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Anxiety and Performance: An Endogenous Learning-by-Doing Model

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Abstract. In this paper we show that a standard economic model, the endogenous learning-by-doing model, captures several major themes from the anxiety literature in psychology. In our model, anxiety is a fully endogenous construct which can be separated naturally into its cognitive and physiological components. As such, our results are directly comparable with hypotheses and evidence from psychology. We show that anxiety can serve a motivating function, which suggests potential applications in the principal-agent literature.

> It doesn't take much technique to roll a 1.68 inch ball along a smooth, level surface into, or in the immediate vicinity of, a 4.5 inch hole. With no pressure on you, you can do it one-handed most of the time. But there is always pressure on the shorter putts... 90 percent of the rounds I play in major championships, I play with a bit of a shake.
>
> Jack Nicklaus, quoted in Patmore (1986, p. 75).

1. Introduction

Economic decision-making often involves considerable anxiety. Despite this, anxiety research in economics has been fairly limited, with recent pioneering contributions by Loewenstein (1987) and Caplin and Leahy (2001). In contrast, psychologists have amassed a substantial theoretical and empirical literature analyzing the effects of anxiety on "performance", where the latter may refer to reaction times, information processing, athletic performance, and diverse other activities. For economists interested in incorporating the effects of anxiety into formal economic models, the problem is that most of these psychological "theories" are really non-deductive theoretical "frameworks" which seem difficult to formalize in terms of explicit utility maximization.

The literature on endogenous learning-by-doing (ELBD) or "experimentation" includes Prescott (1972), Grossman, Kihlstrom, and Mirman (1977), Fusselman and Mirman (1993), and Mirman, Samuelson, and Urbano (1993), among others. In this paper, we use a simplified version of the ELBD model in Rauh and Seccia (2003) to investigate the relationship between anxiety and performance. Specifically, we consider the decision problem of an agent performing a task which extends over two periods, where performance depends on skill, effort, and a stochastic productivity shock. At the start of the first period, the agent is uncertain about her skill level, and we define *anxiety* as the agent's subjective evaluation of the residual uncertainty corresponding to the second period, valued in terms of expected utility. Hence, anxiety is an *anticipatory* emotion as in Caplin and Leahy. At the start of the second period, the agent observes her first period performance and updates her prior belief about her skill. So effort plays two distinct roles in this model: it increases performance directly, but it also impacts on anxiety by influencing the informativeness of the signal.

We show that the ELBD model captures several major themes from the theoretical and empirical psychology literature. E.g., in our model anxiety can serve a *motivating function:* increases in anxiety can motivate the agent to increase effort, thereby improving expected performance. This is also a cornerstone of the *processing efficiency theory* in cognitive psychology. Furthermore, under certain conditions effort and expected performance at the optimum are hill-shaped in anxiety-related parameters, which is known as the *inverted-U hypothesis* or *Yerkes-Dodson Law* in psychology.

Similarly, Caplin and Leahy consider a two-period model where anxiety is conceptualized as a psychological state depending on the first period outcome and the unresolved uncertainty corresponding to the second period. As in our model, the agent chooses the first period action with an eye towards the first period outcome, as well as anxiety about the second period. The main element in their *psychological expected utility theory* (PEUT) is an exogenous map ϕ which assigns a psychological state to the current outcome and expected second period lottery.

The PEUT offers several advantages over our model. It is substantively far more general and can accomodate the full range of anticipatory emotions, not just anxiety. It is also highly amenable for applications, which Caplin and Leahy demonstrate in the context of portfolio choice and the equity premium puzzle. However, anxiety remains a "black box" in their theory and the map ϕ has little structure apart from continuity, making comparisons with the psychology literature difficult. This is problematic, since many applications require some specification for ϕ, as illustrated by their asset pricing example, and the PEUT provides little guidance on this. In contrast, anxiety is a fully endogenous construct in the ELBD model since the information processing aspect is well-articulated. Furthermore, we show that our anxiety concept can be naturally separated into its cognitive and physiological components, making our results directly comparable with the theoretical and empirical psychology literature. One can therefore view the paper as an attempt to put some empirically testable structure on ϕ.

The rest of the paper is organized as follows. In section 2 we briefly survey the relevant psychology literature. In section 3 we develop the model and in sections 4 and 5 present our comparative statics results. Section 6 concludes.

2. The Psychology Literature on Anxiety and Performance

In this section, we briefly survey some of the hypotheses, theories, and evidence on the anxiety-performance relationship in the psychology literature. This survey is far from exhaustive, not only for reasons of space, but also because much of the psychology literature is essentially non-economic in nature.[1] This section is heavily indebted to the highly readable surveys by Woodman and Hardy (2001) and Zaichkowsky and Baltzell (2001).

We begin by defining terms, essentially conforming to the definitions in Woodman and Hardy.[2]

> Anxiety is generally accepted as being an unpleasant emotion... Researchers in mainstream psychology have suggested that anxiety might have at least two distinguishable components: a mental component normally termed *cognitive anxiety* or *worry*, and a physiological component normally termed *somatic anxiety* or *physiological arousal.*
>
> Woodman and Hardy, p. 290-291.

In this paper, *arousal* is distinguished by its significant (although not exclusive) physiological component, and often subconscious nature: "indications of autonomic arousal and unpleasant feeling states such as nervousness and tension" [Morris,

[1] E.g., the *theory of ironic processes of mental control* is based on the observation that if one consciously attempts to *not* think of a white bear, it becomes difficult not to. In general, this theory posits the existence of a monitoring process which identifies and highlights actions with negative consequences. Normally, this focuses the agent to avoid such actions, but when the agent is under pressure, this focus becomes excessive, causing them to be carried out. In other words, the agent says to herself "whatever you do, don't do x" and then she does x. Although interesting, this theory has little relevance for the model in this paper.

[2] All quotations in this paper are exact, except for terms in square brackets [...].

Davis, and Hutchings (1981, p. 541)]. High arousal may lead to an elevated heart rate, shaky hands, and other physical symptoms, some of which can be objectively measured, although psychologists also use self-report scales.[3] Arousal also contains a psychological component. E.g., Janelle, Singer, and Williams (1999) found that anxious subjects often focus on irrelevant internal and external information. Since this may have a significant subconscious aspect, we include such effects in our conceptualization of arousal. The second component of anxiety, *cognitive anxiety*, is a mental component synonymous with worry: "negative expectations and cognitive concerns about oneself, the situation at hand, and potential consequences" (Morris *et al.*, p. 541). It is distinguished from arousal by being a largely conscious mental state associated with information processing.

Inverted-U Hypothesis

The *inverted-U hypothesis*, often taught in introductory psychology courses, states that performance is hill-shaped in anxiety or one of its components. Hence, increases in anxiety improve performance when anxiety is low, but impair performance when anxiety is high. (A stronger version requires a symmetric or even quadratic relationship.) Although numerous studies support the inverted-U hypothesis, the empirical evidence is mixed. See Zaichkowsky and Baltzell (p. 326-328) for a survey. As a hypothesis about the relationship between performance and *anxiety*, it has been criticized for conceptualizing the latter as a one-dimensional construct, as opposed to consisting of different components. Like the IZOF and cusp catastrophe models discussed below, it does not *explain* the anxiety-performance relationship. Finally, it fails to take account of individual differences across agents.

[3] One may also distinguish between *objective* physiological arousal and its *perception* (the latter is sometimes called *somatic anxiety*), but we do not make that distinction here.

Individualized Zones of Optimal Functioning (IZOF)

The IZOF model was developed by Yuri Hanin from data collected on elite Soviet athletes. It is similar to the inverted-U hypothesis, except that the optimal level of anxiety is a *zone*, rather than a singleton; i.e., the relationship is plateau- rather than hill-shaped. The goal of the applied psychologist is to find this zone through repeated observation and help the agent attain it ("get in the zone"). The criticisms of the IZOF model are similar to those of the inverted-U hypothesis:

> More seriously, Hanin's IZOFs constitute what is essentially an individual difference "theory" without any individual difference variables... Consequently, despite some encouraging applied data, IZOF remains an intuitive applied tool that, as yet, has little theoretical value.
>
> Woodman and Hardy, p. 295.

Cusp Catastrophe Model

Fazey and Hardy (1988) model the anxiety-performance relationship as a standard cusp catastrophe. As in this paper, they conceptualize anxiety as consisting of two components: arousal and cognitive anxiety. When cognitive anxiety is low, the relationship between performance and arousal is an inverted-U. When cognitive anxiety is high, however, a discontinuity develops exhibiting hysteresis. At first, increases in arousal improve performance, but at some point a small increase in arousal leads to a disproportionately large drop in performance (catastrophe), after which performance is decreasing in arousal. A large reduction in arousal is necessary to re-attain the high, pre-catastrophe level of performance (hysteresis). Along the other axis, increases in cognitive anxiety improve performance when arousal is low, but impair it when arousal is high. Once again, the cusp catastrophe model is not a theory, but a mathematically precise hypothesis. Woodman and Hardy (p. 298) discuss several recent empirical studies supporting some of the predictions of the cusp catastrophe model, including the inverted-U relationship between performance and arousal when cognitive anxiety is low and hysteresis when it is high.

We now turn to theories which purport to *explain* the anxiety-performance relationship. Note that all of the "theories" surveyed below are not really theories in the deductive sense, but rather non-deductive theoretical "frameworks".

The Processing Efficiency Theory

The empirical psychology literature on the anxiety-performance relationship is quite rich, with numerous contrary findings. Eysenck and Calvo (1992) developed the *processing efficiency theory* to address these varied results and, in particular, to explain how anxiety could improve performance.

> One is concerned with the explanation of the relationship between anxiety and performance, taking into account not only the data regarding the negative effects of anxiety, but also trying to reconcile them with those findings indicating a lack of effect (or even a positive one).
>
> Eysenck and Calvo, p. 410.

