta] - \Pr_G[\text{win}|u, \theta]$, $\Pr_F[q|\theta] = \Pr_G[q|\theta]$, and $\Pr_F[\theta] = \Pr_G[\theta] = 1/2$ for $q \in \{u, s\}$, $\theta \in \{b, g\}$.*

Using this definition, we now show that embarrassment aversion implies lower risk premia on low probability gambles than on high probability gambles in each of the three pure strategy equilibria.

Proposition 3 *Suppose $v' > 0$, $v'' < 0$, and $v''' > 0$ and consider a symmetric pair of gambles F and G where $\Pr_F[\text{win}] = \Pr_G[\text{lose}] < 1/2$. In any given pure strategy equilibrium the risk premia π_θ are lower for the F gamble than for the G gamble.*

These results are for given equilibria, but the same equilibrium might not prevail for high and low probability gambles. The following proposition shows that the tendency to favor lower probability gambles remains even when different equilibria are considered in that there is a stronger tendency for equilibria with gambling to exist for low than high probability gambles. In particular, if an equilibrium exists in which some type does not gamble for a low probability gamble, then an equilibrium with as little or less gambling exists for a high probability gamble. And if an equilibrium exists in which some types gamble for a high probability gamble, then an equilibrium exists with as much or more gambling for a low probability gamble. These results assume that the net expected monetary values of the two gambles are the same, e.g., a gamble with a 20% chance of winning \$100 is priced at \$20 and a gamble with an 80% chance of winning \$100 is priced at \$80.

Proposition 4 *Suppose $v' > 0$, $v'' < 0$, and $v''' > 0$ and consider a symmetric pair of gambles F and G with respective prices z_F and z_G where $E_F[x|\theta] - z_F = E_G[x|\theta] - z_G$ and $\Pr_F[\text{win}] = \Pr_G[\text{lose}] < 1/2$. (i) If a neither-gamble equilibrium exists for F then it exists for G . (ii) If a separating equilibrium exists for F then it or a neither-gamble equilibrium exists for G . (iii) If a separating equilibrium exists for G then it or a both-gamble equilibrium exists for F . (iv) If a both-gamble equilibrium exists for G then it exists for F .*

This proposition shows the robustness of the results to different equilibria, and also provides the straightforward prediction that, for fixed expected monetary payoffs, low probability gambles are more frequently taken than high probability gambles. Since measuring exact risk premia requires measuring willingness

to pay, and since the decision maker has an incentive to strategically manipulate such information, this substantially simplifies testing the model.¹⁸

3.1 Extension: Outcome observed even if gamble refused

So far we have assumed that if the gamble is refused the observer does not learn the outcome of the gamble. But in some cases the outcome is known regardless of whether the gamble is taken or not, e.g., a stock price rises or falls regardless of whether a particular decision maker chooses to buy the stock. In such cases the decision maker will be evaluated based on the outcome whether she takes the gamble or not, e.g., purchasing a stock that does poorly indicates poor skill at evaluating stocks, but so does not purchasing a stock that does well. Since no information is revealed about skill if a gamble that only involves performance skill is refused, we are interested in gambles with an evaluation skill component. For simplicity we restrict attention to the case of pure evaluation skill.

If a decision maker is expected to gamble if and only if she has good news about the probability of success, i.e., there is a separating equilibrium, then the decision maker looks wise for refusing a gamble that loses and looks foolish for refusing a gamble that wins. Therefore the updating process works in the opposite direction as with accepted gambles. This is most clear when a gamble is symmetric.

Definition 2 *A gamble F with pure evaluation skill is symmetric if $\Pr[\text{win}|s, g] - \Pr[\text{win}|u, g] = \Pr[\text{win}|u, b] - \Pr[\text{win}|s, b]$ and $\Pr[g] = \Pr[b] = 1/2$.*

Consider a symmetric gamble with pure evaluation skill where a separating equilibrium is being played and the gamble is refused. The observer expects the decision maker has a bad signal $\theta = b$, so the expected skill conditional on x is, for $x \neq x'$,

$$\begin{aligned} \Pr[s|x, b] &= \Pr[s] + \frac{\Pr[x|s, b] - \Pr[x|u, b]}{\Pr[x|b]} \Pr[s] \Pr[u] \\ &= \Pr[s] + \frac{\Pr[x'|s, g] - \Pr[x'|u, g]}{\Pr[x|b]} \Pr[s] \Pr[u]. \end{aligned} \quad (10)$$

Looking at the first line of (10), since the gamble is more likely to lose if a decision maker with a bad signal is skilled rather than unskilled, $\Pr[\text{win}|s, b] < \Pr[\text{win}|u, b]$, refusing a gamble that wins is embarrassing, $\Pr[s|\text{win}, b] < \Pr[s]$. Now consider the second line of (10). For $\Pr[\text{win}] = 1/2$, symmetry implies that $\Pr[x|b] = \Pr[x'|g]$, so $\Pr[s|x, b] = \Pr[s|x', g]$, i.e., the negative (positive) updating from refusing a gamble that wins (loses) is the same as that from taking a gamble that loses (wins), and the risk premium is zero. However for $\Pr[\text{win}] < 1/2$, symmetry implies that $\Pr[\text{win}|b] < \Pr[\text{lose}|g]$, so $\Pr[s|\text{win}, b] < \Pr[s|\text{lose}, g]$, i.e., it is more embarrassing to refuse a gamble that wins than to take a gamble that loses. And for $\Pr[\text{win}] > 1/2$, the reverse pattern holds, $\Pr[s|\text{win}, b] > \Pr[s|\text{lose}, g]$. Therefore, as shown in the first part of the following proposition, gambles with a low probability of success have negative risk premia even as

¹⁸In our model the decision maker has the usual incentive to underestimate willingness to pay so as to avoid paying too much, and can also have an incentive to overestimate willingness to pay so as to signal private information about her skill when θ is correlated with q .

gambles with a high probability of success have more standard positive risk premia. For low probability gambles losing is expected so the negative updating is limited if the gamble is taken and it loses. But for the same reason if the gamble is refused and it wins, the observer will infer that the decision maker had low quality information so the negative updating of skill is more substantial. The decision maker therefore has a reputational incentive to accept a low probability gamble.

In the separating equilibrium the decision maker is judged whether she takes the gamble or not. But for pooling equilibria where both types take the same action, the decision maker avoids judgement as long as she does not deviate from the equilibrium. For instance, if both types are expected to take the gamble, then the success or failure of the gamble provides no information on the quality of the decision maker’s information and hence on her skill. If the decision maker deviates and refuses to take the gamble then the observer will, following our D1 refinement, infer that the decision maker has an unfavorable $\theta = b$ signal, so the outcome is informative of the quality of this signal. Consequently, unless the financial incentives are sufficiently strong, an embarrassment averse decision maker prefers to stick with the pooling equilibrium. In particular, we find that the average risk premium is always negative. If we consider the neither-gamble equilibrium then again it is safer for the decision maker to pool rather than risk standing out and losing, so in this case the risk premium is always positive.

Proposition 5 *Consider a pure evaluation skill gamble where the outcome is observed even if the gamble is refused. (i) Suppose $v' > 0$, $v'' < 0$, and $v''' > 0$ and the gamble is symmetric. In the separating equilibrium the average risk premium π is negative (positive) for $\Pr[\text{win}] < (>)1/2$. (ii) Suppose $v' > 0$ and $v'' < 0$. In the both-gamble (neither-gamble) equilibrium the risk premia π_θ are always negative (positive).*

Note that part (ii) implies that for a fair gamble, $E[x] = z$, the both-gamble and neither-gamble equilibria will often coexist since each type of decision maker finds it least risky to take the same action as the other type.¹⁹ The strong sensitivity of behavior in this environment to observer expectations can be interpreted as implying a greater role for social influences such as “peer pressure”.

3.2 Extension: Observer uninformed of probability of success

So far we have assumed that the observer knows the unconditional probability of success, $\Pr[\text{win}]$, but clearly this is not always true, e.g., a manager might know the difficulty of a project, but the manager’s boss might be in the dark. When the observer is uninformed of the odds of the gamble we would naturally expect the decision maker to favor high rather than low probability gambles, the opposite of what we have found so far. Understanding this case helps clarify our main results and also provides a clear prediction that can separate skill signaling from behavioral models where the decision maker always favors low probability gambles.

To model this case assume there are two gambles F and G where $\Pr_F[\text{win}] < \Pr_G[\text{win}]$ and where the decision maker faces one of the two gambles, each with positive probability, but the observer does not

¹⁹Since $E[x|g] \geq E[x|b]$, we cannot be sure that both equilibria coexist based on the risk premia results unless the bet is sufficiently small.

know which. If, as we have assumed so far, the decision maker has a private signal θ for each gamble that is informative of the probability of success, then the decision maker has two dimensions of private information so there are a large number of possible separating and partially separating equilibria. Therefore, for simplicity, we revert to the model in the introduction and assume that the decision maker does not have any private information other than knowing which gamble is being faced.

With this assumption, we refer to the separating equilibrium as the equilibrium where the G gamble is taken and the F gamble is refused and the both-gamble (neither-gamble) equilibrium as the equilibrium where either (neither) gamble is taken.

Proposition 6 *Suppose $v' > 0$ and the decision maker faces one of two gambles F or G where $\Pr_F[\text{win}] < \Pr_G[\text{win}]$ and there is no private information θ for either gamble. If the decision maker knows which gamble she faces but the observer only knows that each gamble is faced with positive probability, then in any pure strategy equilibrium the risk premium is higher for the F gamble than for the G gamble.*

This result follows from just reinterpreting our base model with one gamble with private information $\theta \in \{b, g\}$ as a model with two gambles where $\theta = b$ corresponds to gamble F and $\theta = g$ corresponds to gamble G . In our base model the risk premium is always higher for type $\theta = b$, so this implies that the risk premium is higher for gamble F . The fact that our base model can be reinterpreted in this way highlights the importance of being careful in tests of skill signaling. If, for instance, $\Pr[\text{win}|g] = 3/8$, $\Pr[\text{win}|b] = 1/8$, and $\Pr[\text{win}] = 1/4$, we predict that the risk premium is higher for type b facing $\Pr[\text{win}|b] = 1/8$ than for type g facing $\Pr[\text{win}|g] = 3/8$. But we also predict that the risk premium is lower for each type than in the paired symmetric gamble where $\Pr[\text{win}|b] = 5/8$, $\Pr[\text{win}|g] = 7/8$, and $\Pr[\text{win}] = 3/4$. Our main argument in this paper that lower probability gambles are favored refers to this latter conclusion.

In addition to these two extensions on observability of refused gambles and observability of the probability of success, clearly a number of other extensions are possible, many of which appear in related contexts in the career concerns literature and may also be related to behavioral anomalies. For instance, the decision maker might be able to choose whether to report that a gamble was taken, e.g., people often choose whether to inform friends and associates of gambling or stock market outcomes. In such cases the absence of a report may be interpreted as a sign of failure, thereby giving an extra incentive to take risky actions. The presence of multiple gambles also changes the information structure substantially.²⁰ Rather than pursuing more of these extensions we now consider how our results relate to the literature on behavioral anomalies in risk-taking.