After citing some evidence suggesting a minor role for arousal, Eysenck and Calvo focus exclusively on cognitive anxiety. As in most anxiety theories, the processing efficiency theory posits that anxiety induces worry, which creates "cognitive interference" by using up scarce attentional resources. This reduces *processing efficiency*, defined as the speed and ease with which information is processed, leading to a reduction in performance. The direct effect of anxiety is therefore always negative. Unlike most anxiety theories, however, the agent can take certain actions to mitigate its adverse effects. In particular, Eysenck and Calvo stress that anxiety also serves a motivational function, inducing the agent to consider measures to avoid the adverse consequences of poor performance. In particular, the agent may increase effort, provided the probability of success is perceived to be sufficiently high. Hence, although anxiety may be an unpleasant state which reduces processing efficiency, it can increase performance by inducing greater effort. If, however, the probability of success is perceived to be low, the agent may decrease effort, reducing performance still further. Woodman and Hardy (p. 308) survey supporting evidence, including support for the prediction that an increase in cognitive anxiety can lead to an increase in effort and performance.

One is immediately struck by the essentially *economic* nature of this theory, where the choice of effort involves an explicit cost-benefit calculation based on the probability of success. Indeed, the ELBD model in this paper will closely parallel the processing efficiency theory in several respects.

Reversal Theory

Another theory which might explain some of the divergent empirical findings is Apter's *reversal theory*. Reversal theory emphasizes the dynamic and inconsistent aspects of human nature, positing that agents can suddenly switch between "meta-motivational" states.

> In a telic state (i.e., a state in which individuals are goal-oriented and express purpose), individuals tend to be fairly serious, with a preference for low arousal. Conversely, in a paratelic state (i.e., a state in which individuals are oriented toward the sensations associated with their behavior), individuals tend to be fairly spontaneous, with a preference for high arousal... Reversal theory further posits that performers can rapidly change (reverse) from one metamotivational state to another.
>
> Woodman and Hardy, p. 300.

Presumably, increases in anxiety would reduce performance in a telic state, and improve it in a paratelic state. However,

> There does not appear to be an obvious theoretical reason for proposing that pleasant feelings about one's level of physiological arousal should lead to better performance... Although the notion of reversals is interesting, reversal theory has been limited by its lack of theory in relation to performance... As such, reversal theory does not offer a great deal in terms of explaining *how* and *why* anxiety might affect motor performance.
>
> *ibid*, p. 300.

Questions

Woodman and Hardy close their survey with the following questions (among others):

> • How do cognitive anxiety and physiological arousal (or somatic anxiety) exert their influence on performance (or performance-related variables)?
>
> • What role, if any, does effort play in delaying drops in performance or in curtailing the magnitude of such decrements?
>
> • Does effort moderate the effects of cognitive anxiety on performance?
>
> • What personality and individual variables influence IZOFs?
> *ibid*, p. 312.

To the above list, we would add the following:

• What is the relationship between performance and the components of anxiety? Inverted-U, IZOF, or cusp catastrophe? In particular, why does anxiety sometimes enhance performance, and other times inhibit it?

• What conditions encourage peak performance?

• Why and how do reversals occur, and what is the relationship with performance?

In this paper, we show that the ELBD model can be used to effectively address these issues.

3. The Model

> The agents who usually populate economic models have little doubt about "who they are": they know their own abilities and basic preferences... Psychology, by contrast, gives a central role to the process of learning about oneself and to individuals' struggle with their own identity...
> Bénabou and Tirole (2002)

We consider the decision problem of an agent engaged in a task which extends over two periods, $t = 1, 2$. Her performance in period t is given by

$$\pi_t = \theta e_t + \epsilon_t, \tag{1}$$

where θ is talent or skill, e_t is period t effort, and ϵ_t is an unobservable productivity shock. At the start of the first period, the agent is uncertain about the value of θ, either because the task itself is unfamiliar to some degree, or because ability can vary across repetitions of the task due to changes in environmental conditions and/or the agent's physical or mental state. This *self-doubt* will be the source of the agent's anxiety in our model. The parameter $R \equiv \theta_H - \theta_L$ indicates the level of *cognitive anxiety* because it measures the agent's "cognitive concerns about oneself" as discussed in the previous section. Although θ is uncertain, its value is fixed from the outset. *A priori*, the agent believes θ is either high θ_H or low θ_L with equal probability, where $0 < \theta_L < \theta_H$. Let $\bar{\theta} = (1/2)(\theta_H + \theta_L)$, *ex ante* expected skill. Throughout the paper, we assume $\bar{\theta} < 2/R$ for reasons which will become clear.

Recall that arousal is largely physiological and subconscious in nature, producing such effects as shaky hands as in the introductory quote. In addition to actual physical trembles, arousal may also produce mental "trembles" in decision-making arising from a subconscious focus on irrelevant internal and external information. Due to its subconscious nature, arousal enters the model through the stochastic productivity shock and we posit that an increase in arousal increases the volatility of such trembles. (Indeed, in section 5 below we assume ϵ_t has a quadratic density and arousal is parameterized by its curvature.) Arousal is therefore exogenous, which seems reasonable in the short-run, although over time the agent may learn to better control its effects. Furthermore, there may be a deeper underlying process jointly determining arousal and cognitive anxiety, making them correlated to some degree, although we neglect this by modeling them as separate exogenous parameters. In particular, nothing precludes a psychological state in which cognitive anxiety is high but arousal is low ("cool under pressure"). Note that the productivity shock may also incorporate *external* factors such as the effects of the weather or the actions of other agents. For the purposes of this section, we make the following assumptions.

Assumptions 1. *(i) ϵ_t is i.i.d. across periods and uncorrelated with θ. (ii) The* *cumulative distribution function F for ϵ_t is representable by a continuous probability density function f with support $[-1,1]$. (iii) f is positive on $(-1,1)$ and symmetric about 0. The latter implies ϵ_t has mean zero.*

In this section, we distinguish between two different types of f. *Type 1* densities correspond to low arousal. They are roughly hill-shaped and represent low volatility situations where with high probability ϵ_t takes values close to the mean, which is zero. *Type 2* densities correspond to high arousal. They are roughly U-shaped and represent high volatility situations where with high probability ϵ_t takes extreme values in the tails, close to -1 or 1.

Definition 1. *We say f is* **type 1** *(***type 2***) if it is increasing (decreasing) on $[-1,0)$.*[4]

At the start of the second period, the agent observes her first period performance and updates according to Bayes' rule. The posterior belief that $\theta = \theta_H$ is given by

$$\rho(e_1, \pi_1 \,|\, \theta_H, \theta_L) = \frac{f_H}{f_H + f_L}, \tag{2}$$

where $f_H \equiv f(\pi_1 - \theta_H e_1)$ and $f_L \equiv f(\pi_1 - \theta_L e_1)$ are the conditional densities. Hence, effort plays two distinct roles in this model: it affects performance directly as in (1), but it also affects the second period belief as in (2). The latter represents the ELBD aspect of the model.

Let

$$\begin{aligned} L_1 &\equiv \theta_L e_1 - 1 \qquad L_2 \equiv \theta_L e_1 + 1 \\ H_1 &\equiv \theta_H e_1 - 1 \qquad H_2 \equiv \theta_H e_1 + 1. \end{aligned} \tag{3}$$

An effort level satisfying $e_1 < 2/R$ is called *non-fully-revealing*, since information remains incomplete for signals $\pi_1 \in [H_1, L_2]$, where $H_1 < L_2$. Effort levels satisfying

[4] Throughout the paper, we use the terms "increasing" and "decreasing" in their strict senses.

$e_1 \geq 2/R$ are called *fully-revealing*, since the agent can fully infer θ for almost all signals (i.e., with probability 1). We call $2/R$ the *threshold level* of effort.[5]

The agent's utility in period t is given by

$$u_t = \pi_t - (1/2)e_t^2; \tag{4}$$

performance minus the quadratic disutility of effort. Although the agent suffers from anxiety, she is "rational" in the usual sense that she chooses $e_1(\theta_H, \theta_L)$ and $e_2(\pi_1 \,|\, \theta_H, \theta_L)$ to maximize the prior expectation of $u_1 + u_2$.

We first consider the second period problem. To make the model tractable, we assume second period effort is a binary choice variable. Specifically, the agent can only choose between high effort $e_2 = \theta_H$ or low effort $e_2 = \theta_L$.[6] Let $E(\theta \,|\, I_2)$ denote the expectation of θ given the second period information set I_2, which includes $\{e_1, \pi_1\}$. The expected payoff of choosing $e_2 = \theta_i$ is therefore $E(\theta \,|\, I_2)\theta_i - (1/2)\theta_i^2$, where $i = H, L$. Assuming indifference is resolved in favor of high effort, the agent chooses $e_2 = \theta_H$ iff

$$E(\theta \,|\, I_2) \geq \bar{\theta} \iff \rho \geq 1/2, \tag{5}$$

where we have used the fact that

$$\theta_H = \bar{\theta} + (R/2) \quad \text{and} \quad \theta_L = \bar{\theta} - (R/2). \tag{6}$$

Since

$$E(\theta \,|\, I_2) = \rho R + \bar{\theta} - (R/2), \tag{7}$$

the second period value function is

$$\begin{aligned} V_2(e_1, \pi_1 \,|\, \bar{\theta}, R) \;=\; & 1_{\rho \geq 1/2}\left(\rho R + \frac{\bar{\theta}}{2} - \frac{3R}{4}\right)\left(\bar{\theta} + \frac{R}{2}\right) + \\ & 1_{\rho < 1/2}\left(\rho R + \frac{\bar{\theta}}{2} - \frac{R}{4}\right)\left(\bar{\theta} - \frac{R}{2}\right), \end{aligned} \tag{8}$$

[5] The feasibility of full revelation is unimportant for our results. One could assume effort is constrained from above by some constant less than the threshold.

[6] In the working paper version, available from the authors upon request, we allowed second period effort to be a continuous choice variable. The resulting model is substantially more complicated than the one analyzed here, but simulations show that the results are qualitatively similar.

where $1_{\rho \geq 1/2}$ is the indicator function which equals one when $\rho \geq 1/2$ and zero otherwise, and $1_{\rho < 1/2}$ is defined similarly.