²⁰Multiple gambles are important in the fallacy of large numbers (Samuelson, 1963), the house-money effect (Thaler and Johnson, 1990), and the disposition effect (Shefrin and Statman, 1995).

4 Relation to prospect theory and other models

4.1 Prospect Theory

As discussed in the introduction, these results are closely related to the key phenomena of loss aversion, probability weighting, and framing effects that have been identified in the prospect theory literature. Since the connections with loss aversion and framing effects are straightforward, we will concentrate on clarifying the connection between skill signaling and probability weighting. We are interested in determining what probability weighting function will be estimated from data on risk taking if the decision maker is concerned in part with looking skilled, but is modeled as only concerned about the monetary payoffs.

Our results on risk premia can be related to the probability weighting function by finding the different certainty equivalents c for different values of $p = \Pr[\textit{win}]$ in a gamble and then inferring what weighted probability w of winning would have induced a risk neutral decision maker unconcerned with embarrassment aversion (or other factors) to choose that certainty equivalent.²¹ Setting w such that $c = w(\textit{win}) + (1 - w)(\textit{lose})$, the weighting function is then

$$w = \frac{c - \textit{lose}}{\textit{win} - \textit{lose}}. \quad (11)$$

In our model with embarrassment aversion but without probability weighting (i.e., $w = p$) the certainty equivalent is just $c = p(\textit{win}) + (1 - p)(\textit{lose}) - \pi$. So if the probability weighting function is estimated as $U = Y$ assuming no embarrassment aversion, but the true utility function is $U = Y + v(\Pr[s|\Omega])$ with embarrassment aversion but without probability weighting, then the probability weights are estimated as

$$w = p - \frac{\pi}{\textit{win} - \textit{lose}}. \quad (12)$$

Note that overweighting is found if the risk premium is negative and underweighting is found if it is positive. Moreover, the amount of overweighting or underweighting is in direct proportion to the risk premium, so Proposition 3 implies that there will be disproportionate weight on low probabilities relative to high probabilities, which is consistent with prospect theory.²²

To see this in more detail, consider the example from Section 2 where $v = -1/x$ and set $\textit{lose} = 0$ and $\textit{win} = 10$ so that the imputed probability weighting function is

$$w(p) = p - \frac{v(1/2) - (pv(\Pr[s|\textit{win}]) + (1 - p)v(\Pr[s|\textit{lose}]))}{10}. \quad (13)$$

As shown in Figure 2(a), the decision maker appears to underweight both high and low probability gambles, and to underweight high probability gambles more strongly. Note that there is a ‘‘certainty effect’’

²¹Rather than assuming risk neutrality when estimating the probability weighting function, the prospect theory literature sometimes follows the more complicated approach of disentangling the predictions of the probability weighting function and the convex-concave utility function assumed in original prospect theory (Wu and Gonzalez, 1996).

²²Recall from the discussion in the introduction that prospect theory treats gambles in gains and losses differently, and the pattern of weighting across the two-domains leads to the ‘‘four-fold pattern’’. Our model generates this same pattern by considering only the probability of success, i.e., we do not care about the signs of \textit{win} or \textit{lose} but only require that $\textit{win} > \textit{lose}$.

(Kahneman and Tversky, 1979) or discontinuity at $p = 1$ in that the decision maker is especially wary of gambles that are near sure things, but such fears disappear at the limit of $p = 1$.²³ Such an effect arises because the skill estimate $\Pr[s|lose]$ approaches 0 as $\Pr[win]$ goes to 1, so the decision maker becomes increasingly worried about this small chance of extreme embarrassment. If we change the assumption in the example so that the skill gap is half as large, $\Pr[win|s] - \Pr[win|u] = \Pr[win]\Pr[lose]$, then losing is not so embarrassing for large $\Pr[win]$ and this discontinuity disappears.²⁴ Figure 2(b) shows this case where we have also reduced the financial stakes, setting $lose = 0$ and $win = 1$. As seen from (12) and (13), lower stakes tend to accentuate the degree of underweighting, which in this case compensates for the smaller skill gap.

Figure 2(c) shows the similar case as Figure 2(a), with the difference that we now allow the decision maker to have some private information regarding her skill, $\Pr[s|g] = .55$ and $\Pr[s|b] = .45$ where $\Pr[win|s, \theta] - \Pr[win|u, \theta] = 2\Pr[win]\Pr[lose]$, $\Pr[s] = \Pr[u]$ as before, and $\Pr[g] = \Pr[b]$. Now, instead of there just being more relative weight on a low probability gamble than on its symmetric high probability gamble, the gain from showing off confidence in one's skill implies there is overweighting of low probability gambles. Pictured is the imputed weighting function for the separating equilibrium based on the average risk premium.²⁵ The weights for the both-gamble and neither-gamble equilibrium have the same pattern but are lower since the choice to gamble or not is less revealing of the decision maker's private information. Note that the general pattern of this function with its discontinuity at $p = 0$ and $p = 1$ tracks that found in Kahneman and Tversky (1979).

An example of pure evaluation skill where the outcome is observed even if the gamble is refused is shown in Figure 2(d). Similar to the case of Figure 2(b), we assume a more moderate skill gap, $\Pr[win|s, g] - \Pr[win|u, g] = \Pr[win|u, b] - \Pr[win|s, b] = \Pr[win]\Pr[lose]$, set $lose = 0$ and $win = 1$, and make the symmetry assumption that $\Pr[g] = \Pr[b]$. The figure shows the imputed weighting function for the separating equilibrium where, as implied by Proposition 4, there is overweighting for $\Pr[win] < 1/2$ and underweighting for $\Pr[win] > 1/2$. This pattern is similar to that in Tversky and Kahneman (1992), with the exception that they find underweighting starting at a point below $p = 1/2$. Recall that behavior in this version of the model is particularly susceptible to multiple equilibria since both types want to pool with each other when monetary incentives are weak. In the both-gamble equilibrium the risk premium is always negative while in the neither-gamble equilibrium it is always positive, implying that there is always overweighting in the former equilibrium and always underweighting in the latter equilibrium.

We have chosen the parameters in these examples for their simplicity and their ability to generate probability weighting functions that track the canonical forms in Kahneman and Tversky (1979) and Tversky and Kahneman (1986). However, other parameters can generate functions that differ significantly

²³The certainty effect can also arise from a simple fear of being cheated, in that it is much easier to demand payment of an amount promised with 100% certainty than an amount promised with 99% certainty. Our model partially captures this concern in that determining when a winning gamble will really be paid can be interpreted as evaluation skill, so that failure to be paid is an embarrassing indication of poor judgement.

²⁴In this case $\Pr[s|lose]$ approaches 1/4 rather than 0 as $\Pr[win]$ goes to 1.

²⁵Since the risk premia are different for types b and g , the imputed weighting functions are also different.

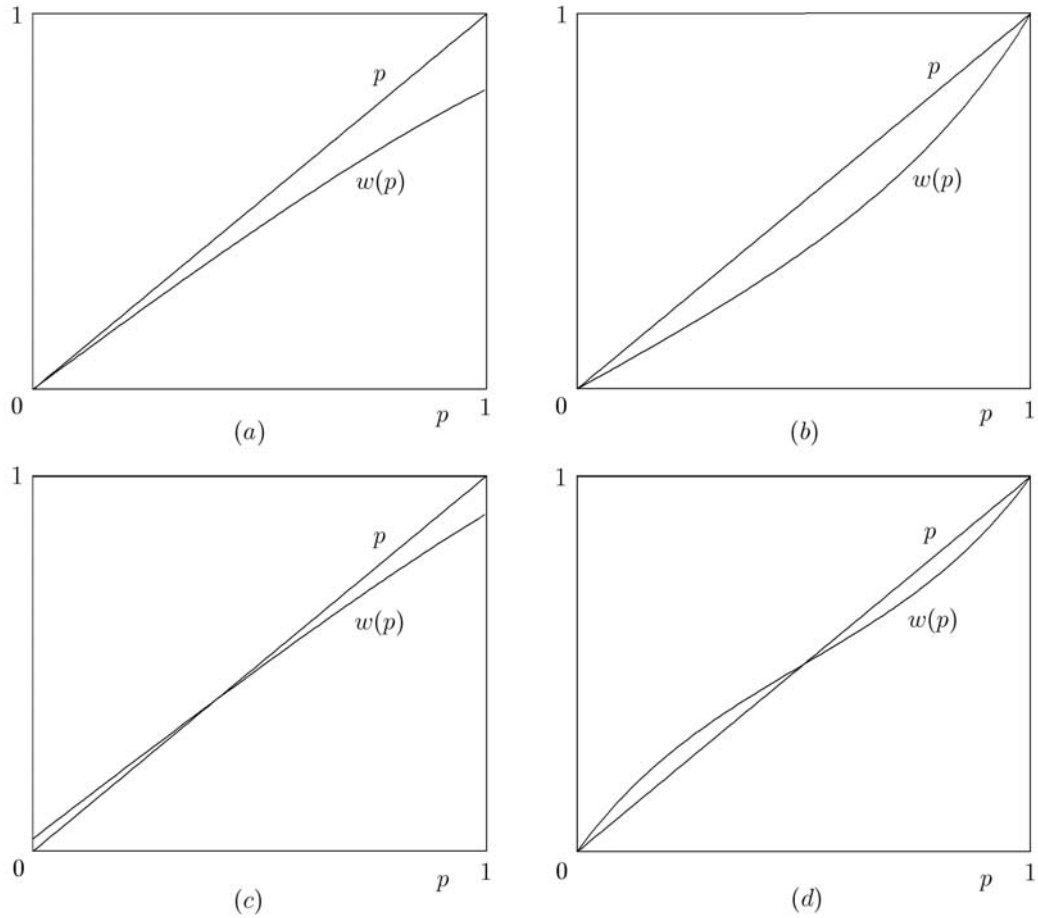


Figure 2: Imputed probability weighting functions if skill signaling is omitted, $p = \Pr[\text{win}]$

from these, and the prospect theory literature itself has found considerable variety. From a skill signaling perspective, the restrictions on the shape of these functions are (for the base model) those implied by the risk premia results of Propositions 2 and 3 – that sufficiently low probability gambles are overweighted in absolute terms if the decision maker has private information about her skill, and that low probability gambles are overweighted in relative terms quite generally. Note from (13) that the probability weights need not be positive when the risk of embarrassment is significant and the monetary incentives are small, e.g., in some cases a decision maker might prefer to avoid a zero-cost gamble even if all the outcomes are non-negative.²⁶

Behavior consistent with prospect theory is often found in managerial decision-making and other environments where skill signaling is likely to be important. Our results indicate that it is an open question

²⁶Negative probability weights are found by Gneezy, List, and Gonzalez (2006), who label the phenomenon the “uncertainty effect”. We discuss negative weights further in our comparison of skill signaling and rank-dependent utility.

whether behavioral or strategic models best capture such behavior. The results also raise the question of whether the prospect theory behavior observed in experimental settings might have a skill signaling component. Unlike the early social psychology experiments, most prospect theory experiments do not explicitly involve skill, so the role of skill signaling would appear to be limited.²⁷ However, the behavior observed in experiments might reflect rule-of-thumb strategies based on skill signaling that are not appropriate for pure games of chance, but that are generally advisable given that most real world gambles involve both skill and chance.²⁸ Consistent with this view, people often behave as if there is a skill component to pure chance outcomes (Langer, 1975), so they may still feel embarrassment or loss of self-image at losing. Another possibility is that experimental tests of prospect theory have not fully controlled for skill. In particular, most tests starting with Kahneman and Tversky (1979) involve hypothetical gambles so it is not clear whether subjects in these experiments should imagine that losing would reflect unfavorably on them. Consistent with this perspective, recent experiments using real gambles without a skill component²⁹ find that probability weighting weakens or disappears (Laury and Holt, 2002; Harbaugh, Krause, and Vesterlund, 2002a; Harbaugh, Krause, and Vesterlund, 2002b; Bosch-Domenech and Silvestre, 2003).³⁰

4.2 Other models

We have emphasized the relation between skill signaling and prospect theory because of the close connections that we find and because both models are so widely used in economics and related disciplines. We now consider several other models in less detail.

Achievement motivation Skill signaling can be seen as a formalization of key aspects of Atkinson’s (1957) model of achievement motivation, one of the primary psychological models of risk taking before prospect theory.³¹ Atkinson notes that different probability gambles convey different information about skill, and argues that people will be most afraid of gambles with an equal probability of success or failure

²⁷The phenomena of loss aversion and framing can also arise in contexts without explicit uncertainty, in which case the role of skill is likely to be small.

²⁸Such arguments are often used to explain anomalous behavior when the experimental design is close to familiar real world environments, but predicted behavior differs from what would be appropriate in such environments, e.g., Halevy and Feltkamp (2005) make such an argument regarding uncertainty aversion.

²⁹Of course, completely eliminating skill from an experiment is difficult since a subject’s decision to participate involves an evaluation that the likely payoff will be higher than the opportunity cost of time. Even conditional on participation, evaluation skill arises regarding subject estimates that the odds really are as reported and that they really will get paid if they win. Performance skill also arises if some subjects are more likely to understand the instructions than other subjects. Given these uncertainties, subjects with large earnings have favorable information about their judgment to report to friends and family, and subjects with low earnings have good reason to feel foolish.

³⁰Harbaugh, Krause, and Vesterlund (2002a) find that adults weight gambles linearly but children tend to underweight low probability gambles and overweight high probability gambles. Reverse probability weighting by children could reflect insufficient understanding that success is less impressive when the gamble is easy. Relatedly, as shown in Section 3.2, reverse probability weighting arises when the odds of the gamble are treated as private information.

³¹I thank Tatiana Kornienko for noting the connection between the model in this paper and the early social psychology models. See Coombs, Dawes, and Tversky (1970) for a discussion of the early psychological models of risk taking.

since they are most revealing of ability. Experiments found evidence of this behavior, but also found a strong tendency to favor long-shots relative to sure things, behavior that was considered to be outside of the model’s predictions (Atkinson, Bastian, Earl, and Litwin, 1960; Atkinson and Litwin, 1960).³² Applying our results on skill signaling to Atkinson’s model, this inability to explain probability weighting is due to use of a simple piece-wise linear function to represent a decision maker’s utility from what effectively are different skill estimates.

In particular, Atkinson assumes that the utility gain from winning a gamble and the utility loss from losing a gamble are both linear functions of the probability of success. Letting the constants $m_s > 0$ and $m_f > 0$ represent the respective motives to gain success and avoid failure, the decision maker’s utility from a gamble with a p chance of success is assumed to be $pm_s(1-p) + (1-p)m_f(-p)$ where $1-p$ is the utility from success and $-p$ is the utility from failure. Rearranging, the utility from the gamble is $(m_s - m_f)p(1-p)$ which for $m_s > m_f$ is maximized at $p = 1/2$, and for $m_s < m_f$ is maximized for p as close as possible to 0 or 1. Therefore, those with a stronger motive to achieve should prefer gambles with a near equal probability of success, and those with a stronger motive to avoid failure should prefer more extreme gambles.

Atkinson’s utility function maps directly into our model when estimated skill from winning and losing is linear in the probability of success,³³ when the decision maker does not have any private information about her own skill or about the gamble, and when v is a piecewise linear function with a kink at $1/2$. If $m_s > m_f$ then this corresponds to the case where v is convex, and if $m_s < m_f$ then this corresponds to our case where v is concave and decision makers are risk averse with respect to their reputations. The key difference with our model is that we assume that $v''' > 0$ so that the slope of v is increasingly steep for lower skill estimates as seen in Figure 1(b). Such downside risk aversion implies that decision makers want to avoid gambles where success is expected and tend to favor long shots. Incorporating a standard utility function with downside risk aversion into Atkinson’s original model allows it to explain the observed pattern of favoring long-shots relative to near sure things, and thereby realize Atkinson’s insight that there is “little embarrassment in failing” at difficult tasks and a great “sense of humiliation” in failing at easy tasks.

Self-esteem Following the career concerns literature, we have presented the model primarily in terms of the decision maker having a concern for the esteem of others, but if the decision maker feels utility in maintaining self-esteem, the observer in our model can be seen as the decision maker herself. The idea that self-esteem is affected by the outcome of risky decisions, and that people may avoid risk to avoid loss of self-esteem, dates back at least to James (1890), who defined self-esteem as the ratio of success to “pretensions”, and noted that self-esteem could be raised “as well by diminishing the denominator as increasing the numerator.” Of course, avoiding a situation where success is not assured can also reveal

³²The experiments used shuffle board and ring-toss games in which the subjects could choose from what distance to play. The theoretical model predicted that subjects who were afraid of failure would prefer long or close distances to intermediate distances. Most choices were for long distances with a low probability of success.

³³This holds under our assumptions when, as in Figure 1(a), the skill gap is proportional to $\Pr[\text{win}] \Pr[\text{lose}]$.

unfavorable information to one's self and others.³⁴ Our analysis formalizes the tradeoff between the sure loss in esteem from such avoidance and the uncertain loss from taking a chance and losing, and shows how this tradeoff changes with the probability of success.

The empirical literature supports the idea that self-esteem matters (Diener and Diener, 1995) and that many people are risk averse with respect to self-esteem (Falk, Huffman, and Sunde, 2006). A concern for self-esteem could reflect a direct preference (Koszegi, 2006), or as Benabou and Tirole (2002) note, it could be instrumental in that high self-esteem makes it less costly to convey a favorable image to others, in which case our arguments about skill signaling apply directly.³⁵ However, in some cases the relation between self-esteem and skill signaling can be more complicated. Benabou and Tirole (2002) consider self-esteem in a model where a decision maker's self-knowledge about ability affects her incentive to take costly actions, and find conditions under which higher confidence is a motivating factor that leads to higher effort. They also find conditions under which a decision maker will want to motivate herself, so that self-esteem is desirable, and conditions under which a lack of confidence is particularly damaging, which can be interpreted as risk aversion with respect to self-esteem.³⁶ Higher self-esteem is not always desirable in their model, however, since it can sometimes lead to slacking off.

Self-handicapping The question of how different probabilities of success reveal different information about ability is central to the theory of self-handicapping in which people deliberately lower the odds of success so as to reduce the loss in self-esteem or public image due to failure (Jones and Berglas, 1978).³⁷ Our model formalizes the implicit assumption in this literature that losing at lower probability gambles is less damaging to estimated ability. Not addressed in this literature is that self-handicapping makes losing more frequent even as it makes losing less painful, so it is unclear why people should prefer to self-handicap. We explicitly address this tradeoff and show that downside risk aversion and/or private information about skill can explain a preference for gambles with a lower probability of success. Benabou and Tirole (2002) also address self-handicapping as a strategy to maintain confidence, and model it as taking an inefficient action that completely avoids revealing ability. Our results indicate why actions that reduce the probability of success, but still allow for some chance of revealing high ability, are an attractive self-handicapping strategy.

Rank-dependent utility When individual probabilities are weighted differently from their actual (or subjective) probabilities, a decision maker will sometimes choose a stochastically dominated gamble (Fishburn, 1978). To avoid this problem, Quiggin (1982) develops an alternative weighting model that reweights

³⁴Self-signaling in this context requires some form of intrapersonal asymmetric information (Benabou and Tirole, 2002 and 2004; Bodner and Prelec, 2003).

³⁵They note that there may be a purely hedonic value from thinking highly of oneself.

³⁶Note that risk-aversion with respect to self-esteem implies that the decision maker prefers to have less self-knowledge of her abilities, but more accurate self-knowledge can be desirable in that it improves decision-making (Rauh, 2006). In such cases the decision maker is essentially risk-loving rather than risk-averse with respect to self-esteem.

³⁷The literature considers both esteem and self-esteem as factors and finds that self-handicapping is more common in public situations (Kolditz and Arkin, 1982).

the entire probability distribution rather than individual probabilities. This rank-dependent utility approach is adopted by Tversky and Kahneman (1992) in their model of cumulative prospect theory. Monetary outcomes are not the only contributors to utility in our model, so a decision maker might in fact prefer a gamble that is stochastically dominated in the monetary dimension. That is, from a skill signaling perspective, there is no reason to assume that a weighting function based on rank-dependent utility is a better predictor of behavior than the weighting function used in original prospect theory.

This paper considers the simple case of choosing between a gamble with two outcomes and a fixed amount, in which case both forms of probability weighting are typically equivalent. But even in this case, stochastic dominance in the monetary dimension can be violated when the risk premia due to embarrassment aversion are sufficiently large or the monetary values sufficiently small that the imputed probability weights are negative. The decision maker then prefers to avoid a free gamble even when all outcomes are positive, i.e., the decision maker prefers the stochastically dominated choice of not gambling. For instance, if a gamble if both outcomes of a free gamble are positive but close to zero, a decision maker might prefer to avoid the bet if she thinks it is too risky in terms of revealing a lack of skill. Likewise, if a manager is considering a project where the direct financial benefits to the manager stochastically dominate the direct financial costs, the manager might still avoid the project if the danger of embarrassment is sufficiently high.

Regret theory Skill signaling is also related to models of regret theory in which the utility function includes wealth and an additively separable regret function that is increasing in the difference between the realized and unrealized outcomes (Bell, 1982; Loomes and Sugden, 1982). For instance, a stock might be bought because of the regret that would arise if it was not purchased and did well. This is related to our result that the success of a refused gamble reflects unfavorably on the decision maker. Regret theory can explain prospect theory’s weighting function if the regret function is less concave for negative outcomes than for corresponding negative outcomes, a condition very similar to our assumption of decreasing concavity with respect to skill estimates.