In the first period, the agent chooses $e_1 \geq 0$ to maximize

$$U(e_1 \,|\, \bar{\theta}, R) = \bar{\theta} e_1 - (1/2) e_1^2 + E(V_2 \,|\, I_1), \tag{9}$$

where

$$E(V_2 \,|\, I_1) = (1/2) \int_{H_1}^{H_2} V_2 \, f_H \, d\pi_1 + (1/2) \int_{L_1}^{L_2} V_2 \, f_L \, d\pi_1 \tag{10}$$

and I_1 is the first period information set. Proposition 1 below states that the precise form of $E(V_2 \,|\, I_1)$ depends on whether f is type 1 or 2. This is because type 1 and 2 densities represent different informational environments, as explained in the following lemma. When f is type 1, higher signals imply $\rho \geq 1/2$, but when f is type 2, this is not necessarily so.

Lemma. *If $0 < e_1 < 2/R$, then on the interval $[H_1, L_2]$: (i) if f is type 1,*

$$\rho \geq 1/2 \iff \pi_1 \geq \bar{\theta} e_1. \tag{11}$$

(ii) If f is type 2,

$$\rho \geq 1/2 \iff \pi_1 \leq \bar{\theta} e_1. \tag{12}$$

The proof of the following proposition involves a straightforward but lengthy evaluation of the integral in (10), using the lemma (see the appendix).

Proposition 1. *For $0 \leq e_1 \leq 2/R$, (i) if f is type 1,*

$$E(V_2 \,|\, I_1) = (1/2)\bar{\theta}^2 - (3/8)R^2 + (1/2)R^2 F(Re_1/2). \tag{13}$$

(ii) If f is type 2,

$$E(V_2 \,|\, I_1) = (1/2)\bar{\theta}^2 + (5/8)R^2 - (1/2)R^2 \left[F(Re_1/2) + F(1 - Re_1)\right]. \tag{14}$$

In the present context, we define the *value of information* by

$$E\left[V_2(e_1^*, \pi_1 \,|\, \bar{\theta}, R) \,\middle|\, I_1\right] - E\left[V_2(\bar{\theta}, \pi_1 \,|\, \bar{\theta}, R) \,\middle|\, I_1\right], \tag{15}$$

where e_1^* is optimal first period effort. To interpret this, note that $\bar{\theta}$ is the *myopic optimum*: the optimal solution when the agent is unconcerned about generating information for the second period.[7] Any deviation from $e_1 = \bar{\theta}$ will reduce expected first period utility. The expression in (15) is the expected second period benefit from choosing e_1^* rather than $\bar{\theta}$. Since first period effort is a purely informational variable in the second period, this expected benefit represents the agent's subjective evaluation of the benefit of improved information. The *cost of information* is the reduction in expected first period utility as a result of choosing e_1^* rather than $\bar{\theta}$. Since the second term in (15) is a constant, we could re-write the objective function in (9) as

$$\bar{\theta}e_1 - (1/2)e_1^2 + \text{the value of information.} \tag{16}$$

To motivate our anxiety construct, we recall that anxiety is generally regarded as a "negative emotion" and is therefore associated with the economic concept of utility. As Caplin and Leahy emphasize, anxiety is *anticipatory* in nature, and often connected with future uncertainties. In our model, the agent anticipates the start of the second period, when she may have to make a decision based on incomplete information. The subjective evaluation of the residual uncertainty is given by

$$E\left[V_2(2/R, \pi_1 \,|\, \bar{\theta}, R) \,\middle|\, I_1\right] - E\left[V_2(e_1, \pi_1 \,|\, \bar{\theta}, R) \,\middle|\, I_1\right]. \tag{17}$$

We therefore define *anxiety* $A(e_1 \,|\, R)$ by the expression in (17).

Remarks

(i) Although cognitive anxiety appears explicitly in (17), arousal is only implicitly represented by f. In particular, in this section we distinguish broadly between type 1 (low arousal) and type 2 (high arousal) densities. In section 5 below, arousal will appear in (17) as an explicit parameter. (ii) Note that anxiety is zero when effort

[7] Note that if $\bar{\theta} \geq 2/R$ then $e_1 = \bar{\theta}$ is trivially optimal, since the myopic optimum is also fully-revealing.

is fully-revealing. Evidently, the agent is not anxious about the possibility that θ could be low, only that there might be some residual uncertainty at the start of the second period. Hence, in our model anxiety is connected with residual uncertainty, not with the possibility of negative outcomes. Clearly, this is a limitation of our theory. (iii) Finally, our definition of anxiety is founded in standard expected utility theory. Although this may not be desirable in and of itself, it does mean that we can draw on an established set of techniques to analyze the model.

Since the first term in (17) is a constant, we can re-write (9) as

$$U(e_1 \,|\, \bar{\theta}, R) = \bar{\theta} e_1 - (1/2)e_1^2 - A(e_1 \,|\, R). \tag{18}$$

From now on, we take (18) to be the agent's objective function. Proposition 2 is immediate from proposition 1.

Proposition 2. *For $0 \le e_1 \le 2/R$: (i) if f is type 1,*

$$A(e_1 \,|\, R) = (R^2/2)\big[1 - F(Re_1/2)\big]. \tag{19}$$

(ii) If f is type 2,

$$A(e_1 \,|\, R) = (R^2/2)\big[F(Re_1/2) + F(1 - Re_1) - 1\big]. \tag{20}$$

4. Type 1 Comparative Statics with the MLRP

In this section, we derive some fairly general comparative statics results assuming f is type 1 and satisfies the usual *monotone likelihood ratio property* (MLRP) (type 2 densities generally do not satisfy the MLRP).

Assumptions 2. *(i) f is continuously differentiable on $(-1, 1)$. (ii) f'/f and $(1 - F)/f$ are decreasing on $(0, 1)$ (strict MLRP). (iii) $\lim_{\epsilon \to 1} -f'(\epsilon)/f(\epsilon) > 3$.*

Proposition 3 below deals with the relationship between anxiety, effort, and cognitive anxiety. The first part states that anxiety is decreasing in effort, which is a fundamental assumption of the processing efficiency theory. The second part states that anxiety is hill-shaped in cognitive anxiety: increases in R increase anxiety when R is low, but reduce it when R is high. This is similar to reversal theory, in which anxiety is sometimes interpreted negatively (telic state) and sometimes positively (paratelic state). Recall that reversal theory stresses the inconsistent aspect of human nature, and these differing interpretations of anxiety occur because of changes in the agent's metamotivational state. In contrast, in our theory changes in R are associated with changes in the informational content of the signal. The intuition is given in Figure 1 below.

Figure 1 Goes Here

Figure 1 depicts the conditional densities f_H and f_L for a given level of effort. The difference between the conditional means is $\theta_H e_1 - \theta_L e_1 = Re_1$. An increase in e_1 or R would further separate the conditional densities, allowing the signal to better discriminate between the two possible states. Hence, an increase in R has two effects: it increases the uncertainty associated with the second period, increasing anxiety, but it also makes it easier for the signal to differentiate between the two states, reducing anxiety. Looking back at (19), these two effects are now clearly evident: an increase in R holding $x \equiv Re_1/2$ inside F constant increases anxiety, but an increase in x holding R outside F fixed reduces it.

Proposition 3. *When effort is non-fully-revealing: (i) anxiety is decreasing in effort. (ii) If* $x = Re_1/2$*, there exists an* $\bar{x} > 0$ *such that* $\partial A/\partial R > 0$ *when* $0 \leq x < \bar{x}$*,* $\partial A/\partial R = 0$ *at* $x = \bar{x}$*, and* $\partial A/\partial R < 0$ *when* $\bar{x} < x < 1$*.*

We now turn to optimal effort. The next proposition shows that *anxiety matters*, in the sense that the agent generally does not choose the myopic optimum, but instead manipulates her effort level to cope with anxiety. Since U is strictly concave in effort, the agent's maximization problem has a unique solution e_1^* characterized by the first-order condition

$$\frac{\partial U}{\partial e_1} = \bar{\theta} - e_1 - \frac{\partial A}{\partial e_1} = 0. \tag{21}$$

when it is interior. Since $e_1 = \bar{\theta}$ strictly dominates $e_1 = 0$, the only other possible solution is full revelation, $e_1 = 2/R$. At an interior solution $\partial A/\partial e_1 < 0$ by proposition 3(i), so optimal effort exceeds the myopic optimum. This is the classical ELBD result: increasing effort above the myopic optimum reduces expected first period utility, but increases overall utility by improving second period information.

The second part of Proposition 4 states that, assuming an interior solution, the relationship between effort and cognitive anxiety is hill-shaped; *i.e., the inverted-U hypothesis obtains.* Furthermore, the possibility of increasing effort in response to an increase in cognitive anxiety is the distinguishing characteristic of the processing efficiency theory:

> Processing efficiency theory states that cognitive anxiety (a negative emotion) can have a negative cognitive effect... while serving *a positive motivational function* (increased effort).
>
> Woodman and Hardy, p. 306.

In our model, increases in cognitive anxiety at first increase anxiety, and the agent is therefore motivated to increase effort. Eventually, however, increases in cognitive anxiety are informative, thereby reducing anxiety, so the agent reduces effort to save on the information cost.

Proposition 4. *(i) If $\bar{\theta} < 2/R$ then optimal effort e_1^* exceeds the myopic optimum. (ii) Let $x^* \equiv Re_1^*/2$. There exists a constant $\bar{x} > 0$ such that the interior solution satisfies $\partial e_1^*/\partial R > 0$ when $0 < x^* < \bar{x}$, $\partial e_1^*/\partial R = 0$ at $x^* = \bar{x}$, and $\partial e_1^*/\partial R < 0$ when $\bar{x} < x^* < 1$. A similar statement applies to expected first period performance at the optimum, since the latter is optimal effort scaled by $\bar{\theta}$.*

5. Quadratic Comparative Statics

In the previous section, we focused on the type 1 case only, and did not address the relationship between effort and arousal. Recall that in this paper we formalize the effects of arousal in terms of subconscious trembles in performance, both physical and mental. We therefore measure the effects of arousal by the volatility of ϵ. In this section, we assume f belongs to a quadratic family of densities whose volatility can be parameterized, allowing us to study the effort-arousal relationship. This will facilitate comparisons with, for example, the cusp catastrophe model. In particular, we assume

$$f(\epsilon_t \,|\, a) = \begin{cases} \frac{3(1+a\epsilon_t^2)}{2(3+a)} & -1 \leq \epsilon_t \leq 1 \\ 0 & \text{otherwise,} \end{cases} \tag{22}$$

where $a \geq -1$. Note that $\partial^2 f/\partial\epsilon_t^2 = 3a/(3+a)$, which is increasing in a. When $a < 0$, f is type 1 and satisfies the MLRP. As a increases, starting from -1, f is transformed from a standard hill-shaped density into the uniform distribution at $a = 0$. When $a > 0$, f is type 2 and increases in a make it progressively more U-shaped. Hence, the parameter a determines the volatility of ϵ_t and measures the effects of arousal. The cumulative distribution function of ϵ_t is

$$F(\epsilon_t \,|\, a) = \begin{cases} 0 & -\infty < \epsilon_t < -1 \\ \frac{(1+\epsilon_t)(a\epsilon_t^2 - a\epsilon_t + 3 + a)}{2(3+a)} & -1 \leq \epsilon_t \leq 1 \\ 1 & \text{otherwise.} \end{cases} \tag{23}$$

Straightforward substitution into proposition 2 yields

Proposition 5. *(i) When* $-1 \leq a \leq 0$,

$$A(e_1 \,|\, R, a) = \frac{R^2}{32(3+a)}\big(24 + 8a - 12Re_1 - aR^3e_1^3\big). \tag{24}$$

(ii) When $a > 0$,

$$A(e_1 \,|\, R, a) = \frac{R^2}{32(3+a)}\Big[24 + 8a - 12(1+2a)Re_1 + 24aR^2e_1^2 - 7aR^3e_1^3\Big]. \tag{25}$$

Now anxiety depends explicitly on both cognitive anxiety and arousal.

Proposition 6 states that when arousal is low, anxiety is decreasing in effort as before. But when arousal is high, anxiety becomes cubic in effort as evident from (25), where anxiety is increasing in effort over some region. In contrast, the processing efficiency theory *assumes* anxiety is always decreasing in effort. The intuition for the non-monotonicity of anxiety is illustrated in Figure 2.

Figure 2 Goes Here

When $a > 0$, the conditional densities are U-shaped, so with high probability the signal will occur in the tails of one of the conditional densities. The outer tails at the extreme ends are already separated, so it can be more informative to reduce effort, pulling the conditional means closer together. This would further separate the inner tails, making signals between the conditional means more informative. Hence, a reduction in effort can improve information.

Proposition 6. *(anxiety and effort) When effort is non-fully-revealing: (i) anxiety is decreasing in effort when* $-1 \leq a < 7/2$ *and nonincreasing when* $a = 7/2$*. (ii) If* $a > 7/2$*, there exists* $0 < y_1 < y_2 < 2$ *such that anxiety is decreasing in effort when* $y \equiv Re_1 < y_1$*, increasing when* $y_1 < y < y_2$*, and decreasing when* $y_2 < y < 2$*.*

The statements of the next two propositions are somewhat involved, and are illustrated in Figure 3.

Figure 3 Goes Here

Panel A depicts the relationship between anxiety, cognitive anxiety, and arousal when $e_1 = 1$. Panels B and C depict cross-sections in arousal when R is 1.2 and 1.8, respectively. Recall that in the processing efficiency theory (as well as most anxiety theories in psychology), anxiety is assumed to create "cognitive interference" by using up scarce attentional resources. In our model, when $a < 0$ increases in arousal increase anxiety by interfering with Bayesian inference. This is evident from

Figure 1, where an increase in a would flatten the conditional densities, reducing the region of fully-revealing signals. In B, anxiety is hill-shaped in arousal, albeit with a sharp (non-differentiable) peak. Our model therefore continues to exhibit the telic/paratelic distinction of reversal theory, where increases in the components of anxiety are sometimes viewed positively, sometimes negatively. More formally,

Proposition 7. *(anxiety and arousal) When* $0 < e_1 < 2/R$*: (i) if* $-1 \leq a < 0$*, anxiety is increasing in arousal. (ii) If* $a > 0$*, anxiety is decreasing in arousal when* $0 < Re_1 \equiv y < 10/7$ *as in panel B and increasing when* $10/7 < y < 2$ *as in C.*

Panels D and E depict the relationship between anxiety and cognitive anxiety when $a = -1$ and 200, respectively. In D, the relationship is hill-shaped as in the previous section but in E there is an initial small hill-shape joined to a second, more prominent one.

Proposition 8. *(anxiety and cognitive anxiety) When effort is non-fully-revealing, there exists* $\bar{a} > 0$ *such that (i) anxiety is hill-shaped in cognitive anxiety for all* $-1 \leq a < \bar{a}$*. (ii) For* $a > \bar{a}$*, there exists* $0 < \bar{y} < 2$ *such that anxiety is hill-shaped in cognitive anxiety on* $[0, \bar{y}]$ *and on* $[\bar{y}, 2]$*.*[8]

We use the following passage to summarize our anxiety construct.

> [Anxiety] should be viewed as a multidimensional construct that contains a physiological arousal component and a cognitive interpretation-appraisal component. Furthermore the cognitive interpretation-appraisal component consists of a cognitive appraisal of one's physiological arousal..., negative affect associated with one's cognitive appraisal of increased arousal..., and positive affect associated with one's cognitive appraisal of increased arousal (paratelic state of excitement).
>
> Gould and Udry (1994, p. 479).

In the psychology literature, the terms "component" and "multidimensional construct" are used informally. In our theory, anxiety is literally a *function* of two parameters which measure the agent's cognitive anxiety (self-doubt) and arousal.

[8] More precisely, if $a > \bar{a}$ and $y \equiv Re_1$, there exists $0 < y_1 < \bar{y} < y_2 < 2$ such that $\partial A/\partial R > 0$ for $0 \leq y < y_1$ and $\bar{y} < y < y_2$ and $\partial A/\partial R < 0$ for $y_1 < y < \bar{y}$ and $y_2 < y < 2$.

This function contains an objective cognitive component (Bayesian updating), as well as a subjective appraisal component (utility), and therefore corresponds to the "cognitive interpretation-appraisal component" in the above passage. The agent is fully cognizant of the parameters R and a and the roles they play in information processing, and will choose her effort level accordingly. This resembles the "cognitive appraisal of one's physiological arousal" requirement. Finally, in our theory increases in arousal and cognitive anxiety can either increase or decrease anxiety, corresponding to negative and positive "affect", respectively.

We now turn to optimal effort. The proof of the following proposition is routine and therefore omitted. When $-1 \leq a \leq 0$ the objective function is strictly concave on $[0, 2/R]$, so optimal effort is the local maximizer given in (26) and (27), provided it is available on $[0, 2/R]$. When $a > 0$, however, U can assume a cubic form with a local maximum followed by a local minimum. The interior maximizer is given by (28) and (29) below, which must be compared with the threshold to find the global maximizer.

Proposition 9. *(optimal effort) (i) When $-1 \leq a \leq 0$ optimal effort is given by*

$$local^{a\leq 0}(\bar{\theta}, R, a) = \frac{1}{6aR^5}\left[32(3+a) - \sqrt{N(\bar{\theta}, R, a)}\right], \tag{26}$$

where

$$N(\bar{\theta}, R, a) = 1024(3+a)^2 - 48aR^5[8\bar{\theta}(3+a) + 3R^3], \tag{27}$$

provided $local^{a\leq 0} \leq 2/R$. Otherwise, the threshold is the unique optimum. (ii) If $a > 0$ then when it exists, the interior maximizer is

$$local^{a>0}(\bar{\theta}, R, a) = \frac{1}{42aR^5}\left[32(3+a) + 48aR^4 - 4\sqrt{P(\bar{\theta}, R, a)}\right], \tag{28}$$

where

$$P(\bar{\theta}, R, a) = 64(3+a)^2 + 24aR^4(3+a)(8 - 7\bar{\theta}R) + 9aR^8(2a-7). \tag{29}$$

In that case, the global maximizer is either the expression in (28) and (29) or the threshold.

We illustrate the above proposition in Figure 4.

Figure 4 Goes Here

Panel A depicts the 3-dimensional relationship between optimal effort, arousal, and cognitive anxiety when $\bar{\theta} = 1$. In B-E we plot cross-sections in arousal for successively higher values of R and in F-H cross-sections in R for successively higher values of a.[9] Recall that Figure 4 also depicts the qualitative behavior of expected first period performance at the optimum.

We first note the variety of anxiety-performance relationships in Figure 4. Given the rich empirical record discussed in section 2, we view this as a *strength* of our theory.

> The [processing efficiency] theory emerged from Eysenck's dissatisfaction with theorists' simplistic conceptualization of anxiety-performance relationships... most anxiety theories are based on anxiety-induced cognitive interference, such that anxiety uses up attentional resources. These theories typically predict that high-anxious individuals will perform less well than low-anxious individuals...
>
> Woodman and Hardy, p. 306.

Indeed, Figure 4 displays several major themes from the theoretical and empirical psychology literature. The monotonically decreasing relationship predicted in the last sentence of the previous quote is depicted in panel C. In D,F, and G we have roughly the inverted-U hypothesis, and in E the IZOF model, where the set of maximizers is a zone rather than a singleton. The explanation for this array of anxiety-performance relationships is that changes in the components of anxiety can have several different effects. The main effect of an increase in cognitive anxiety when the latter is low is to increase the uncertainty associated with the second period which can serve a motivating function as in panels F and G. However, increases in

[9] In Panels B-E R equals 0.4, 0.6, 1.3, and 1.34, respectively, and in F-H a equals -1, 1, and 5, respectively.

cognitive anxiety when the latter is high can also improve the informativeness of the signal, causing the agent to reduce effort to conserve on information costs, or because lower effort is more informative, as we saw in section 4. Similarly, increases in arousal can create “cognitive interference” which can also serve a motivating function as in D and E, but it can also improve the informativeness of the signal. The variety of anxiety-performance relationships therefore reflects the variety of relationships between anxiety and its components.