A distinguishing prediction of regret theory relative to prospect theory is that a decision maker will pay to avoid learning the outcome of a refused gamble (Bell, 1983). Skill signaling implies the related prediction that, for $v'' < 0$, the decision maker prefers to keep the observer rather than herself in the dark. However, the predictions can be quite similar since skill signaling can also involve signaling to one’s self, and since one factor in regret might be external incentives similar to career concerns. In fact, Bell (1982) suggests that “the evaluation of others, one’s bosses for example, may be an important consideration” in regret but regret theory does not formally model how such evaluations would be affected by different gambles. By explicitly allowing for some decision makers to be more skilled than others, we are able to show why losing high probability gambles is more embarrassing than losing low probability gambles and to analyze the different equilibria that can result.

Disappointment aversion To most simply capture the Allais paradox and related phenomena, Gul (1991) develops a model of disappointment aversion in which decision makers receive extra utility from

outcomes that are higher than the certainty equivalent of the gamble, and extra disutility from outcomes that are worse than the certainty equivalent.³⁸ This differs from regret theory in which concern over unrealized outcomes drives behavior. Our model only has two outcomes, and in equilibrium success of an accepted gamble always leads to a higher skill estimate than failure, so the predictions of disappointment aversion and skill signaling are quite similar over the domain we consider. In particular, Gul (1991) finds that, for a two-outcome gamble, disappointment aversion implies a probability weighting function equivalent to that in Quiggin (1982) and Tversky and Kahneman (1992).

5 Conclusion

Simple economic models are often poor predictors of behavior. The behavioral approach to this failing, as exemplified by prospect theory, is to include perceptual and cognitive biases that interfere with rational decision making. In this paper our approach is to enrich the economic model by including strategic and informational effects that are often omitted. In recent decades this strategic approach has shown how a wide range of seemingly irrational behaviors may in fact be quite reasonable, e.g., the signaling literature shows how wasteful displays can be individually rational, the folk theorem and reputation literatures show that cooperative behavior does not require altruism, the career concerns literature shows that people are often right to care about sunk costs, and the information cascades literature shows how rational individuals can inefficiently herd. This paper uses insights from the social psychology and career concerns literatures to show that the key behavioral anomalies associated with prospect theory are also predicted by a model of rational skill signaling in environments where decision makers care about appearing skilled. The results indicate that prospect theory behavior observed in many economic, financial, and managerial decisions might be confounded with effects due to skill signaling.

Given the similar predictions of prospect theory and skill signaling in such environments, it is not always necessary to distinguish between the theories. For instance, if consumers and investors want to avoid looking foolish in front of friends and family, one modeling choice is to assume loss aversion rather than formally incorporate the information flows. However, as seen from the range of behaviors analyzed in the career concerns literature, the predictions of skill signaling are sensitive to the information and incentive environments, so distinguishing between the theories is often important. In particular, this paper shows that prospect theory's prediction of loss aversion can be reversed when private information on skill allows for dare-taking behavior. It also shows that prospect theory's predicted pattern of probability weighting is reversed when observers are uninformed of the probabilities. These differences indicate when it is important to explicitly model the information flows, and also provide a basis for testing the relative explanatory power of the behavioral and strategic approaches to understanding risk.

³⁸Related to disappointment aversion and to skill signaling is Neilson's (2002) model of victory and defeat in which the decision maker receives higher state-dependent utility from an outcome when the outcome is more favorable and rare relative to the other outcomes, and lower state-dependent utility when it is less favorable and rare. Neilson shows how the Allais paradox and a number of other risk anomalies can be explained by such a model.

6 Appendix

We first prove the following lemma which is used in the proofs of Propositions 3 and 5.

Lemma 1: Consider a distribution $P(x)$ on $x \in \{l, w\} \subset [0, 1]$, and a distribution $Q(x)$ on $x \in \{L, W\} \subset [0, 1]$. If $L < l < W < w$, $W - L \geq w - l$, $\Pr_P[w] = \Pr_Q[L] < 1/2$, and $E_P(x) \geq E_Q(x)$, then $E_P[v(x)] \geq E_Q[v(x)]$ for all v such that $v' > 0$, $v'' < 0$, and $v''' > 0$.

Proof: We apply Whitmore's (1970) third-order stochastic dominance result that, for $v' > 0$, $v'' < 0$, $v''' > 0$, the distribution $P(x)$ on $[0, 1]$ is weakly preferred to the distribution $Q(x)$ on $[0, 1]$ if

$$E_P(x) \geq E_Q(x) \tag{14}$$

and

$$\int_0^a \int_0^b (Q(x) - P(x)) dx db \geq 0 \tag{15}$$

for all $a \in [0, 1]$. Condition (14) is a condition of the lemma. Regarding condition (15), note that

$$\begin{aligned} P(x) &= 0 && \text{for } x < l \\ P(x) &= \Pr_P[l] && \text{for } l \leq x < w \\ P(x) &= 1 && \text{for } x \geq w \end{aligned}$$

and

$$\begin{aligned} Q(x) &= 0 && \text{for } x < L \\ Q(x) &= \Pr_Q[L] && \text{for } L \leq x < W \\ Q(x) &= 1 && \text{for } x \geq W \end{aligned}$$

so $Q(x) = P(x) = 0$ for $x < L$, $Q(x) > P(x)$ for $L \leq x < l$, $Q(x) < P(x)$ for $l \leq x < W$, $Q(x) > P(x)$ for $W \leq x < w$, and $Q(x) = P(x) = 1$ for $x \geq w$. Therefore if (15) holds for $a = W$ then it must hold for all a . Checking this case,

$$\begin{aligned} \int_0^W \int_0^b (Q(x) - P(x)) dx db &= \int_0^W \left(b \Pr_Q[L] I_{[L, W]} - b \Pr_P[l] I_{[l, w]} \right) db \\ &= \Pr_Q[L] \frac{(W - L)^2}{2} - \Pr_P[l] \frac{(W - l)^2}{2} \end{aligned} \tag{16}$$

where I is the index function.³⁹

Using the condition $\Pr_P[w] = \Pr_Q[L]$, and its implication that $\Pr_P[l] = \Pr_Q[W]$, condition (14) can be written

$$\Pr_Q[L](w - L) \geq \Pr_P[l](W - l).$$

³⁹Geometrically, the distribution functions are flat on $[L, W]$ and $[l, w]$ respectively, so the areas under them are increasing linear functions with slopes $\Pr[L]$ and $\Pr[l]$ respectively, so the comparison is between a triangle with base $W - L$ and height $\Pr[L](W - L)$ and a triangle with base $W - l$ and height $\Pr[l](W - l)$.

Therefore, from (16), condition (15) holds if condition (14) holds and if

$$\begin{aligned}
& (W - L)^2 \geq (w - L)(W - l) \\
\iff & (W - L)^2 \geq ((w - l) + (l - L))((W - L) - (l - L)) \\
\iff & (W - L)^2 \geq (W - L)^2 - (l - L)^2
\end{aligned} \tag{17}$$

where the final implication holds by the condition of the lemma that $W - L \geq w - l$. ■

Proof of Proposition 1: Conditions (4) and (5) for the separating equilibrium are straightforward since there are no off-equilibrium-path outcomes. Regarding the both-gamble equilibrium, D1 requires that if one type is willing to deviate to not gambling for a larger range of observer actions, i.e., estimates of the decision maker's skill, the observer believes that any such deviation comes from that type. By assumption, $\Pr[s|g] \geq \Pr[s|b]$ so any estimated skill y from not gambling is in the range $Y = [\Pr[s|b], \Pr[s|g]]$. Given such a y a decision maker prefers to gamble iff

$$E[x|\theta] - z \geq v(y) - E[v(\Pr[s|x])|\theta]. \tag{18}$$

Let $Y_\theta \subset Y$ be the subset of Y such that (18) does not hold for type θ , i.e., the $y \in Y$ such that type θ will deviate. Since $E[x|\theta]$ and $E[v(\Pr[s|x])|\theta]$ are both increasing in θ , for $v' > 0$ there are three possibilities, $Y_g \subsetneq Y_b \subset Y$, $Y_g = Y_b = \emptyset$, and $Y_g = Y_b = Y$. In the first case D1 implies that $y = \Pr[s|b]$ and the both-gamble equilibrium exists iff condition (8) holds. In the second case D1 does not restrict beliefs, neither type will deviate for any beliefs, and (8) holds. In the third case D1 does not restrict beliefs, either type will deviate for any beliefs, and (8) does not hold. So an equilibrium surviving D1 holds iff (8) holds.

Now consider the neither-gamble equilibrium. In this case the observer will combine his beliefs about which type deviated with the outcome x of the gamble, and the probability of x depends on θ . Estimated skill $y(x)$ is in the range $Y(x) = [\min_\theta \Pr[s|x, \theta], \max_\theta \Pr[s|x, \theta]]$.⁴⁰ A decision maker prefers to gamble iff

$$E[x|\theta] - z \geq v(\Pr[s]) - E[v(y(x))|\theta]. \tag{19}$$

Let $Y = Y(win) \times Y(lose)$ and let $Y_\theta \subset Y$ be the subset of Y such that (19) does not hold. First suppose $y(win) \geq y(lose)$. Then $E[v(y(x))|\theta]$ is increasing in θ since $\Pr[win|g] > \Pr[win|b]$. Since $E[x|\theta]$ is also increasing in θ , type g has more incentive to deviate. Now suppose $y(lose) > y(win)$, in which case $E[v(y(x))|\theta]$ is decreasing in θ . For any F , the largest gap $E[v(y(x))|b] - E[v(y(x))|g]$ is when, for pure evaluation skill, $Y(win) = \Pr[s|win, b]$ and $Y(lose) = \Pr[s|lose, b]$. However, by assumption (6), $E[x|\theta] - E[v(y(x))|\theta]$ is always increasing in θ even for this case, so type g always has more incentive to deviate. Therefore we again have three cases $Y_b \subsetneq Y_g \subset Y$, $Y_b = Y_g = \emptyset$, and $Y_b = Y_g = Y$, and by the equivalent arguments as for the both-gamble equilibrium, an equilibrium surviving D1 holds iff (9) holds. ■

Proof of Proposition 2: (i-a) In the separating equilibrium, the risk premium for type θ is $\pi_\theta = v(\Pr[s|b]) - \Pr[win|\theta]v(\Pr[s|win, g]) - \Pr[lose|\theta]v(\Pr[s|lose, g])$. Considering $\Pr[s|lose, g]$ from (3), as

⁴⁰Recall that $\Pr[s|x, \theta]$ need not be monotonic in θ for a gamble with evaluation skill. Also note that it is straightforward to show that if the observer places positive probability on each type having gambled, payoffs are still in $Y(x)$.