An advantage of our formal approach is that the ELBD model distinguishes between these competing hypotheses. Take, for example, the inverted-U and IZOF hypotheses: the latter obtains when *ex ante* expected skill and/or cognitive anxiety are relatively high. Indeed, the difference between D and E is that R is higher in the latter. The myopic optimum and the threshold are closer, making the information cost of full revelation smaller. To see the difference, we return to panels 3F and 3G which depict anxiety evaluated at optimal effort as a function of arousal for the same values of R as in 4D and 4E, respectively. In 3F, anxiety reaches its maximum for some level of arousal between 2 and 4. In 3G, the information cost of full revelation is lower so the agent switches discontinuously to the threshold, effectively “cutting off” the region of maximum anxiety in 3F. As arousal increases further, non-fully-revealing effort levels become more informative and eventually the agent discontinuously reduces effort below the threshold to conserve on the information cost.

We now compare our results with the cusp catastrophe model. Recall that in the latter the relationship between performance and arousal is an inverted-U when cognitive anxiety is low, but discontinuous when cognitive anxiety is high. Our results in 4D and 4E are similar, except that our discontinuity does not involve hysteresis. However, in the cusp catastrophe model performance is increasing in cognitive anxiety when arousal is low, and decreasing when arousal is high. In contrast, the inverted-U hypothesis generally obtains in our model, as in 4F and 4G. Overall, our model generates a wider variety of anxiety-performance relationships.

Some theories stress the importance of idiosyncratic factors for the anxiety-performance relationship. In particular, the crux of the IZOF hypothesis is that the zone of optimal functioning varies across individuals and the goal of the applied psychologist is to find it. However, the IZOF hypothesis itself cannot be used for that purpose, since it contains no idiosyncratic parameters. Another example is the butterfly catastrophe model in Hardy (1990), where self-confidence is a "bias factor" affecting the anxiety-performance relationship. Zaichkowsky and Baltzell (p. 331-333) discuss other potentially relevant factors, including:

> It is also assumed that optimal arousal is dependent on the skill level of the performer. This view comes largely from observations indicating that novice or low-skilled athletes perform poorly under pressure conditions when arousal is high, whereas experienced or highly skilled athletes tend to excel with pressure is highest (and arousal is modest).
> *ibid*, p. 332.

In Figure 5, we plot optimal effort as a function of $\bar{\theta}$ and a when $R = 1.34$.

Figure 5 Goes Here

We observe that when *ex ante* expected skill is low, effort and performance are largely decreasing in arousal, but when $\bar{\theta}$ is high, maximum effort and performance occur for medium values of arousal. This agrees with the above passage, provided we interpret "pressure" as high cognitive anxiety. Furthermore, the length of the IZOF is increasing in $\bar{\theta}$. Hence, high-skilled agents require lower levels of arousal to discontinuously jump to the threshold and can withstand higher levels of arousal before discontinuously cutting effort.

6. Conclusions

In this paper, we have shown that the ELBD model captures several major themes from the anxiety literature in psychology. As in the processing efficiency theory, anxiety can serve a motivating function and changes in the components of anxiety can have positive or negative “affect” as in reversal theory. The combination of these two forces can make anxiety hill-shaped in its components, leading to an inverted-U relationship with respect to effort and expected performance. Furthermore, the IZOF hypothesis obtains when *ex ante* expected skill and/or cognitive anxiety are relatively high. Finally, the relationship between *ex ante* expected skill and the IZOF is broadly consistent with hypotheses and anecdotal evidence in the sports psychology literature.

Since anxiety can be manipulated and is partly determined by the institutional environment, our theory should have applications in the principal-agent literature and specifically to organizational management and design. E.g., consider the annual performance review process common to many organizations. As Milgrom and Roberts (1992) point out,

> Most employees end up being evaluated highly, and so the rankings carry little useful information. The problem is that the individual managers bear the (personal) costs of assigning low ratings, and it is difficult to compensate them for these costs. In fact, to the extent that the system by which the managers are evaluated rewards employee development, the managers may in effect bear extra costs when they grade employees as poor performers.
> (p. 370).

Furthermore, if managers made fine distinctions among employees, the latter would have an incentive to attempt to politically influence the process, with all the attending opportunity costs (*influence costs*). But this raises the question: given that performance reviews are costly and generate little useful information, why are they used? According to our model, there may be an optimal level of cognitive anxiety and the mere prospect of an annual performance review may generate enough concern on the employee's part to provide a motivating function. Reducing the risk associated with the evaluations ensures that cognitive anxiety is not too high, which could adversely impact performance.

In the broader principal-agent literature, proposition 13 in Grossman and Hart (1983) [coined the *informativeness principle* by Milgrom and Roberts (1992)] states that the principal should never condition on uninformative noise because it only increases the agent's risk premium. However, this result relies crucially on the assumption of no wealth effects and the principal may prefer a stochastic incentive mechanism otherwise. According to our theory, an increase in *pure noise* (an increase in a) can serve a motivating function, so the principal may want to manipulate it. This clearly has implications for the *monitoring intensity principle* (*ibid*, p. 226), where monitoring is modeled as a costly activity which reduces noise.

Appendix

Proof of Lemma

Assume $0 < e_1 < 2/R$ and f is type 1. One can visualize the proof using Figure 1 above. By symmetry,

$$f(\bar{\theta}e_1 - \theta_L e_1) = f(Re_1/2) = f(-Re_1/2) = f(\bar{\theta}e_1 - \theta_H e_1), \qquad (A1)$$

so f_L and f_H cross at $\bar{\theta}e_1$. Since f_L is decreasing on $(\theta_L e_1, \bar{\theta}e_1]$ and f_H is increasing on $[\bar{\theta}e_1, \theta_H e_1)$, this crossing point is unique, and the rest follows. The type 2 case is similar. ∎

Proof of Proposition 1

We first check that the proposition holds when $e_1 = 0$ and $e_1 = 2/R$. In the former case, $\rho = 1/2$ for $-1 < \pi_1 < 1$ so $E(V_2 \mid I_1) = (1/2)\bar{\theta}^2 - (1/8)R^2$ from (8) and (10), which agrees with (13) and (14). If $e_1 = 2/R$ then the signal π_1 reveals the state with probability 1 at the start of the second period. With probability 1/2: $\theta = \theta_H$, $e_2 = \theta_H$, and $V_2 = (1/2)\theta_H^2$. Also with probability 1/2: $\theta = \theta_L$, $e_2 = \theta_L$, and $V_2 = (1/2)\theta_L^2$. Using (6),

$$E(V_2 \mid I_1) = (1/4)(\theta_H^2 + \theta_L^2) = (1/2)\bar{\theta}^2 + (1/8)R^2, \qquad (A2)$$

which agrees with (13) and (14).

Now assume $0 < e_1 < 2/R$. We first re-write (10) as

$$E(V_2 \,|\, I_1) = (1/2) \int_{L_1}^{H_2} V_2 \, (f_H + f_L) \, d\pi_1. \tag{A3}$$

We now work with the integrand in (A3).

$$\begin{aligned}
&V_2 \, (f_H + f_L) = \\
&1_{\rho \geq 1/2} \left(\bar{\theta} + \frac{R}{2} \right) \left\{ R f_H + \left(\frac{\bar{\theta}}{2} - \frac{3R}{4} \right) (f_H + f_L) \right\} + \\
&1_{\rho < 1/2} \left(\bar{\theta} - \frac{R}{2} \right) \left\{ R f_H + \left(\frac{\bar{\theta}}{2} - \frac{R}{4} \right) (f_H + f_L) \right\}.
\end{aligned} \tag{A4}$$

Simplifying,

$$\begin{aligned}
&V_2 \, (f_H + f_L) = (1/2) \Bigg[\\
&1_{\rho \geq 1/2} \left(\bar{\theta} + \frac{R}{2} \right)^2 f_H + 1_{\rho \geq 1/2} \left(\bar{\theta} + \frac{R}{2} \right) \left(\bar{\theta} - \frac{3R}{2} \right) f_L + \\
&1_{\rho < 1/2} \left(\bar{\theta} - \frac{R}{2} \right) \left(\bar{\theta} + \frac{3R}{2} \right) f_H + 1_{\rho < 1/2} \left(\bar{\theta} - \frac{R}{2} \right)^2 f_L \Bigg].
\end{aligned} \tag{A5}$$

Substituting into (A3),

$$\begin{aligned}
E(V_2 \,|\, I_1) = (1/4) \Bigg[&\left(\bar{\theta} + \frac{R}{2} \right)^2 \int_{H_1}^{H_2} 1_{\rho \geq 1/2} \, f_H \, d\pi_1 + \\
&\left(\bar{\theta} + \frac{R}{2} \right) \left(\bar{\theta} - \frac{3R}{2} \right) \int_{L_1}^{L_2} 1_{\rho \geq 1/2} \, f_L \, d\pi_1 + \\
&\left(\bar{\theta} - \frac{R}{2} \right) \left(\bar{\theta} + \frac{3R}{2} \right) \int_{H_1}^{H_2} 1_{\rho < 1/2} \, f_H \, d\pi_1 + \\
&\left(\bar{\theta} - \frac{R}{2} \right)^2 \int_{L_1}^{L_2} 1_{\rho < 1/2} \, f_L \, d\pi_1.
\end{aligned} \tag{A6}$$

Assume f is type 1. By the lemma, $\rho \geq 1/2$ on $[\bar{\theta} e_1, H_2)$ and $\rho < 1/2$ on $(L_1, \bar{\theta} e_1)$. Hence,

$$\int_{H_1}^{H_2} 1_{\rho \geq 1/2} \, f_H \, d\pi_1 = 1 - F(\bar{\theta} e_1 - \theta_H e_1) = 1 - F(-R e_1/2)$$

$$\int_{L_1}^{L_2} 1_{\rho\geq 1/2}\, f_L\, d\pi_1 = 1 - F(\bar{\theta}e_1 - \theta_L e_1) = 1 - F(Re_1/2)$$

$$\int_{H_1}^{H_2} 1_{\rho< 1/2}\, f_H\, d\pi_1 = F(\bar{\theta}e_1 - \theta_H e_1) = F(-Re_1/2)$$

$$\int_{L_1}^{L_2} 1_{\rho< 1/2}\, f_L\, d\pi_1 = F(\bar{\theta}e_1 - \theta_L e_1) = F(Re_1/2). \qquad (A7)$$

By symmetry,

$$F(Re_1/2) = 1 - F(-Re_1/2). \qquad (A8)$$

Substituting into (A6) and simplifying finishes the type 1 case.

Now assume f is type 2. By the lemma, $\rho \geq 1/2$ on $[H_1, \bar{\theta}e_1]$ and (L_2, H_2) and $\rho < 1/2$ on (L_1, H_1) and $(\bar{\theta}e_1, L_2]$. Hence,

$$\begin{aligned}
\int_{H_1}^{H_2} 1_{\rho\geq 1/2}\, f_H\, d\pi_1 &= F(\bar{\theta}e_1 - \theta_H e_1) + 1 - F(L_2 - \theta_H e_1) \\
&= F(-Re_1/2) + 1 - F(1 - Re_1) \\
\int_{L_1}^{L_2} 1_{\rho\geq 1/2}\, f_L\, d\pi_1 &= F(\bar{\theta}e_1 - \theta_L e_1) - F(H_1 - \theta_L e_1) \\
&= F(Re_1/2) - F(Re_1 - 1) \\
\int_{H_1}^{H_2} 1_{\rho< 1/2}\, f_H\, d\pi_1 &= F(L_2 - \theta_H e_1) - F(\bar{\theta}e_1 - \theta_H e_1) \\
&= F(1 - Re_1) - F(-Re_1/2) \\
\int_{L_1}^{L_2} 1_{\rho< 1/2}\, f_L\, d\pi_1 &= 1 - F(\bar{\theta}e_1 - \theta_L e_1) + F(H_1 - \theta_L e_1) \\
&= 1 - F(Re_1/2) + F(Re_1 - 1). \qquad (A9)
\end{aligned}$$

By symmetry,

$$F(1 - Re_1) = 1 - F(Re_1 - 1). \qquad (A10)$$

Using the other symmetry relationship (A8), substituting into (A6), and simplifying finishes the type 2 case. ▮

Proof of Proposition 3

(i) is clear by inspection. To prove (ii), we calculate

$$\partial A/\partial R = -(R^2/2) f(Re_1/2)(e_1/2) + R[1 - F(Re_1/2)]. \qquad (A11)$$

This can be re-written as

$$(1/R)(\partial A/\partial R) = -(1/2)f(x)x + [1 - F(x)], \tag{A12}$$

where $x \equiv Re_1/2$ and $0 \leq x < 1$. Hence,

$$\text{sign}\ \frac{\partial A}{\partial R} = \text{sign}\left[\frac{1-F(x)}{f(x)} - \frac{x}{2}\right]. \tag{A13}$$

Since f is type 1, $f(0) > 0$. Hence,

$$\frac{1-F(0)}{f(0)} = \frac{1}{2f(0)} > 0. \tag{A14}$$

Furthermore,

$$\lim_{x\to 1} \frac{1-F(x)}{f(x)} < 1/3. \tag{A15}$$

This is clear if $f(1) > 0$ and follows by L'Hospital's Rule otherwise. Since $(1-F)/f$ is continuous and decreasing, the result follows. ▮

Proof of Proposition 4

We have already proved (i). Differentiating the first-order condition in (21),

$$-\frac{\partial e_1^*}{\partial R} + \frac{R^3}{8}f'(Re_1^*/2)\left(R\frac{\partial e_1^*}{\partial R} + e_1^*\right) + \frac{3R^2}{4}f(Re_1^*/2) \equiv 0. \tag{A16}$$

Solving,

$$\frac{\partial e_1^*}{\partial R} = \frac{R^3 e_1^* f'(Re_1^*/2) + 6R^2 f(Re_1^*/2)}{8 - R^4 f'(Re_1^*/2)}. \tag{A17}$$

Since $Re_1^*/2 > 0$ (the myopic optimum strictly dominates zero effort) and f is type 1, the denominator is positive. Hence, the sign equals the sign of the numerator and we therefore consider

$$\text{sign}\left[\frac{3}{x} + \frac{f'(x)}{f(x)}\right], \tag{A18}$$

where $0 < x < 1$. Since f is type 1, $f'(0)/f(0) = 0$ and the expression in (A18) blows up as $x \to 0$. By assumption $\lim_{x\to 1} f'(x)/f(x) < -3$, so (A18) becomes negative as $x \to 1$. Since the expression in (A18) is continuous and decreasing, the result follows. ∎

Proof of Proposition 6

The case $-1 \le a < 0$ has already been considered. When $a = 0$ (uniform distribution),

$$A(e_1 \mid R) = (R^2/8)(2 - Re_1), \tag{A19}$$

and the result is clear. Turning to the case $a > 0$,

$$\frac{\partial A}{\partial e_1} = \frac{3R^3}{32(3+a)}\left[-4(1+2a) + 16aRe_1 - 7aR^2e_1^2\right]. \tag{A20}$$

The sign of this depends on the behavior of

$$-7ay^2 + 16ay - 4(1+2a), \tag{A21}$$

where $y = Re_1$. The maximizer for (A21) is $y = 8/7$, and its maximum is nonpositive when $a \le 7/2$. When $a > 7/2$, the roots of (A21) are

$$y_1 = \frac{8 - 2\sqrt{2-(7/a)}}{7} > 0 \quad \text{and} \quad y_2 = \frac{8 + 2\sqrt{2-(7/a)}}{7} < 2. \tag{A22}$$

Since the expression in (A21) is quadratic, the result follows. ∎

Proof of Proposition 7

When $-1 \le a < 0$,

$$\frac{\partial A}{\partial a} = \frac{3R^3e_1(4 - R^2e_1^2)}{32(3+a)^2}, \tag{A23}$$

whose sign depends on $4 - R^2e_1^2$ which is positive when effort is non-fully-revealing. When $a > 0$,

$$\frac{\partial A}{\partial a} = \frac{3R^3e_1}{32(3+a)^2}(-20 + 24Re_1 - 7R^2e_1^2). \tag{A24}$$

The sign of (A24) depends on $-20 + 24y - 7y^2$, which is negative for $0 \le y < 10/7$ and positive for $10/7 < y < 2$. ∎

Proof of Proposition 8

Note that assumptions 2(iii) only holds on a subinterval of $[-1, 0]$ so we must prove the result *ab ovo*. When $-1 \leq a < 0$,

$$\frac{\partial A}{\partial R} = \frac{R}{32(3+a)}(48 + 16a - 36Re_1 - 5aR^3e_1^3), \tag{A25}$$

so we investigate the behavior of

$$p_1(y) = 48 - 36y + a(16 - 5y^3), \tag{A26}$$

where $y \equiv Re_1$ and $0 < y < 2$. We have $p_1(0) = 16a + 48 > 0$, $p_1(2) = -24(1+a) \leq 0$, and p_1 is decreasing from $y = 0$ to the critical point $y = \sqrt{-12/5a}$, after which it is increasing. When $-1 \leq a < -3/5$, $\sqrt{-12/5a} < 2$ and $p_1(2) \leq 0$. When $-3/5 \leq a < 0$, $\sqrt{-12/5a} \geq 2$, and the result follows. The case $a = 0$ is easily analyzed. When $a > 0$,

$$\frac{\partial A}{\partial R} = \frac{R}{32(3+a)}\left[48 - 36e_1R + a(16 - 72e_1R + 96e_1^2R^2 - 35e_1^3R^3\right], \tag{A27}$$

so we analyze

$$p_2(y) = 48 - 36y + a(16 - 72y + 96y^2 - 35y^3). \tag{A28}$$

Once again, we have $p_2(0) = 48 + 16a > 0$ and $p_2(2) = -24(1 + a) < 0$. Now, if p_2 has one real root, then A will be hill-shaped in R. If it has three between 0 and 2, it will consist of two successive hills joined together. (If there are two, the transition point is an inflection point.) Since p_2 has the usual sideways-S shape, it will have three real roots iff there exists a negative local minimum, so that two of the roots are on either side of the local minimizer. The local minimizer is

$$\frac{32 - 2\sqrt{46 - (105/a)}}{35}. \tag{A29}$$

Substituting this into p_2, after a fair amount of algebra we get

$$\begin{aligned} p_3(a) = {} & (528/35) + (48/35)\sqrt{46 - (105/a)} + (4496/1225)a - \\ & (736/1225)a\sqrt{46 - (105/a)}. \end{aligned} \tag{A30}$$

Letting $z^2 = 46 - (105/a)$, the above is equivalent to

$$p_4(z) = (37776/35) - (528/35)z^2 - (48/35)z^3. \tag{A31}$$

Clearly, p_4 has a unique positive zero $\bar{z} \approx 6.67$. Furthermore, $p_4 > 0$ for all $0 \leq z < \bar{z}$ and $p_4 < 0$ for all $\bar{z} < z$. Hence, there exists a unique $\bar{a} > 0$ such that $\bar{z}^2 = 46 - (105/\bar{a})$, and which serves the same role for p_3. $\blacksquare$

References

BÉNABOU, R. AND J. TIROLE, "Self-Knowledge and Self-Regulation: An Economic Approach," in I. Brocas and J. Carrillo, eds., *The Psychology of Economic Decisions: Volume One: Rationality and Well-Being* (New York: Oxford University Press, 2002), 137-167.