$\Pr[\text{win}|g]$ goes to 0 and $\Pr[\text{lose}|g]$ goes to 1, $\Pr[\text{lose}|s, g]$ and $\Pr[\text{lose}|u, g]$ must both go to 1 unless the ratio $\Pr[s|g]/\Pr[u|g]$ becomes arbitrarily large, which is not possible for fixed $\Pr[s|g] - \Pr[s|b]$, so $\Pr[s|\text{lose}, g]$ goes to $\Pr[s|g]$. Therefore π_g goes to $v(\Pr[s|b]) - v(\Pr[s|g])$ as $\Pr[\text{win}|g]$ goes to zero, as does π_b since $\Pr[\text{win}|b] < \Pr[\text{win}|g]$. For $v' > 0$ and fixed $\Pr[s|g] > \Pr[s|b]$, $v(\Pr[s|b]) < v(\Pr[s|g])$ so $\pi_b, \pi_g < 0$ for sufficiently small $\Pr[\text{win}|g] > \Pr[\text{win}|b]$. In the both-gamble equilibrium, the risk premium for type θ is $\pi_\theta = v(\Pr[s|b]) - \Pr[\text{win}|b]v(\Pr[s|\text{win}]) - \Pr[\text{lose}|b]v(\Pr[s|\text{lose}])$, which by the same arguments goes to $v(\Pr[s|b]) - v(\Pr[s]) < 0$ as $\Pr[\text{win}|g] > \Pr[\text{win}|b]$ goes to zero, so $\pi_b, \pi_g < 0$ for sufficiently small $\Pr[\text{win}|g] > \Pr[\text{win}|b]$. In the neither-gamble equilibrium, the risk premium for type θ is $\pi_\theta = v(\Pr[s]) - \Pr[\text{win}]v(\Pr[s|\text{win}, g]) - \Pr[\text{lose}]v(\Pr[s|\text{lose}, g])$, which by the same arguments goes to $v(\Pr[s]) - v(\Pr[s|g]) < 0$ as $\Pr[\text{win}|g] > \Pr[\text{win}|b]$ goes to zero, so $\pi_b, \pi_g < 0$ for sufficiently small $\Pr[\text{win}|g] > \Pr[\text{win}|b]$.

(i-b) For fixed $\Pr[\text{win}|\theta]$, as $\Pr[s|g] - \Pr[s|b]$ goes to 1, $\Pr[s|x, g]$ goes to 1 from (3). Therefore, in the separating equilibrium, as $\Pr[s|g] - \Pr[s|b]$ goes to 1, π_θ goes to $v(0) - v(1) < 0$. Similarly, in the both-gamble equilibrium, as $\Pr[s|g] - \Pr[s|b]$ goes to 1, π_θ goes to $v(0) - \Pr[\text{win}]v(\Pr[s|\text{win}]) - \Pr[\text{lose}]v(\Pr[s|\text{lose}]) < 0$. And in the neither-gamble equilibrium, as $\Pr[s|g]$ goes to 1, π_θ goes to $v(\Pr[s]) - v(1) < 0$.

(ii) For fixed $\Pr[\text{win}|\theta]$, as $\Pr[s|g] - \Pr[s|b]$ goes to 0, $\Pr[s|x, g]$ goes to $\Pr[s|x]$. Therefore in any of the equilibria, as $\Pr[s|g] - \Pr[s|b]$ goes to 0, π_θ goes to $v(\Pr[s]) - \Pr[\text{win}|\theta]v(\Pr[s|\text{win}]) - \Pr[\text{lose}|\theta]v(\Pr[s|\text{lose}])$, and the average premium π goes to $v(\Pr[s]) - \Pr[\text{win}]v(\Pr[s|\text{win}]) - \Pr[\text{lose}]v(\Pr[s|\text{lose}]) < 0$ where the inequality follows from $v'' < 0$. ■

Proof of Proposition 3: Starting with the separating equilibrium, from (4) and (5) the risk premium for type θ for gamble F is $\pi_{F, \theta} = v(\Pr_F[s|b]) - \Pr_F[\text{win}|\theta]v(\Pr_F[s|\text{win}, g]) - \Pr_F[\text{lose}|\theta]v(\Pr_F[s|\text{lose}, g])$ and for gamble G is $\pi_{G, \theta} = v(\Pr_G[s|b]) - \Pr_G[\text{win}|\theta]v(\Pr_G[s|\text{win}, g]) - \Pr_G[\text{lose}|\theta]v(\Pr_G[s|\text{lose}, g])$. Since $\Pr_G[s|b] = \Pr_F[s|b]$ by symmetry, $\pi_{F, \theta} < \pi_{G, \theta}$ if

$$\begin{aligned} & \Pr_F[\text{win}|\theta]v(\Pr_F[s|\text{win}, g]) + \Pr_F[\text{lose}|\theta]v(\Pr_F[s|\text{lose}, g]) \\ > & \Pr_G[\text{win}|\theta]v(\Pr_G[s|\text{win}, g]) + \Pr_G[\text{lose}|\theta]v(\Pr_G[s|\text{lose}, g]). \end{aligned} \quad (20)$$

Applying Lemma 1, let $l = \Pr_F[s|\text{lose}, g]$, $w = \Pr_F[s|\text{win}, g]$, $L = \Pr_G[s|\text{lose}, g]$, and $W = \Pr_G[s|\text{win}, g]$, and let, for type θ , $\Pr_P[l] = \Pr_F[\text{lose}|\theta]$, $\Pr_P[w] = \Pr_F[\text{win}|\theta]$, $\Pr_Q[L] = \Pr_G[\text{lose}|\theta]$, and $\Pr_Q[W] = \Pr_G[\text{win}|\theta]$. Noting that $\Pr_F[\text{win}|\theta] = \Pr_G[\text{lose}|\theta]$ by the symmetry assumption, the condition $\Pr_P[w] = \Pr_Q[L] < 1/2$ is satisfied. Therefore (20) holds for type θ if

$$\Pr_G[s|\text{lose}, g] < \Pr_F[s|\text{lose}, g] < \Pr_G[s|\text{win}, g] < \Pr_F[s|\text{win}, g], \quad (21)$$

$$\Pr_G[s|\text{win}, g] - \Pr_G[s|\text{lose}, g] \geq \Pr_F[s|\text{win}, g] - \Pr_F[s|\text{lose}, g], \quad (22)$$

and

$$\begin{aligned} & \Pr_F[\text{win}|\theta] \Pr_F[s|\text{win}, g] + \Pr_F[\text{lose}|\theta] \Pr_F[s|\text{lose}, g] \\ \geq & \Pr_G[\text{win}|\theta] \Pr_G[s|\text{win}, g] + \Pr_G[\text{lose}|\theta] \Pr_G[s|\text{lose}, g]. \end{aligned} \quad (23)$$

Regarding condition (21), symmetry implies $\Pr_F[q|\theta] = \Pr_G[q|\theta]$ so, from (3), $\Pr_G[s|lose, g] < \Pr_F[s|lose, g]$ is equivalent to

$$\begin{aligned} & \Pr_G[s|g] \Pr_G[u|g] \left(\frac{\Pr_G[lose|s, g] - \Pr_G[lose|u, g]}{\Pr_G[lose|g]} - \frac{\Pr_F[lose|s, g] - \Pr_F[lose|u, g]}{\Pr_F[lose|g]} \right) < 0 \\ \iff & \left(\Pr_G[win|s, g] - \Pr_G[win|u, g] \right) \left(\Pr_G[lose|g] - \Pr_F[lose|g] \right) < 0 \end{aligned} \quad (24)$$

where we have used the symmetry restriction $\Pr_G[win|s, g] - \Pr_G[win|u, g] = \Pr_F[win|s, g] - \Pr_F[win|u, g] > 0$ and the inequality follows from $\Pr_F[lose|g] > \Pr_G[lose|g]$. Similarly, $\Pr_G[s|win, g] < \Pr_F[s|win, g]$ is equivalent to

$$\begin{aligned} & \Pr_G[s|g] \Pr_G[u|g] \left(\frac{\Pr_G[win|s, g] - \Pr_G[win|u, g]}{\Pr_G[win|g]} - \frac{\Pr_F[win|s, g] - \Pr_F[win|u, g]}{\Pr_F[win|g]} \right) < 0 \\ \iff & \left(\Pr_G[win|s, g] - \Pr_G[win|u, g] \right) \left(\Pr_F[win|g] - \Pr_G[win|g] \right) < 0 \end{aligned} \quad (25)$$

which holds by the same arguments. Finally, $\Pr_F[s|lose, g] < \Pr_G[s|win, g]$ is equivalent to

$$\begin{aligned} & \Pr_G[s|g] \Pr_G[u|g] \left(\frac{\Pr_F[lose|s, g] - \Pr_F[lose|u, g]}{\Pr_F[lose|g]} - \frac{\Pr_G[win|s, g] - \Pr_G[win|u, g]}{\Pr_G[win|g]} \right) < 0 \\ \iff & - \left(\Pr_G[win|s, g] - \Pr_G[win|u, g] \right) \left(\Pr_G[win|g] + \Pr_F[lose|g] \right) < 0, \end{aligned} \quad (26)$$

which again holds.

Regarding condition (22), the LHS equals

$$\begin{aligned} & \Pr_G[s|g] \Pr_G[u|g] \left(\frac{\Pr_G[win|s, g] - \Pr_G[win|u, g]}{\Pr_G[win|g]} - \frac{\Pr_G[lose|s, g] - \Pr_G[lose|u, g]}{\Pr_G[lose|g]} \right) \\ = & \Pr_G[s|g] \Pr_G[u|g] \left(\frac{\Pr_G[win|s, g] - \Pr_G[win|u, g]}{\Pr_G[win|g]} - \frac{\Pr_G[win|u, g] - \Pr_G[win|s, g]}{\Pr_G[lose|g]} \right) \\ = & \Pr_G[s|g] \Pr_G[u|g] \frac{\left(\Pr_G[lose|g] (\Pr_G[win|s, g] - \Pr_G[win|u, g]) - \right.}{\Pr_G[win|g] \Pr_G[lose|g]} \\ & \left. \Pr_G[win|g] (\Pr_G[win|u, g] - \Pr_G[win|s, g]) \right) \\ = & \Pr_G[s|g] \Pr_G[u|g] \frac{\Pr_G[win|s, g] - \Pr_G[win|u, g]}{\Pr_G[win|g] \Pr_G[lose|g]} \end{aligned} \quad (27)$$

and similarly the RHS equals

$$\Pr_F[s|g] \Pr_F[u|g] \frac{\Pr_F[win|s, g] - \Pr_F[win|u, g]}{\Pr_F[win|g] \Pr_F[lose|g]}. \quad (28)$$

Therefore, since $\Pr_G[q|g] = \Pr_F[q|g]$ and $\Pr_F[win|s, \theta] - \Pr_F[win|u, \theta] = \Pr_G[win|s, \theta] - \Pr_G[win|u, \theta]$ by symmetry, (22) is equivalent to

$$\begin{aligned} & \frac{\Pr_G[win|s, g] - \Pr_G[win|u, g]}{\Pr_G[win|g] \Pr_G[lose|g]} \geq \frac{\Pr_F[win|s, g] - \Pr_F[win|u, g]}{\Pr_F[win|g] \Pr_F[lose|g]} \\ \iff & \Pr_F[win|g] \Pr_F[lose|g] \geq \Pr_G[win|g] \Pr_G[lose|g] \\ \iff & \Pr_F[win|g] (1 - \Pr_F[win|g]) \geq \Pr_G[win|g] (1 - \Pr_G[win|g]) \\ \iff & \left(\Pr_G[win|g] - \Pr_F[win|g] \right) \left(\Pr_F[win|g] + \Pr_G[win|g] - 1 \right) \geq 0 \end{aligned} \quad (29)$$

where the final inequality holds since $\Pr_G[\text{win}|g] > \Pr_F[\text{win}|g]$ for $\Pr_G[\text{win}] > \Pr_F[\text{win}]$ by the $\Pr_F[g] = \Pr_G[g]$ symmetry assumption and since $\Pr_F[\text{win}|g] > \Pr_F[\text{win}]$, $\Pr_G[\text{win}|g] > \Pr_G[\text{win}]$, and $\Pr_F[\text{win}] + \Pr_G[\text{win}] = 1$.