CAPLIN, A.S. AND J. LEAHY, "Psychological Expected Utility Theory and Anticipatory Feelings," *Quarterly Journal of Economics* 116 (2001), 55-80.

EYSENCK, M.W. AND M.G. CALVO, "Anxiety and Performance: the Processing Efficiency Theory," *Cognition and Emotion* 6 (1992), 409-434.

FAZEY, J.A. AND L. HARDY, *The Inverted-U Hypothesis: a Catastrophe for Sport Psychology* (British Association of Sport Sciences Monograph No. 1) (Leeds, England: National Coaching Foundation, 1988).

FUSSELMAN, J.M. AND L.J. MIRMAN, "Experimental Consumption for a General Class of Disturbance Densities," in R. Becker et al., eds., *General Equilibrium, Growth, and Trade. The Legacy of Lionel McKenzie* (Economic theory, Econometrics, and Mathematical Economics Series Vol. 2) (London: Academic Press, 1993), 367-392.

GOULD, D. AND E. UDRY, "Psychological Skills for Enhancing Performance: Arousal Regulation Strategies," *Medicine and Science in Sports and Exercise* 26 (1994), 478-485.

GROSSMAN, S.J. AND O.D. HART, "An Analysis of the Principal-Agent Problem," *Econometrica* 51 (1983), 7-46.

GROSSMAN, S.J., R.E. KIHLSTROM, AND L.J. MIRMAN, "A Bayesian Approach to the Production of Information and Learning By Doing," *Review of Economic Studies* 44 (1977), 533-547.

HARDY, L., "A Catastrophe Model of Anxiety and Performance," in J.G. Jones and L. Hardy, eds., *Stress and Performance in Sport* (Chichester, England: Wiley, 1990), 81-106.

JANELLE, C., R. SINGER, AND M. WILLIAMS, "External Distraction and Attentional Narrowing: Visual Search Evidence," *Journal of Sport and Exercise Psychology* 21 (1999), 70-91.

LOEWENSTEIN, G., "Anticipation and the Valuation of Delayed Consumption," *Economic Journal* 97 (1987), 666-684.

MILGROM, P. AND J. ROBERTS, *Economics, Organization, and Management* (Englewood Cliffs, New Jersey: Prentice-Hall, 1992).

MIRMAN, L.J., L. SAMUELSON, AND A. URBANO, "Monopoly Experimentation," *International Economic Review* 34 (1993), 549-563.

MORRIS, L., D. DAVIS, AND C. HUTCHINGS, "Cognitive and Emotional Components of Anxiety: Literature Review and Revised Worry-Emotional Scale," *Journal of Educational Psychology* 75 (1981), 541-555.

Patmore, A., *Sportsmen Under Stress* (London: Stanley Paul, 1986).

Prescott, E., "The Multi-Period Control Problem Under Uncertainty," *Econometrica* 40 (1972), 1043-1058.

Rauh, M.T. and G. Seccia, "Experimentation, Full Revelation, and the Monotone Likelihood Ratio Property," *Journal of Economic Behavior and Organization* (forthcoming), (2004).

Woodman, T. and L. Hardy, "Stress and Anxiety," in R.N. Singer, H.A. Hausenblas, and C.M. Janelle, eds., *Handbook of Sport Psychology*, 2nd edition (New York: Wiley, 2001), 290-318.

Zaichkowsky, L.D. and A. Baltzell, "Arousal and Performance," in R.N. Singer, H.A. Hausenblas, and C.M. Janelle, eds., *Handbook of Sport Psychology*, 2nd edition (New York: Wiley, 2001), 319-339.

Figure 1. Separating the conditional densities in the type 1 case.

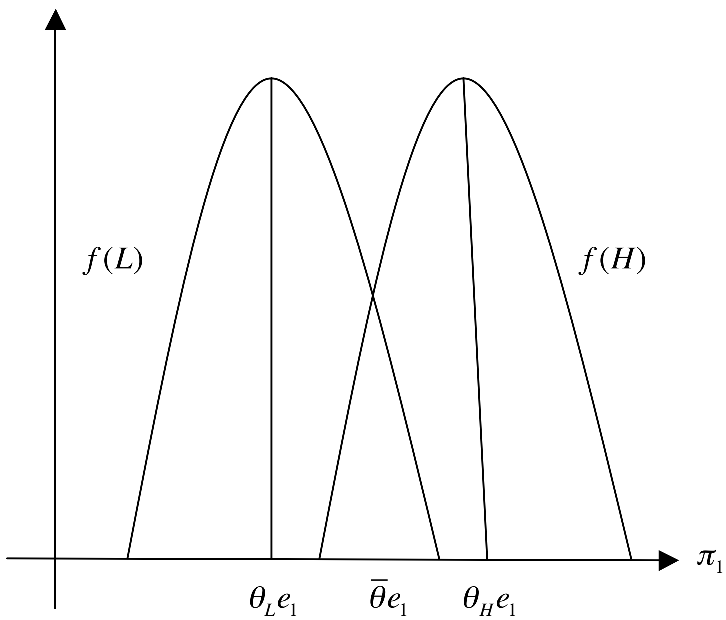


Figure 2. Separating the inner tails in the type 2 case.