Regarding condition (23), note that it holds with equality for $\theta = g$ since $\Pr_F[g] = \Pr_G[g]$. Regarding $\theta = b$, note that the equality condition for $\theta = g$ implies

$$\begin{aligned}
& \Pr_F[\text{win}, g] \Pr_F[s|\text{win}, g] + \Pr_F[\text{lose}, g] \Pr_F[s|\text{lose}, g] \\
= & \Pr_G[\text{win}, g] \Pr_G[s|\text{win}, g] + \Pr_G[\text{lose}, g] \Pr_G[s|\text{lose}, g] \\
\iff & \left(\left(\Pr_F[\text{win}, g] - \Pr_F[\text{win}, b] \right) + \Pr_F[\text{win}, b] \right) \Pr_F[s|\text{win}, g] \\
& + \left(\left(\Pr_F[\text{lose}, g] - \Pr_F[\text{lose}, b] \right) + \Pr_F[\text{lose}, b] \right) \Pr_F[s|\text{lose}, g] \\
= & \left(\left(\Pr_G[\text{win}, g] - \Pr_G[\text{win}, b] \right) + \Pr_G[\text{win}, b] \right) \Pr_G[s|\text{win}, g] \\
& + \left(\left(\Pr_G[\text{lose}, g] - \Pr_G[\text{lose}, b] \right) + \Pr_G[\text{lose}, b] \right) \Pr_G[s|\text{lose}, g] \\
\iff & \left(\Pr_F[\text{win}, g] - \Pr_F[\text{win}, b] \right) \Pr_F[s|\text{win}, g] + \left(\Pr_F[\text{lose}, g] - \Pr_F[\text{lose}, b] \right) \Pr_F[s|\text{lose}, g] \\
& - \left(\Pr_G[\text{win}, g] - \Pr_G[\text{win}, b] \right) \Pr_G[s|\text{win}, g] - \left(\Pr_G[\text{lose}, g] - \Pr_G[\text{lose}, b] \right) \Pr_G[s|\text{lose}, g] \\
= & \Pr_G[\text{win}, b] \Pr_G[s|\text{win}, g] + \Pr_G[\text{lose}, b] \Pr_G[s|\text{lose}, g] \\
& - \Pr_F[\text{win}, b] \Pr_F[s|\text{win}, g] - \Pr_F[\text{lose}, b] \Pr_F[s|\text{lose}, g]. \tag{30}
\end{aligned}$$

Therefore the RHS of the last term is negative, implying (23) holds for $\theta = b$, if

$$\begin{aligned}
& \left(\Pr_F[\text{win}, g] - \Pr_F[\text{win}, b] \right) \Pr_F[s|\text{win}, g] + \left(\Pr_F[\text{lose}, g] - \Pr_F[\text{lose}, b] \right) \Pr_F[s|\text{lose}, g] \\
\leq & \left(\Pr_G[\text{win}, g] - \Pr_G[\text{win}, b] \right) \Pr_G[s|\text{win}, g] + \left(\Pr_G[\text{lose}, g] - \Pr_G[\text{lose}, b] \right) \Pr_G[s|\text{lose}, g] \\
\iff & \left(\Pr_F[\text{win}|g] - \Pr_F[\text{win}|b] \right) \Pr_F[s|\text{win}, g] - \left(\Pr_F[\text{win}|g] - \Pr_F[\text{win}|b] \right) \Pr_F[s|\text{lose}, g] \\
\leq & \left(\Pr_G[\text{win}|g] - \Pr_G[\text{win}|b] \right) \left(\Pr_G[s|\text{win}, g] - \Pr_G[s|\text{lose}, g] \right) \\
\iff & \left(\Pr_F[s|\text{win}, g] - \Pr_F[s|\text{lose}, g] \right) \leq \left(\Pr_G[s|\text{win}, g] - \Pr_G[s|\text{lose}, g] \right) \tag{31}
\end{aligned}$$

where we have used the symmetry restrictions that $\Pr_F[g] = \Pr_G[g] = 1/2$ and that $\Pr_F[\text{win}|g] - \Pr_F[\text{win}|b] = \Pr_G[\text{win}|g] - \Pr_G[\text{win}|b]$ and where the final inequality holds by (22). Therefore (23) holds for $\theta = b$ as well and $\pi_{F,\theta} < \pi_{G,\theta}$.

Now considering the both-gamble equilibrium, from (8) $\pi_{F,\theta} = \Pr_F[s|b] - (\Pr_F[\text{win}|\theta]v(\Pr_F[s|\text{win}]) + \Pr_F[\text{lose}|\theta]v(\Pr_F[s|\text{lose}]))$ and $\pi_{G,\theta} = \Pr_G[s|b] - (\Pr_G[\text{win}|\theta]v(\Pr_G[s|\text{win}]) + \Pr_G[\text{lose}|\theta]v(\Pr_G[s|\text{lose}]))$, so since $\Pr_F[s|b] = \Pr_G[s|b]$ by symmetry, $\pi_{F,\theta} < \pi_{G,\theta}$ if

$$\begin{aligned}
& \Pr_F[\text{win}|\theta]v(\Pr_F[s|\text{win}]) + \Pr_F[\text{lose}|\theta]v(\Pr_F[s|\text{lose}]) \\
> & \Pr_G[\text{win}|\theta]v(\Pr_G[s|\text{win}]) + \Pr_G[\text{lose}|\theta]v(\Pr_G[s|\text{lose}]). \tag{32}
\end{aligned}$$

If the gamble is pure evaluation skill then no information is revealed about type from the outcome, $\Pr_F[s|x] = \Pr_F[s]$ and $\Pr_G[s|x] = \Pr_G[s]$. Therefore since, $\Pr_F[s] = \Pr_G[s]$ by symmetry, the result

holds weakly, $\pi_{F,\theta} = \pi_{G,\theta}$. Now consider if there is some performance skill so $\Pr_F[\text{win}|s] - \Pr_F[\text{win}|u] = \Pr_G[\text{win}|s] - \Pr_G[\text{win}|u] > 0$. Applying Lemma 1 in the same manner as above, and noting that $\Pr_F[\text{win}] = \Pr_G[\text{lose}] < 1/2$, (32) holds if

$$\Pr_G[s|\text{lose}] < \Pr_F[s|\text{lose}] < \Pr_G[s|\text{win}] < \Pr_F[s|\text{win}], \quad (33)$$

$$\Pr_G[s|\text{win}] - \Pr_G[s|\text{lose}] \geq \Pr_F[s|\text{win}] - \Pr_F[s|\text{lose}], \quad (34)$$

and

$$\begin{aligned} & \Pr_F[\text{win}|\theta] \Pr_F[s|\text{win}] + \Pr_F[\text{lose}|\theta] \Pr_F[s|\text{lose}] \\ & \geq \Pr_G[\text{win}|\theta] \Pr_G[s|\text{win}] + \Pr_G[\text{lose}|\theta] \Pr_G[s|\text{lose}]. \end{aligned} \quad (35)$$

Regarding condition (33), and following the same step as (24)-(26), $\Pr_G[s|\text{lose}] < \Pr_F[s|\text{lose}]$ is equivalent to

$$\left(\Pr_G[\text{win}|s] - \Pr_G[\text{win}|u] \right) \left(\Pr_G[\text{lose}] - \Pr_F[\text{lose}] \right) < 0, \quad (36)$$

$\Pr_G[s|\text{win}] < \Pr_F[s|\text{win}]$ is equivalent to

$$\left(\Pr_G[\text{win}|s] - \Pr_G[\text{win}|u] \right) \left(\Pr_F[\text{win}] - \Pr_G[\text{win}] \right) < 0, \quad (37)$$

and $\Pr_F[s|\text{lose}] < \Pr_G[s|\text{win}]$ is equivalent to

$$- \left(\Pr_G[\text{win}|s] - \Pr_G[\text{win}|u] \right) \left(\Pr_G[\text{win}] + \Pr_F[\text{lose}] \right) < 0. \quad (38)$$

These conditions all holds since $\Pr_G[\text{win}|s] > \Pr_G[\text{win}|u]$.

Regarding condition (34), and following the equivalent steps as in (29),

$$\begin{aligned} & \Pr_G[s|\text{win}] - \Pr_G[s|\text{lose}] = \Pr_F[s|\text{win}] - \Pr_F[s|\text{lose}] \\ \iff & \left(\Pr_G[\text{win}] - \Pr_F[\text{win}] \right) \left(\Pr_F[\text{win}] + \Pr_G[\text{win}] - 1 \right) = 0 \end{aligned} \quad (39)$$

which holds by symmetry.

Now checking condition (35), following the equivalent steps as in (30) and (31), the condition holds for type θ if

$$\left(\Pr_F[s|\text{win}] - \Pr_F[s|\text{lose}] \right) \leq \left(\Pr_G[s|\text{win}] - \Pr_G[s|\text{lose}] \right) \quad (40)$$

which holds with equality by (39). So again $\pi_{F,\theta} < \pi_{G,\theta}$.