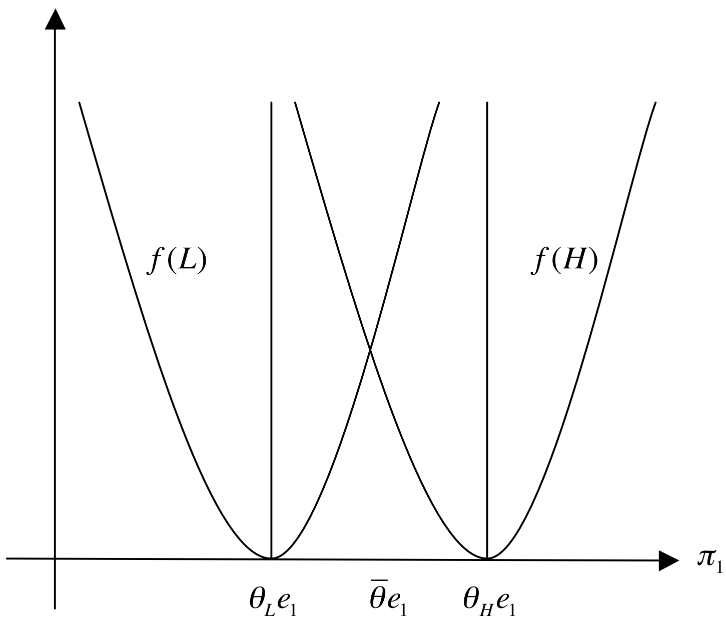


Figure 3

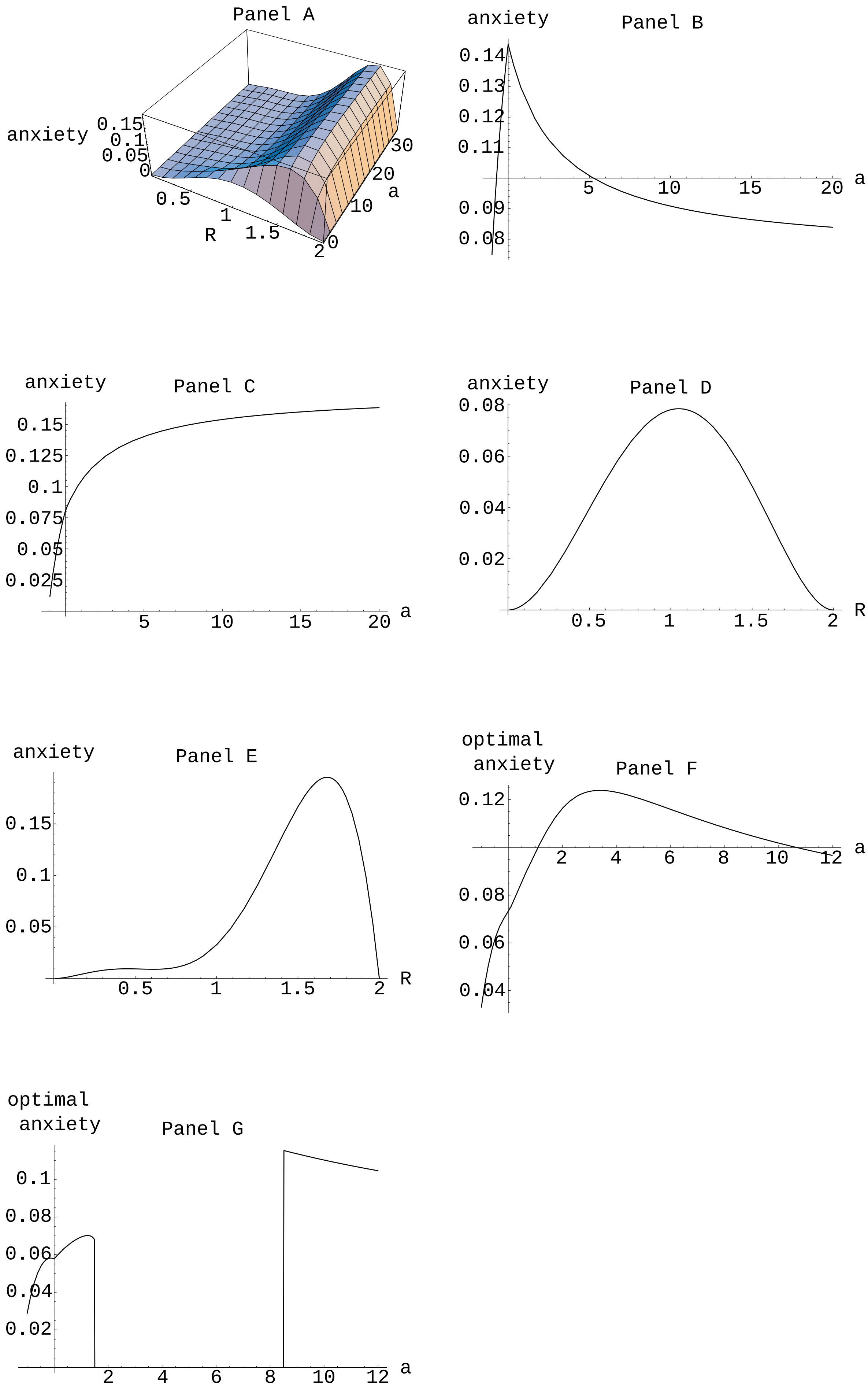
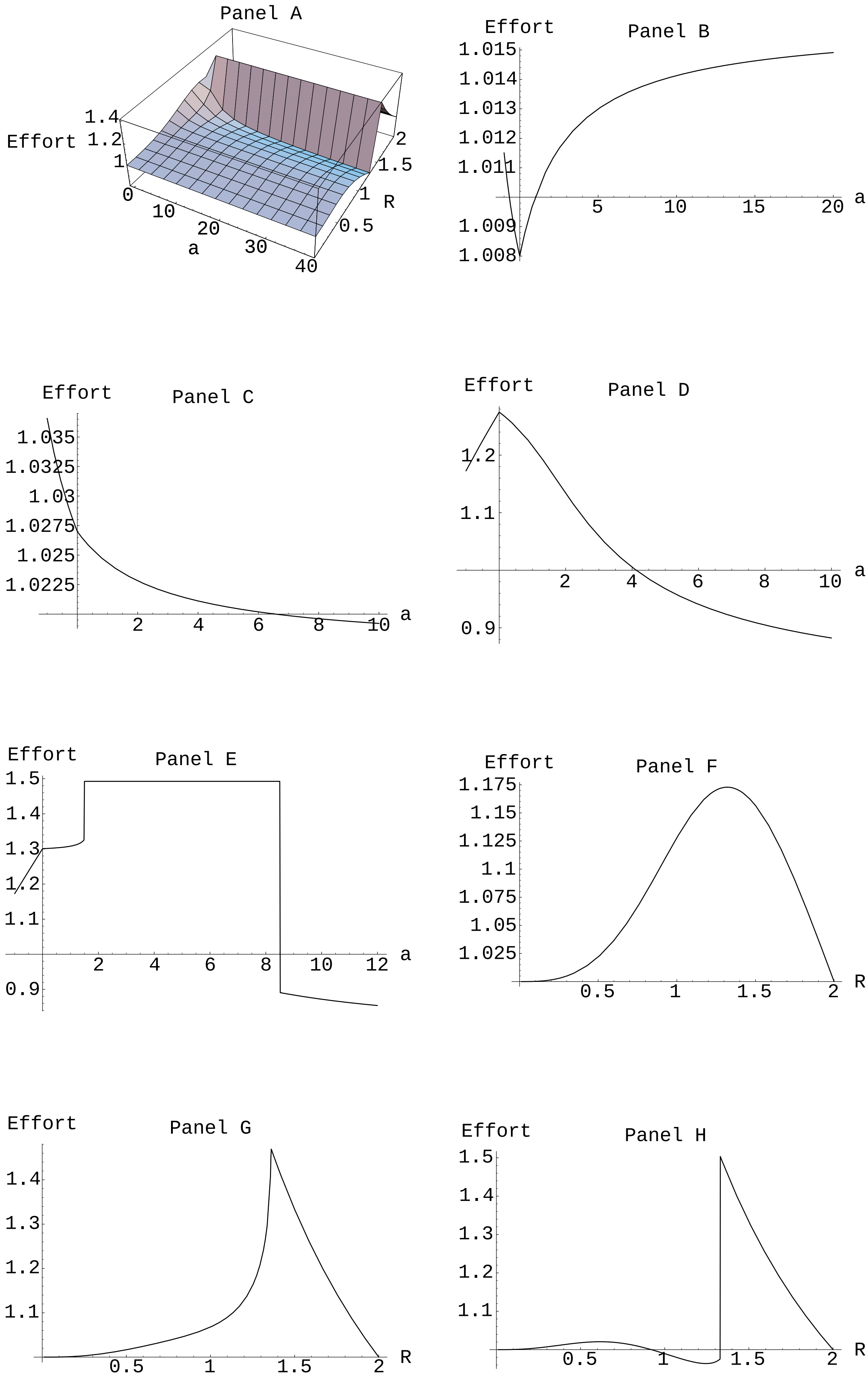
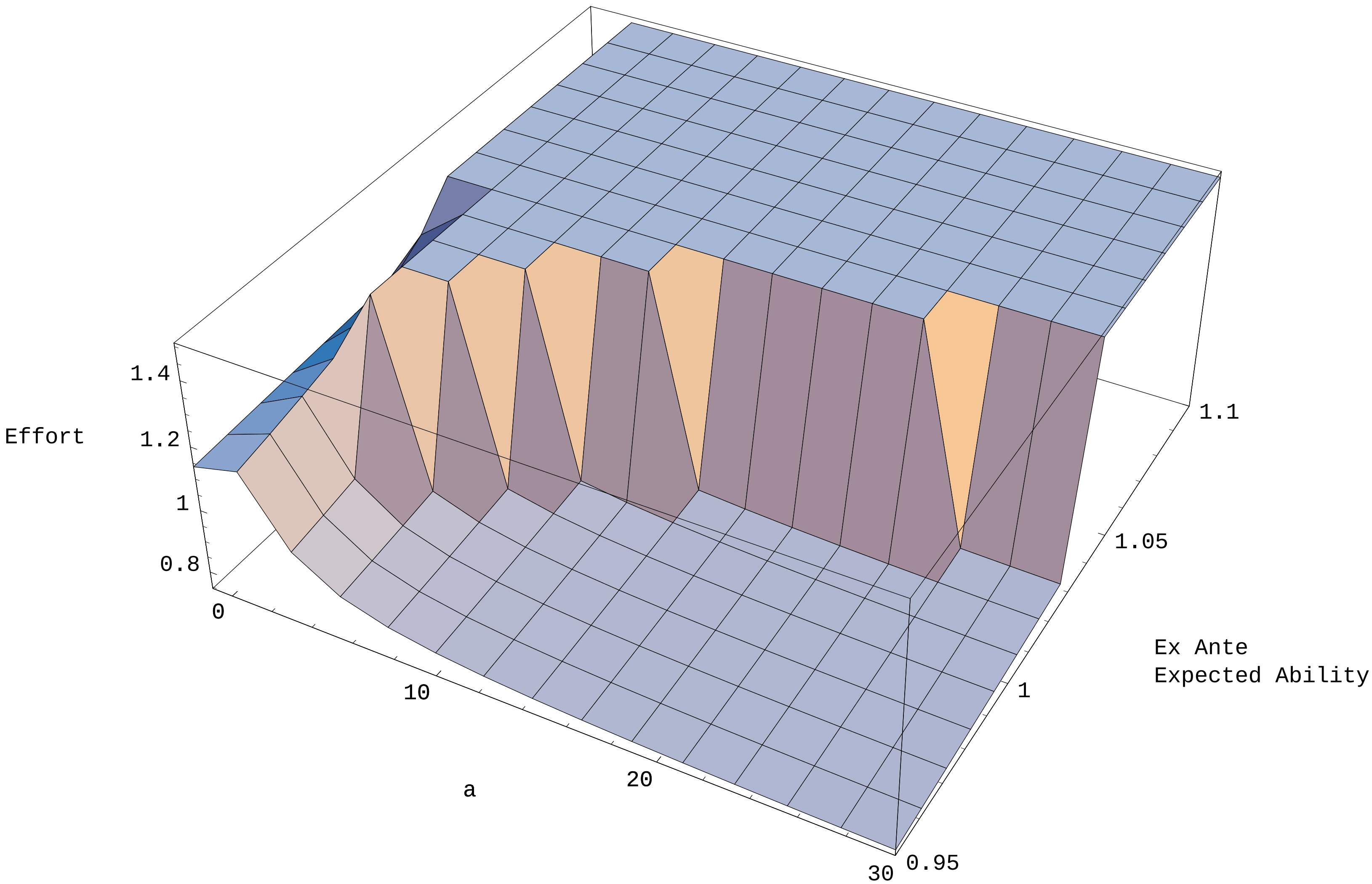

Panel A
anxiety
0.15
0.1
0.05
0
0.5
1
R
1.5
2
0
10
20
30
a
Panel B
anxiety
0.14
0.13
0.12
0.11
0.09
0.08
5
10
15
20
a
Panel C
anxiety
0.15
0.125
0.1
0.075
0.05
0.025
5
10
15
20
a
Panel D
anxiety
0.08
0.06
0.04
0.02
0.5
1
1.5
2
R
Panel E
anxiety
0.15
0.1
0.05
0.5
1
1.5
2
R
Panel F
optimal
anxiety
0.12
0.08
0.06
0.04
2
4
6
8
10
12
a
Panel G
optimal
anxiety
0.1
0.08
0.06
0.04
0.02
2
4
6
8
10
12
a

Figure 4



Panel A
Effort
1.4
1.2
1
0
10
20
a
30
40
0.5
1
R
1.5
2
Panel B
Effort
1.015
1.014
1.013
1.012
1.011
1.009
1.008
5
10
15
20
a
Panel C
Effort
1.035
1.0325
1.03
1.0275
1.025
1.0225
2
4
6
8
10
a
Panel D
Effort
1.2
1.1
0.9
2
4
6
8
10
a
Panel E
Effort
1.5
1.4
1.3
1.2
1.1
0.9
2
4
6
8
10
12
a
Panel F
Effort
1.175
1.15
1.125
1.1
1.075
1.05
1.025
0.5
1
1.5
2
R
Panel G
Effort
1.4
1.3
1.2
1.1
0.5
1
1.5
2
R
Panel H
Effort
1.5
1.4
1.3
1.2
1.1
0.5
1
1.5
2
R

Figure 5


Effort
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0.8
0
10
20
30
a
0.95
1
1.05
1.1
Ex Ante
Expected Ability