Finally, considering the neither-gamble equilibrium, $\pi_{F,\theta} < \pi_{G,\theta}$ holds under the same conditions as (20), which all hold as shown above. ■

Proof of Proposition 4: (i) Existence of a neither-gamble equilibrium for F implies $\pi_{F,g} > E_F[x|g] - z_F$ and $\pi_{F,b} > E_F[x|b] - z_F$. Since $\pi_{F,\theta} < \pi_{G,\theta}$ from Proposition 3 and $E_F[x|\theta] - z_F = E_G[x|\theta] - z_G$,

this implies $\pi_{G,g} > E_G[x|g] - z_G$ and $\pi_{G,b} > E_G[x|b] - z_G$, so a neither-gamble equilibrium exists for G . (ii) Existence of a separating equilibrium for F implies that $\pi_{F,b} > E_F[x|b] - z_F$. Since $\pi_{F,\theta} < \pi_{G,\theta}$ from Proposition 3 and $E_F[x|\theta] - z_F = E_G[x|\theta] - z_G$, this implies $\pi_{G,b} > E_G[x|b] - z_G$. Therefore, if $\pi_{G,g} > E_G[x|g] - z_G$ a neither-gamble equilibrium exists for G , while if $\pi_{G,g} \leq E_G[x|g] - z_G$ a separating equilibrium exists for G . (iii) Existence of a separating equilibrium for G implies $\pi_{G,g} \leq E_G[x|g] - z_G$. Since $\pi_{F,\theta} < \pi_{G,\theta}$ from Proposition 3 and $E_F[x|\theta] - z_F = E_G[x|\theta] - z_G$, this implies $\pi_{F,g} < E_F[x|g] - z_F$. Therefore, if $\pi_{F,b} > E_F[x|b] - z_F$ a separating equilibrium exists for F , while if $\pi_{F,b} \leq E_F[x|b] - z_F$ a both-gamble equilibrium exists for F . (iv) Existence of a both-gamble equilibrium for G implies $\pi_{G,g} \leq E_G[x|g] - z_G$ and $\pi_{G,b} \leq E_G[x|b] - z_G$. Since $\pi_{F,\theta} < \pi_{G,\theta}$ from Proposition 3 and $E_G[x|g] - z_G = E_F[x|g] - z_F$, this implies $\pi_{F,g} \leq E_F[x|g] - z_F$ and $\pi_{F,b} \leq E_F[x|b] - z_F$, so a both-gamble equilibrium exists for F . ■

Proof of Proposition 5: Since the outcome is observed even if the gamble is not taken, in the separating equilibrium the risk premium for type θ is $\pi_\theta = \Pr[\text{win}|\theta]v(\Pr[s|\text{win}, b]) + \Pr[\text{lose}|\theta]v(\Pr[s|\text{lose}, b]) - (\Pr[\text{win}|\theta]v(\Pr[s|\text{win}, g]) + \Pr[\text{lose}|\theta]v(\Pr[s|\text{lose}, g]))$, so the average risk premium $\pi = \pi_b \Pr[b] + \pi_g \Pr[g]$ is negative if

$$\begin{aligned} & \Pr[\text{win}]v(\Pr[s|\text{win}, g]) + \Pr[\text{lose}]v(\Pr[s|\text{lose}, g]) \\ > & \Pr[\text{win}]v(\Pr[s|\text{win}, b]) + \Pr[\text{lose}]v(\Pr[s|\text{lose}, b]). \end{aligned} \quad (41)$$

First suppose $\Pr[\text{win}] < 1/2$. Applying Lemma 1 in the same manner as in Proposition 3, let $l = \Pr[s|\text{lose}, g]$, $w = \Pr[s|\text{win}, g]$, $L = \Pr[s|\text{win}, b]$, and $W = \Pr[s|\text{lose}, b]$, and let $\Pr_P[l] = \Pr_Q[W] = \Pr[\text{lose}]$ and $\Pr_P[w] = \Pr_Q[L] = \Pr[\text{win}]$. Note that $\Pr_P[w] = \Pr_Q[L] < 1/2$. Therefore (41) holds if

$$\Pr[s|\text{win}, b] < \Pr[s|\text{lose}, g] < \Pr[s|\text{lose}, b] < \Pr[s|\text{win}, g], \quad (42)$$

$$\Pr[s|\text{lose}, b] - \Pr[s|\text{win}, b] \geq \Pr[s|\text{win}, g] - \Pr[s|\text{lose}, g], \quad (43)$$

and

$$\begin{aligned} & \Pr[\text{win}] \Pr[s|\text{win}, g] + \Pr[\text{lose}] \Pr[s|\text{lose}, g] \\ \geq & \Pr[\text{win}] \Pr[s|\text{win}, b] + \Pr[\text{lose}] \Pr[s|\text{lose}, b]. \end{aligned} \quad (44)$$

Regarding condition (42), $\Pr[s|\text{win}, b] < \Pr[s|\text{lose}, g]$ is equivalent to

$$\begin{aligned} & \left(\Pr[s|b] + \frac{\Pr[\text{win}|s, b] - \Pr[\text{win}|u, b]}{\Pr[\text{win}|b]} \Pr[s|b] \Pr[u|b] \right) \\ < & \left(\Pr[s|g] + \frac{\Pr[\text{lose}|s, g] - \Pr[\text{lose}|u, g]}{\Pr[\text{lose}|g]} \Pr[s|g] \Pr[u|g] \right) \\ \iff & \frac{\Pr[\text{win}|s, b] - \Pr[\text{win}|u, b]}{\Pr[\text{win}|b]} < \frac{\Pr[\text{lose}|s, g] - \Pr[\text{lose}|u, g]}{\Pr[\text{lose}|g]} \\ \iff & \frac{\Pr[\text{win}|s, b] - \Pr[\text{win}|u, b]}{\Pr[\text{win}|b]} < \frac{\Pr[\text{win}|u, g] - \Pr[\text{win}|s, g]}{\Pr[\text{lose}|g]} \\ \iff & \Pr[\text{lose}|g] > \Pr[\text{win}|b] \iff \Pr[\text{win}|b] + \Pr[\text{win}|g] < 1 \\ \iff & \Pr[\text{win}, b] + \Pr[\text{win}, g] < 1/2 \iff \Pr[\text{win}] < 1/2 \end{aligned} \quad (45)$$

where we have used the pure evaluation skill restriction $\Pr[q|\theta] = \Pr[q]$, the symmetry restrictions that $\Pr[\text{win}|s, g] - \Pr[\text{win}|u, g] = \Pr[\text{win}|u, b] - \Pr[\text{win}|s, b]$ and $\Pr[b] = \Pr[g] = 1/2$, and the evaluation skill condition that $\Pr[\text{win}|s, g] > \Pr[\text{win}|u, g]$. Similarly $\Pr[s|\text{lose}, b] < \Pr[s|\text{win}, g]$ is equivalent to

$$\begin{aligned}
& \left(\Pr[s|b] - \frac{\Pr[\text{win}|s, b] - \Pr[\text{win}|u, b]}{\Pr[\text{lose}|b]} \Pr[s|b] \Pr[u|b] \right) \\
& < \left(\Pr[s|g] + \frac{\Pr[\text{win}|s, g] - \Pr[\text{win}|u, g]}{\Pr[\text{win}|g]} \Pr[s|g] \Pr[u|g] \right) \\
& \iff \frac{\Pr[\text{win}|u, b] - \Pr[\text{win}|s, b]}{\Pr[\text{lose}|b]} < \frac{\Pr[\text{win}|s, g] - \Pr[\text{win}|u, g]}{\Pr[\text{win}|g]} \\
& \iff \Pr[\text{win}|g] < \Pr[\text{lose}|b], \tag{46}
\end{aligned}$$

which again holds for $\Pr[\text{win}] < 1/2$, and $\Pr[s|\text{lose}, g] < \Pr[s|\text{lose}, b]$ is equivalent to

$$\begin{aligned}
& \left(\Pr[s|g] + \frac{\Pr[\text{lose}|s, g] - \Pr[\text{lose}|u, g]}{\Pr[\text{lose}|g]} \Pr[s|g] \Pr[u|g] \right) \\
& < \left(\Pr[s|b] + \frac{\Pr[\text{lose}|s, b] - \Pr[\text{lose}|u, b]}{\Pr[\text{lose}|b]} \Pr[s|b] \Pr[u|b] \right) \\
& \iff -\frac{\Pr[\text{win}|s, g] - \Pr[\text{win}|u, g]}{\Pr[\text{lose}|g]} < \frac{\Pr[\text{win}|u, b] - \Pr[\text{win}|s, b]}{\Pr[\text{lose}|b]} \tag{47}
\end{aligned}$$

which holds since the LHS is negative and the RHS is positive.

Regarding condition (43), note that the LHS equals

$$\begin{aligned}
& \left(\Pr[s|b] + \frac{\Pr[\text{lose}|s, b] - \Pr[\text{lose}|u, b]}{\Pr[\text{lose}|b]} \Pr[s|b] \Pr[u|b] \right) - \\
& \left(\Pr[s|b] + \frac{\Pr[\text{win}|s, b] - \Pr[\text{win}|u, b]}{\Pr[\text{win}|b]} \Pr[s|b] \Pr[u|b] \right) \\
& = \Pr[s] \Pr[u] \frac{\Pr[\text{win}|u, b] - \Pr[\text{win}|s, b]}{\Pr[\text{win}|b] \Pr[\text{lose}|b]} \tag{48}
\end{aligned}$$

and the RHS equals

$$\Pr[s] \Pr[u] \frac{\Pr[\text{win}|s, g] - \Pr[\text{win}|u, g]}{\Pr[\text{win}|g] \Pr[\text{lose}|g]} \tag{49}$$

where in each case we have used the pure evaluation skill restriction $\Pr[q|\theta] = \Pr[q]$. So (43) holds if

$$\begin{aligned}
& \frac{\Pr[\text{win}|u, b] - \Pr[\text{win}|s, b]}{\Pr[\text{win}|b] \Pr[\text{lose}|b]} \geq \frac{\Pr[\text{win}|s, g] - \Pr[\text{win}|u, g]}{\Pr[\text{win}|g] \Pr[\text{lose}|g]} \\
& \iff \Pr[\text{win}|g] \Pr[\text{lose}|g] \geq \Pr[\text{win}|b] \Pr[\text{lose}|b] \\
& \iff \Pr[\text{win}|g](1 - \Pr[\text{win}|g]) \geq \Pr[\text{win}|b](1 - \Pr[\text{win}|b]) \\
& \iff \Pr[\text{win}|g] - \Pr[\text{win}|b] \geq \Pr[\text{win}|g]^2 - \Pr[\text{win}|b]^2 \\
& \iff \Pr[\text{win}|g] - \Pr[\text{win}|b] \geq (\Pr[\text{win}|g] - \Pr[\text{win}|b]) (\Pr[\text{win}|g] + \Pr[\text{win}|b]) \tag{50}
\end{aligned}$$

where we have used the symmetry restriction $\Pr[\text{win}|s, g] - \Pr[\text{win}|u, g] = \Pr[\text{win}|u, b] - \Pr[\text{win}|s, b]$ and where the final inequality holds for $\Pr[\text{win}] < 1/2$ since $\Pr[\text{win}|g] + \Pr[\text{win}|b] < 1$ as shown above.

Finally, regarding condition (44), note that for $\Pr[g] = \Pr[b]$ the pure evaluation skill restriction that $\Pr[s|g] = \Pr[s|b]$ implies $\Pr[s, g] = \Pr[s, b]$, which is equivalent to

$$\begin{aligned}
& \Pr[\text{win}, g] \Pr[s|\text{win}, g] + \Pr[\text{lose}, g] \Pr[s|\text{lose}, g] = \Pr[\text{win}, b] \Pr[s|\text{win}, b] + \Pr[\text{lose}, b] \Pr[s|\text{lose}, b] \\
\iff & \Pr[\text{win}|g] \Pr[s|\text{win}, g] + \Pr[\text{lose}|g] \Pr[s|\text{lose}, g] = \Pr[\text{win}|b] \Pr[s|\text{win}, b] + \Pr[\text{lose}|b] \Pr[s|\text{lose}, b] \\
\iff & (\Pr[\text{win}|g] - \Pr[\text{win}] + \Pr[\text{win}]) \Pr[s|\text{win}, g] + (\Pr[\text{lose}|g] - \Pr[\text{lose}] + \Pr[\text{lose}]) \Pr[s|\text{lose}, g] \\
= & (\Pr[\text{win}|b] - \Pr[\text{win}] + \Pr[\text{win}]) \Pr[s|\text{win}, b] + (\Pr[\text{lose}|b] - \Pr[\text{lose}] + \Pr[\text{lose}]) \Pr[s|\text{lose}, b] \\
\iff & (\Pr[\text{win}|g] - \Pr[\text{win}]) \Pr[s|\text{win}, g] + (\Pr[\text{lose}|g] - \Pr[\text{lose}]) \Pr[s|\text{lose}, g] \\
& - (\Pr[\text{win}|b] - \Pr[\text{win}]) \Pr[s|\text{win}, b] - (\Pr[\text{lose}|b] - \Pr[\text{lose}]) \Pr[s|\text{lose}, b] \\
= & \Pr[\text{win}] \Pr[s|\text{win}, b] + \Pr[\text{lose}] \Pr[s|\text{lose}, b] - \Pr[\text{win}] \Pr[s|\text{win}, g] - \Pr[\text{lose}] \Pr[s|\text{lose}, g]. \tag{51}
\end{aligned}$$

Therefore (44) holds if

$$\begin{aligned}
& (\Pr[\text{win}|b] - \Pr[\text{win}]) \Pr[s|\text{win}, b] + (\Pr[\text{lose}|b] - \Pr[\text{lose}]) \Pr[s|\text{lose}, b] \\
\geq & (\Pr[\text{win}|g] - \Pr[\text{win}]) \Pr[s|\text{win}, g] + (\Pr[\text{lose}|g] - \Pr[\text{lose}]) \Pr[s|\text{lose}, g] \\
\iff & (\Pr[\text{win}] - \Pr[\text{win}|b]) (\Pr[s|\text{lose}, b] - \Pr[s|\text{win}, b]) \\
\geq & (\Pr[\text{win}|g] - \Pr[\text{win}]) (\Pr[s|\text{win}, g] - \Pr[s|\text{lose}, g]) \\
\iff & \Pr[s|\text{lose}, b] - \Pr[s|\text{win}, b] \geq \Pr[s|\text{win}, g] - \Pr[s|\text{lose}, g] \tag{52}
\end{aligned}$$

where we have used the implication from $\Pr[g] = \Pr[b]$ that $\Pr[\text{win}] - \Pr[\text{win}|b] = \Pr[\text{win}|g] - \Pr[\text{win}]$. The final inequality is the same condition as (43), so $\pi < 0$ for $\Pr[\text{win}] < 1/2$. By the same arguments $\pi > 0$ for $\Pr[\text{win}] > 1/2$.

Now consider the both-gamble equilibrium. By the same arguments as in the proof of Proposition 1, the D1 refinement requires that the observer believe an unexpected deviation to not gambling is by type $\theta = b$. Therefore the risk premium π_θ is negative if

$$\begin{aligned}
& \Pr[\text{win}|\theta]v(\Pr[s|\text{win}]) + \Pr[\text{lose}|\theta]v(\Pr[s|\text{lose}]) \\
> & \Pr[\text{win}|\theta]v(\Pr[s|\text{win}, b]) + \Pr[\text{lose}|\theta]v(\Pr[s|\text{lose}, b]). \tag{53}
\end{aligned}$$

Since $\Pr[s|\text{win}] = \Pr[s|\text{lose}] = \Pr[s]$ in the both-gamble equilibrium of a pure evaluation skill gamble, this is equivalent to

$$v(\Pr[s]) > \Pr[\text{win}|\theta]v(\Pr[s|\text{win}, b]) + \Pr[\text{lose}|\theta]v(\Pr[s|\text{lose}, b]). \tag{54}$$

For $\theta = b$ this holds for $v'' < 0$ since $\Pr[\text{win}|b] \Pr[s|\text{win}, b] + \Pr[\text{lose}|b] \Pr[s|\text{lose}, b] = \Pr[s, \text{win}|b] + \Pr[s, \text{lose}|b] = \Pr[s|b] = \Pr[s]$. Using this result, for $v'' < 0$ this also holds for $\theta = g$ if

$$\begin{aligned}
& \Pr[\text{win}|g]v(\Pr[s|\text{win}, b]) + \Pr[\text{lose}|g]v(\Pr[s|\text{lose}, b]) \\
< & \Pr[\text{win}|b]v(\Pr[s|\text{win}, b]) + \Pr[\text{lose}|b]v(\Pr[s|\text{lose}, b]) \\
\iff & (\Pr[\text{win}|g] - \Pr[\text{win}|b])v(\Pr[s|\text{win}, b]) < (\Pr[\text{win}|g] - \Pr[\text{win}|b])v(\Pr[s|\text{lose}, b]) \\
\iff & v(\Pr[s|\text{win}, b]) < v(\Pr[s|\text{lose}, b]) \tag{55}
\end{aligned}$$

where we have used the fact that $\Pr[\text{win}|g] > \Pr[\text{win}|b]$. For $v' > 0$ this last inequality holds if

$$\begin{aligned}
& \Pr[s|\text{win}, b] < \Pr[s|\text{lose}, b] \\
\iff & \Pr[s] + \frac{\Pr[\text{win}|s, b] - \Pr[\text{win}|u, b]}{\Pr[\text{win}|b]} \Pr[s] \Pr[u] \\
& < \Pr[s] - \frac{\Pr[\text{win}|s, b] - \Pr[\text{win}|u, b]}{\Pr[\text{lose}|b]} \Pr[s] \Pr[u] \\
\iff & \frac{\Pr[\text{win}|s, b] - \Pr[\text{win}|u, b]}{\Pr[\text{win}|b]} < -\frac{\Pr[\text{win}|s, b] - \Pr[\text{win}|u, b]}{\Pr[\text{lose}|b]} \tag{56}
\end{aligned}$$

which holds by the pure evaluation skill assumption $\Pr[\text{win}|s, b] < \Pr[\text{win}|u, b]$. Therefore $\pi_\theta < 0$.

Now consider the neither-gamble equilibrium. In this case the D1 refinement requires that the observer believe an unexpected deviation to gambling is by type $\theta = g$ so the risk premium π_θ is positive if

$$\begin{aligned}
& \Pr[\text{win}|\theta]v(\Pr[s|\text{win}]) + \Pr[\text{lose}|\theta]v(\Pr[s|\text{lose}]) \\
& < \Pr[\text{win}|\theta]v(\Pr[s|\text{win}, g]) + \Pr[\text{lose}|\theta]v(\Pr[s|\text{lose}, g]). \tag{57}
\end{aligned}$$

By similar logic as the both-gamble case, for $v'' < 0$ this holds for type $\theta = g$, and for $v'' < 0$ and $v' > 0$ it holds for type $\theta = b$ if

$$\begin{aligned}
& \Pr[s|\text{win}, g] > \Pr[s|\text{lose}, g] \\
\iff & \Pr[s] + \frac{\Pr[\text{win}|s, b] - \Pr[\text{win}|u, b]}{\Pr[\text{win}|b]} \Pr[s] \Pr[u] > \Pr[s] - \frac{\Pr[\text{win}|s, b] - \Pr[\text{win}|u, b]}{\Pr[\text{lose}|b]} \Pr[s] \Pr[u] \\
\iff & \frac{\Pr[\text{win}|s, g] - \Pr[\text{win}|u, g]}{\Pr[\text{win}|b]} > -\frac{\Pr[\text{win}|s, g] - \Pr[\text{win}|u, g]}{\Pr[\text{lose}|b]} \tag{58}
\end{aligned}$$

which holds by the evaluation skill assumption $\Pr[\text{win}|s, g] > \Pr[\text{win}|u, b]$. Therefore $\pi_\theta > 0$. ■

Proof of Proposition 6: Consider a single gamble P and let $F = P(q, x|\theta = b)$ and $G = P(q, x|\theta = g)$. Then applying our results on risk premia for different types facing a single gamble, in the separating equilibrium, the risk premium is higher for the F gamble than the G gamble if, from (4) and (5),

$$\begin{aligned}
& v(\Pr[s|b]) - E[v(\Pr[s|x, g])|b] > v(\Pr[s|b]) - E[v(\Pr[s|x, g])|g] \\
\iff & E[v(\Pr[s|x, g])|b] < E[v(\Pr[s|x, g])|g] \\
\iff & \Pr[\text{win}|b]v(\Pr[s|\text{win}, g]) + \Pr[\text{lose}|b]v(\Pr[s|\text{lose}, g]) \\
& < \Pr[\text{win}|g]v(\Pr[s|\text{win}, g]) + \Pr[\text{lose}|g]v(\Pr[s|\text{lose}, g]) \\
\iff & (\Pr[\text{win}|b] - \Pr[\text{win}|g])v(\Pr[s|\text{win}, g]) \\
& < (\Pr[\text{lose}|g] - \Pr[\text{lose}|b])v(\Pr[s|\text{lose}, g]) \\
\iff & (\Pr[\text{win}|b] - \Pr[\text{win}|g])v(\Pr[s|\text{win}, g]) \\
& < (\Pr[\text{win}|b] - \Pr[\text{win}|b])v(\Pr[s|\text{lose}, g]) \\
\iff & (\Pr[\text{win}|b] - \Pr[\text{win}|g])(v(\Pr[s|\text{win}, g]) - v(\Pr[s|\text{lose}, g])) < 0 \tag{59}
\end{aligned}$$

where the final inequality holds for $v' > 0$ since $\Pr[\text{win}|g] > \Pr[\text{win}|b]$ and $\Pr[s|\text{win}, g] > \Pr[s|\text{lose}, g]$.

In the both-gamble equilibrium, the risk premium is weakly higher for the F gamble than the G gamble if, from (8),

$$\begin{aligned}
& v(\Pr[s|b]) - E[v(\Pr[s|x])|b] \geq v(\Pr[s|b]) - E[v(\Pr[s|x])|g] \\
\iff & E[v(\Pr[s|x])|g] \leq E[v(\Pr[s|x])|b] \\
\iff & \Pr[\text{win}|b]v(\Pr[s|\text{win}]) + \Pr[\text{lose}|b]v(\Pr[s|\text{lose}]) \\
& \leq \Pr[\text{win}|g]v(\Pr[s|\text{win}]) + \Pr[\text{lose}|g]v(\Pr[s|\text{lose}]) \\
\iff & (\Pr[\text{win}|b] - \Pr[\text{win}|g]) (v(\Pr[s|\text{win}]) - v(\Pr[s|\text{lose}])) \leq 0
\end{aligned} \tag{60}$$

where the final inequality holds for $v' > 0$ since $\Pr[\text{win}|g] > \Pr[\text{win}|b]$ and $\Pr[s|\text{win}] \geq \Pr[s|\text{lose}]$.

In the neither-gamble equilibrium, the risk premium is higher for the F gamble than the G gamble if, from (9),

$$v(\Pr[s]) - E[v(\Pr[s|x, g])|b] > v(\Pr[s]) - E[v(\Pr[s|x, g])|g] \tag{61}$$

which holds by the same arguments as for the separating equilibrium. ■

7 References

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